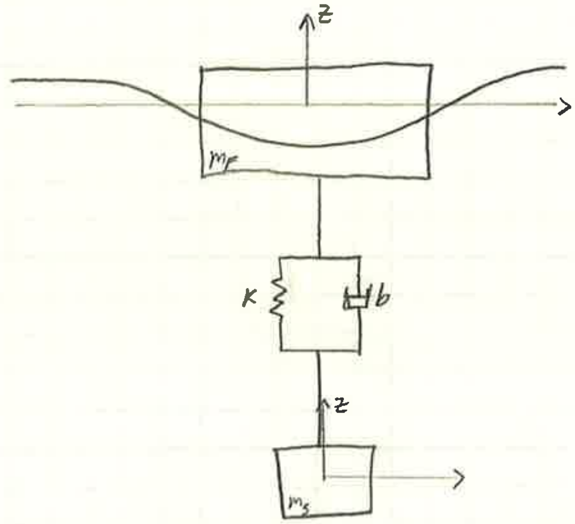
FREQ DOMAIN HEAVE RESPONSE OF SUBMERGED BODY BELOW FLOATING BODY

FLOATING BODY MOTION (RELATIVE TO EQUILIBRIUM)



$$z_3^F(t) = x_3^F(t) = Re[\xi_3 e^{i\omega t}]$$
$$\dot{z}_3^F(t) = \dot{x}_3^F(t) = Re[i\omega \xi_3 e^{i\omega t}]$$
$$\ddot{z}_3^F(t) = \ddot{x}_3^F(t) = Re[-\omega^2 \xi_3 e^{i\omega t}]$$

SUBMERGED BODY MOTION (RELATIVE TO EQUILIBRIUM)

$$z_3^S = x_3^S(t) = Re[\varsigma_3 e^{i\omega t}]$$
$$\dot{z}_3^S = \dot{x}_3^S(t) = Re[i\omega \varsigma_3 e^{i\omega t}]$$
$$\ddot{z}_3^S = \ddot{x}^S(t) = Re[-\omega^2 \varsigma_3 e^{i\omega t}]$$

FORCES ON FLOATING BODY

WEIGHT: $F_3^{F:W}(t) = -m_F g$

HYDROSTATIC: $F_3^{F:HS}(t) = \rho \forall_F g - \rho g S_F Re[\xi_3 e^{i\omega t}]$

RADIATION: $F_3^{F:R}(t) = Re[\xi_3 e^{i\omega t}[\omega^2 a_{33}^F - i\omega b_{33}^F]]$

EXCITING: $F_3^{F:E}(t) = Re[A X_3^F e^{i\omega t}]$

TETHER:
$$F_3^{F:T}(t) = K(z_3^S(t) - z_3^F(t)) + b(\dot{z}_3^S(t) - \dot{z}_3^F(t)) + (\rho \forall_S g - m_S g)$$
$$= Re[\{K(\varsigma_3 - \xi_3) + b i\omega(\varsigma_3 - \xi_3)\} e^{i\omega t}] + (\rho \forall_S g - m_S g)$$
$$= Re[\{\varsigma_3(K + i\omega b) - \xi_3(K + i\omega b)\} e^{i\omega t} + (\rho \forall_S g - m_S g)]$$

FORCES ON SUBMERGED BODY

WEIGHT: $F_3^{S:W}(t) = -m_S g$

HYDROSTATIC: $F_3^{S:HS}(t) = \rho \forall_S g$

RADIATION: $F_3^{S:R} = Re[\varsigma_3 e^{i\omega t}[\omega^2 a_{33}^S - i\omega b_{33}^S]]$

AS SUBMERGENCE $\to \infty$ $b_{33}^S \to$ viscous drag and $a_{33}^S \to a_{33}^S(\omega \to \infty)$

EXCITING: $F_3^{S:E}(t) = Re[A X_3^S e^{i\omega t}]$ ($X_3 \to 0$ AS SUBMERGENCE INCREASES)

TETHER: $F_3^{S:T} = -F_3^{F:T}$

LINEARIZATION OF VISCOUS DRAG

$$b_{33} = \frac{1}{2} C_D S \rho$$

FLOATING BODY EQUATION OF MOTION

$$m_F \ddot{x}_3^F = \sum F_3^F$$

$$Re\left[-m_F \omega^2 \xi_3 e^{i\omega t}\right] = Re\Big\{-m_F g + \rho g \forall - \rho g S_F \xi_3 e^{i\omega t} + \xi_3 (\omega^2 a_{33}^F - i\omega b_{33}^F) e^{i\omega t} + A X_3 e^{i\omega t}$$
$$+ (K + i\omega b)\varsigma_3 e^{i\omega t} - (K + i\omega b)\xi_3 e^{i\omega t}$$
$$+ (\rho \forall_S g - m_S g)\Big\}$$

(1) $$Re\Big\{\left[-m_F \omega^2 + \rho g S_F - (\omega^2 a_{33}^F - i\omega b_{33}^F) + (K + i\omega b)\right]\xi_3 e^{i\omega t} - (K + i\omega b)\varsigma_3 e^{i\omega t}\Big\} = Re\Big\{\underbrace{-m_F g + \rho g \forall^F - m_S g + \rho \forall_S g}_{=0} + A X_3^F e^{i\omega t}\Big\}$$

SUBMERGED BODY EQUATIONS OF MOTION

$$m_S \ddot{x}_3^S = \sum F_3^S$$

$$Re\left[-m_S \omega^2 \varsigma_3 e^{i\omega t}\right] = Re\Big\{\cancel{-m_S g} + \cancel{\rho \forall_S g} + \varsigma_3 e^{i\omega t}(\omega^2 a_{33}^S - i\omega b_{33}^S) + A X_3^S e^{i\omega t}$$
$$-(K + i\omega b)\varsigma_3 e^{i\omega t} + (K + i\omega b)\xi_3 e^{i\omega t}$$
$$-(\cancel{\rho \forall_S g} - \cancel{m_S g})\Big\}$$

(2) $$Re\Big\{-(K + i\omega b)\xi_3 e^{i\omega t} + \left[-m_S \omega^2 - (\omega^2 a_{33}^S - i\omega b_{33}^S) + (K + i\omega b)\right]\varsigma_3 e^{i\omega t}\Big\} = Re\Big\{A X_3^S e^{i\omega t}\Big\}$$

(1) and (2) ARE TWO EQNS FOR TWO UNKNOWNS (ξ_3 and ς_3)

FROM (1)

$$\varsigma_3 = \frac{1}{(K - i\omega b)}\left\{\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]\zeta_3 - A X_3^s\right\} \qquad (3)$$

INSERTING INTO (2)

$$-(K - i\omega b)\zeta_3 + \frac{\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]}{(K - i\omega b)}\zeta_3$$

$$- \frac{A X_3^s}{(K - i\omega b)}\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right] = A X_3^s$$

$$\zeta_3 = \frac{A X_3^s + \frac{A X_3^s}{(K - i\omega b)}\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]}{-(K - i\omega b) + \frac{\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]}{(K - i\omega b)}}$$

$$\zeta_3 = \frac{A X_3^s (K - i\omega b) + A X_3^s\left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]}{-(K - i\omega b)^2 + \left[-m_s\omega^2 - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]\left[-m_s\omega^2 + \rho g S^s - (\omega^2 a_{33}^s - i\omega b_{33}^s) + (K - i\omega b)\right]} \qquad (4)$$

INSERT (4) INTO (3) TO FIND ς_3

<u>LOW FREQUENCY LIMIT</u>

$\omega \to 0$

$$\zeta_3 = \frac{A X_3^s K + A X_3^s K}{-K^2 + [K][\rho g S^s + K]} = \frac{A K[X_3^s + X_3^F]}{K \rho g S^F} = \frac{A[X_3^s + \overset{\frac{F}{m}}{X_3^F}]}{\rho g S^F} \Rightarrow \frac{[m]\left[\frac{kg\,m}{m\,s^2}\right]}{\left[\frac{kg}{m^3}\cdot\frac{m}{s^2}\cdot m^2\right]}$$

$$= [m] \checkmark$$

POWER DISSIPATED IN DAMPER

$$P(t) = F(t)\cdot V(t) = b\left(\dot{z}_3^s(t) - \dot{z}_3^s(t)\right)^2$$

$$= b\left(Re\left\{i w \zeta_3 e^{iwt} - i w \rho_3 e^{iwt}\right\}\right)^2$$

$$= b\left(Re\left\{i w e^{iwt}(\zeta_3 - \rho_3)\right\}\right)^2$$

$\Delta v = \zeta_3 - \rho_3$

$\hookrightarrow$ COMPLEX

$$= b\, Re\left\{[-w \sin wt + i w \cos wt][Re(\Delta v) + i\, Im(\Delta v)]\right\}^2$$

$$= b\, Re\left\{\left(-Re[\Delta v] w \sin wt - Im[\Delta v] w \cos wt\right) + i\left(Re(\Delta v) w \cos wt - Im(\Delta v) w \sin wt\right)\right\}^2$$

$$= b\left(-Re[\Delta v] w \sin wt - Im[\Delta v] w \cos wt\right)^2$$

$$= b\left[\left(Re(\Delta v] w \sin wt\right)^2 + \left(Im[\Delta v] w \cos wt\right)^2 + 2Re(\Delta v] Im(\Delta v) w^2 \cos wt \sin wt\right]$$

$$P(t) = b\left[Re[\Delta v]^2 w^2 \sin^2 wt + Im(\Delta v]^2 w^2 \cos^2 wt + 2Re[\Delta v] Im[\Delta v] w^2 \cos wt \sin wt\right]$$

AVERAGE POWER OVER ONE CYCLE

$$\bar{P} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{b w^2}{T}\int_0^T \left(Re[\Delta v]^2 \sin^2 wt + Im[\Delta v]^2 \cos^2 wt + 2Re[\Delta v] Im[\Delta v] \cos wt \sin wt\right) dt$$

$$= \frac{b w^2}{T}\left[\frac{T}{2} Re[\Delta v]^2 + \frac{T}{2} Im[\Delta v]^2\right] = \underline{\frac{b w^2}{2}\left[Re[\Delta v]^2 + Im[\Delta v]^2\right] = \bar{P}}$$

ALTERNATIVELY, FORMULATE RESULT OF PAGE 3 AS 2 EQNS FOR 2 UNKNOWNS.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \zeta_3 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$A \quad x = B \quad \Rightarrow x = A^{-1} B$$

$$A_{11} = -m_f w^2 + \rho g S_F - (w^2 a_{33}^f - i w b_{33}^f) + (K + i w b)$$

$$A_{12} = -(K + i w b)$$

$$A_{21} = -(K + i w b)$$

$$A_{22} = -m_s w^2 - (w^2 a_{33}^s - i w b_{33}^s) + (K + i w b)$$

$$B_1 = A X_3^f$$

$$B_2 = A X_3^s$$

RELATION OF WAMIT REPORTED QUANTITIES TO THESE DOCUMENTS

$a_{33} = \bar{A}_{33} \rho$

$b_{33} = \bar{B}_{33} \rho \omega$

$X_3 = \rho g \bar{X}_3$

OVERBAR QUANTITIES ARE REPORTED FROM WAMIT IN .1 AND .3 FILES

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