

8

PUMPS AND MOTORS

8.1 CONFIGURATION OF PUMPS AND MOTORS

Two phenomena may be used for pumping. The first phenomenon employs a chamber that is initially attached to an inlet port. The chamber increases its volume to accommodate fluid forced into it by the prevailing fluid pressure at the inlet. The chamber is then connected to an outlet port and reduced in volume to expel the fluid. The discharge pressure is greater than the inlet pressure. Pumps constructed to operate using this phenomenon are *positive displacement* or *hydrostatic* pumps. If leakage and mechanical strength considerations are ignored, then a positive displacement pump can pump fluid at any speed and over any pressure difference. The second phenomenon employs a spinning rotor that can impart a tangential velocity to the fluid. There is no gross flow of fluid tangential to the rotor, so ideally the pressure that could be achieved by converting the tangential velocity to pressure would be:

$$p = \rho \frac{(\omega_m r)^2}{2} \quad (8.1)$$

Pumps employing this phenomenon are *hydrodynamic* pumps.

Fluid power systems operate at pressures such that only positive displacement pumps provide the combination of size, performance, and efficiency that is acceptable. A hydrodynamic pump can operate with a closed discharge for a short time, i.e., before the fluid overheats. On the other hand, closing the discharge of a positive displacement pump generally invites some form of immediate catastrophic failure, for example shearing

of the drive shaft. Consequently, all positive displacement pumps must be protected with relief valves or some form of mechanical device that reduces the positive displacement to zero at some preset pressure.

In this chapter, pumps and motors are generally discussed as if there were no difference between the two functions. This lack of discrimination is generally true and pumps may often be substituted for motors and *vice versa*. It is NOT universally true. For example, a gerotor pump is simpler than a gerotor motor, for reasons that will be explained when discussing that specific geometry. Manufacturers design pumps and motors for specific purposes and it is always advisable to consult with the manufacturer should the intended role of a piece of equipment be changed.

The common patterns of positive displacement pumps employed in fluid power systems are displayed in Figure 8.1 to 8.8. It will be observed that all the patterns shown have some means of connecting the expanding chamber to an inlet port and the contracting chamber to a discharge port. Achieving a graceful transition is not trivial. Excess leakage vs. excess pressure rise must be balanced. We shall discuss this topic later for the axial piston pump in Chapter 9. Both pumps and motors are generally ranked in the order:

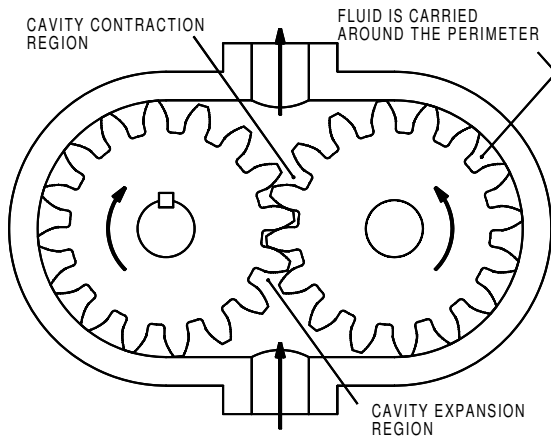


Figure 8.1: Spur gear pump.

SPUR GEAR
INTERNAL GEAR
GEROTOR
VANE
RADIAL PISTON
AXIAL PISTON (SWASH PLATE & BENT AXIS)
SCREW

The initial pumps in the ranking are relatively lower in cost, limited in pressure capability, and lower in efficiency. In addition to pressure capability and efficiency, piston pumps are relatively easily modified to incorporate automatic volume displacement reduction as pressure rises above a set value. Furthermore piston pumps and motors can have their displacement varied during normal operation. This characteristic is made use of in a hydrostatic transmission where motor speed must be varied smoothly (i.e. steplessly) over a wide range.

Spur gear: A spur gear pump is shown in [Figure 8.1](#). The fluid is carried around the perimeter of the gear. The cavity expansion and cavity contraction take place where the gears mesh. It may be observed that the actual volume change is limited to the volume of one inter tooth space. There are practical limitations to the thickness of the gears so the volu-

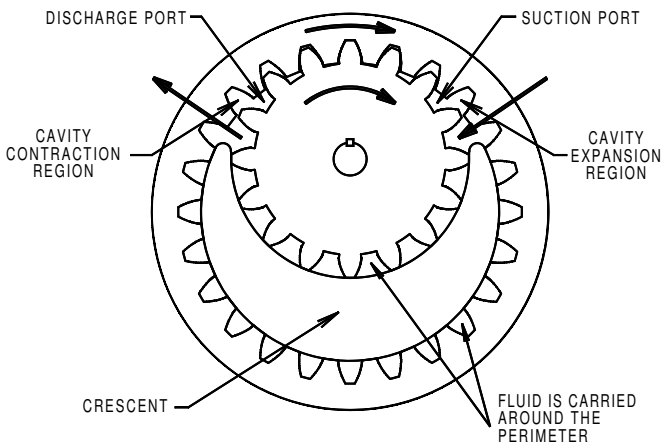


Figure 8.2: Internal gear pump.

metric capacity of spur gear pumps for a given case volume is relatively low.

As with all fluid power pumps or motors, the only elastomeric seals are those around the shaft. The long passage ways in the spur gear pump, for example around the gear perimeter, control leakage quite well. Figure 8.1 has been drawn with realistically shaped teeth. It will be observed that there is significant fluid that is never expelled from the contracting cavity. As discussed in Chapter 2, hydraulic fluids possess elasticity so any dead volume lowers volumetric efficiency as the working pressure increases.

Internal gear: The internal gear pump shown in Figure 8.2 is the second of the variations on gear pumps. For the same number of teeth in the ring gear as the gear of a spur gear pump, the internal gear pump will be more compact. An internal gear pump will also have a slightly smaller dead fluid volume than a spur gear pump because of the tooth meshing geometry.

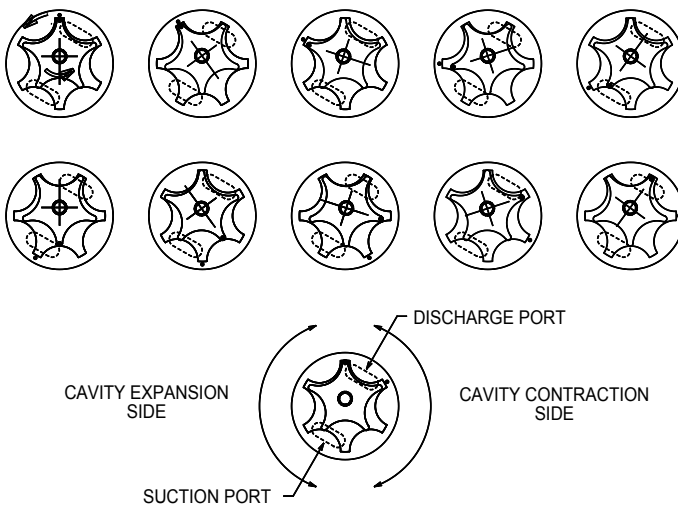


Figure 8.3: Gerotor motor.

Gerotor: The gerotor geometry is a patented form of the internal gear geometry. The word *geometry* is used because the gerotor is one device where the pump and motor may be different.

The gerotor pump is close relative of the internal gear pump. The axes

of the spur and ring gears are offset and fixed in their relative locations. The operation and method of fluid transfer is essentially the same as for the internal gear except that the geometry does not require the crescent insert required in the internal gear pump. This form of pump is commonly used as a charge pump for hydrostatic transmissions (Section 10.3) because it has a minimum number of parts

As with other positive displacement geometries, the fixed axis geometry can be used for a pump or a motor. This simple geometry is seldom used for gerotor *motors* because the addition of a few components can alter the displacement per revolution of the device in a major fashion. A special drive shaft in the form of a diminutive dumbbell is added. This shaft has male splines at both ends, but these splines are curved in the plane of the shaft, that is, the splines are barrel shaped. The ring gear is driven around a fixed axis guided by a fixed circular bearing surface in the pump case. The rotor rotates about an axis that is slightly offset from the ring axis (Figure 8.3). In addition, the rotor orbits around the ring gear axis during operation, hence the need for the dumb bell drive shaft.

The manner in which expanding and contracting cavities are formed during rotation can be seen in Figure 8.3 sequence. The gerotor motor in the orbiting form has one major advantage compared to the simple form with fixed axes. The displacement per revolution is much larger than that which can be obtained with the fixed axis geometry for the same rotor and stator size.

An alternative version of the gerotor motor cuts cylindrical slots in the ring gear and places rollers in these slots. This modification reduces friction and allows the units to be used at much lower speeds than most other motor geometries. The motor is called a *geroller* in this configuration. A typical application would be as a direct drive motor for a wheel. The terminology *low speed high torque* is used for such motors.

Vane: Vane pumps are available in unbalanced and balanced geometries. Figure 8.4 shows the balanced configuration. The unbalanced type is similar but consists of a circular cavity offset from the shaft axis. Consequently, there is an unbalanced radial force on the drive shaft resulting from the differences in pressures in this cavity. For pumps used at low pressures, such an unbalanced force is not a problem, but for the pressures employed in fluid power systems, eliminating unbalanced forces is good practice. The springs shown between the inner edges of the vanes and the rotor are optional. When included they ensure that the outer edges of the vanes and the case are in contact at rest.

Radial piston: Radial piston pumps exist in two main geometries. In

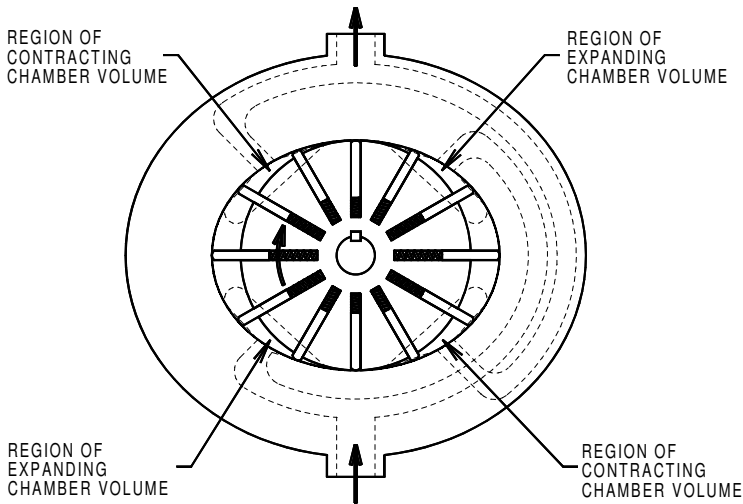


Figure 8.4: Balanced vane pump.

the geometry shown in [Figure 8.5](#), the axis of each piston is fixed in space.

In the other geometry, not shown, the piston axes orbit around the drive shaft axis and piston displacement is caused by the eccentricity of the casing on which the outer ends of the pistons bear. There are no valves and fixed ports are provided in a central cavity surrounding the shaft.

The geometry in [Figure 8.5](#) is shown because it was used by John Deere in many of their tractors. The radial pump shown in [Figure 8.5](#) does not exactly represent a Deere pump because the valve positions have been altered to simplify the presentation. In practice, the valves are aligned with their bores parallel to the drive shaft axis.

The geometry is interesting because it allows stroke control and over-pressure protection. Stroke control is achieved by controlling the pressure in the stroke control cavity. The inward radial projection of the pistons is determined by the equilibrium between the stroke control spring force and the product of stroke control cavity pressure and piston cross section area. Consequently, this geometry of radial piston pump can operate as a controlled variable displacement pump. Facilities may also be provided to raise the stroke control cavity pressure to an upper limiting value such that there is no pumping action if the pump output pressure reaches a preset limiting value. Radial piston pumps commonly employ an even number of

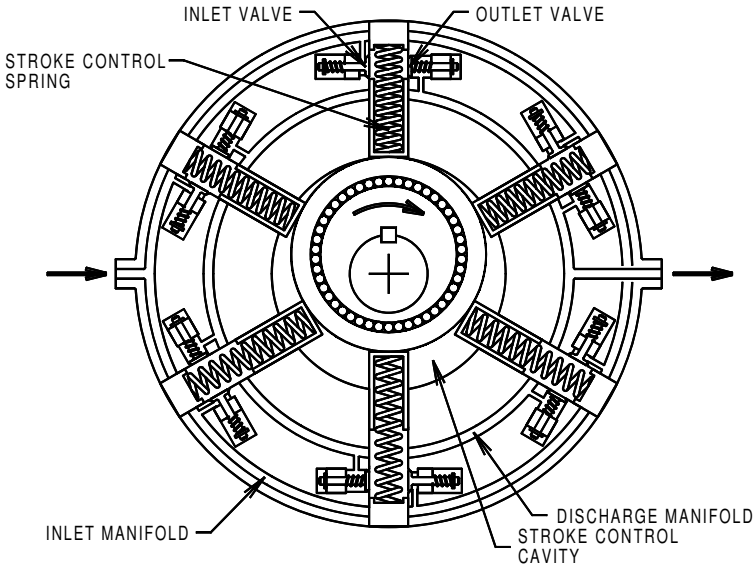


Figure 8.5: Radial piston pump with displacement control.

pistons because boring a pair of cylinders at one setting reduces production cost.

Axial piston, swash plate: In an axial piston pump, a piston is reciprocated by means of motion relative to an inclined plane. Usually the inclined plane is stationary and is called a swash plate. As shown in [Figure 8.6](#), the barrel containing the pistons rotates to give the relative motion between the pistons and the inclined plane. The swash plate angle is limited to 18° for satisfactory mechanical operation.

At top dead center and bottom dead center, the fluid in the cylinder must make a transition from high (or low) to low (or high) pressure. This transition is achieved with the tapered channel, the pressure transition groove. The objectives of this groove are first to ensure that fluid is never trapped in a changing volume that lacks a passage and second to limit communication between the high and low pressure manifolds to a very small passage. The first objective is necessary to avoid excessive pressures developing in the cylinder and the second ensures that efficiency is maintained.

The number and size of grooves that may be observed in a given device may vary. This is another instance where pumps may vary from motors.

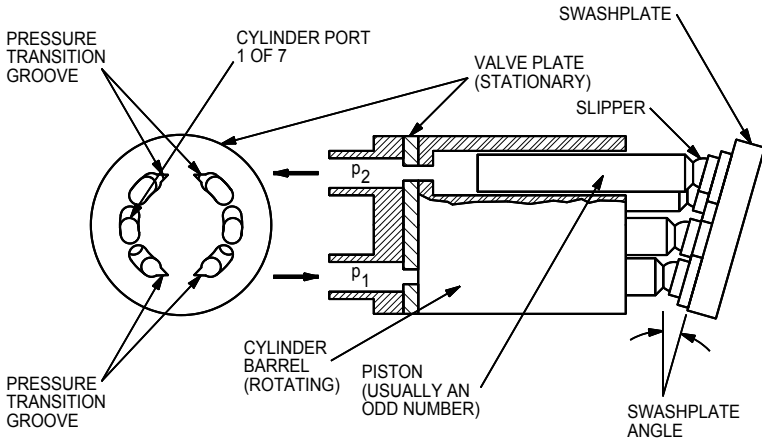


Figure 8.6: Axial piston pump, swash plate type.

A variable displacement pump will generally be constructed to allow the swashplate to move $\pm 18^\circ$ about a null position, thus the top dead center for the pump will switch by 180° on the valve plate. In a motor, reversal is usually permitted, so the direction in which a cylinder approaches top dead center will switch. As will be discussed in [Chapter 9](#), the magnitude of the low pressure on the pump or motor will affect the size of the grooves (p. 247).

It will be shown in Chapter 9 that the variation in flow rate from a pump with an odd number of pistons is much less than that from a pump with an even number. In practice, five, seven, nine, or occasionally 11 pistons are used. Variation in flow rate causes pressure variation and this, in turn, causes noise.

Axial piston, bent axis: The pumping elements of an axial piston pump using the bent axis configuration are essentially the same as those used for an axial piston pump with a swash plate. The major difference results from the means of moving the inclined plane past the pistons. The inclined plane is attached to the pump drive shaft and is normal to that shaft at all times ([Figure 8.7](#)). The inclination is achieved by setting the cylinder block axis and the drive shaft axis at an angle (hence the name bent axis). This angle may be up to 40° . The cylinder block and drive shaft are constrained to rotate together by an internal drive shaft. The ends of this shaft are provided with joints that will allow rotary motion as

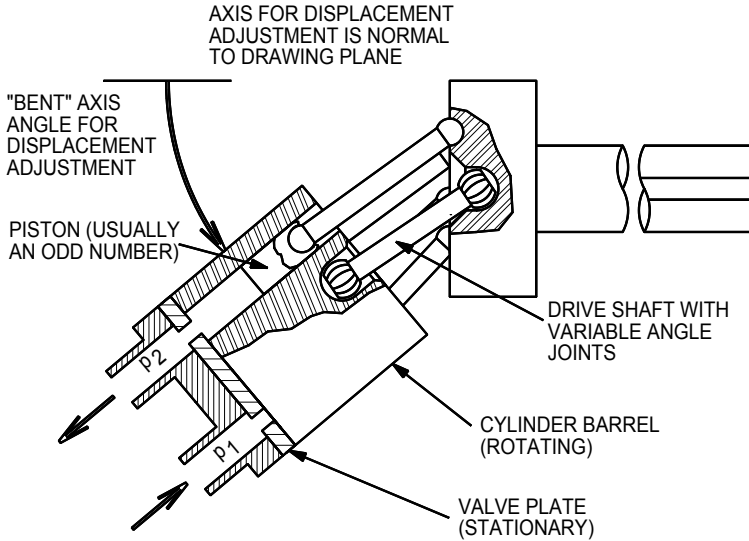


Figure 8.7: Axial piston pump, bent axis type.

the bent axis angle is changed.

Reciprocating motion between the drive shaft plate and the cylinder barrel is transmitted by push rods with ball ends. These ball ends are necessary because the push rods do not remain parallel during rotation.

Screw: The screw pump has been saved until last because of its position in the hierarchy of pressure and cost. In fact, it has much in common with gear pumps. As with these geometries, the screw pump has a fixed displacement and a packet of fluid is moved through the pump against the case from the cavity expansion region to the cavity contraction region.

On the other hand, the screw pump has two beneficial features. Inspection of the specific geometry presented in [Figure 8.8](#) shows that at the discharge there are three contraction regions discharging simultaneously. This means that the screw pump has a very small variation of flow rate with rotation. Consequently, systems using screw pumps have the potential for being much quieter than systems using most other forms of pump.

The second feature is that screw pumps may be designed with long sealing lengths to control leakage and this feature allows the screw pump to develop pressures in excess of most other geometries. The screw pump

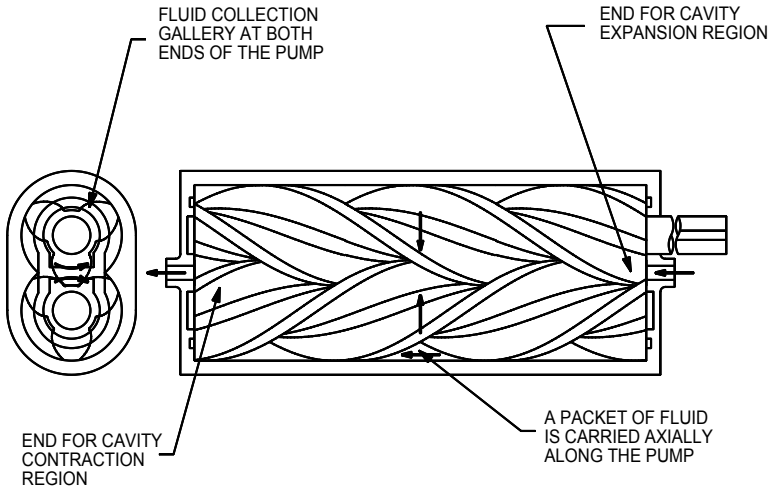


Figure 8.8: Screw pump.

may be found in a wide variety of configurations. The form presented in Figure 8.8 is designed for a high flow rate, but only modest pressure. An alternative configuration uses two square or acme thread form screws with the threads meshed. In this form, there might be 15 to 20 threads along the length of a rotor. Here the discharge per revolution of the pump would be relatively small, but the pressure capability very large.

Two other features that may be encountered are timing gears and more than two rotors. The geometry shown in Figure 8.8 uses involute teeth and does not require timing gears. High pressure pumps using square or similar screws will require timing gears to maintain synchronous rotation. The other feature is use of multiple rotors. An additional slave rotor can be installed parallel to the main drive rotor. The extra rotors improve the displacement of the pump and may cost less than increasing the rotor size. The screw pump is discussed in more detail in Karassik et al. [1].

8.2 PUMP AND MOTOR ANALYSIS

The form of analysis that follows is useful for characterizing existing equipment, but has limited application for the designer of new equipment. In spite of this shortcoming, we believe that the analysis will help a system designer understand the form of curves shown on manufacturer's specifica-

tion sheets. The analysis that will be developed follows that of Merritt [2]. Let us first examine an ideal motor. In such a motor, the following factors would be absent:

LEAKAGE
FRICTION
FLUID COMPRESSIBILITY
CAVITATION

The power output of an ideal motor is:

$$P = T_g \omega_m \quad (8.2)$$

Also for an ideal motor, it is possible to express the power in terms of the flow through the motor and the pressure drop across the motor:

$$P = Q_L p_L \quad (8.3)$$

Note incidentally that both equations imply using consistent units, for example SI units. The two power expressions can be equated to eliminate P yielding:

$$T_g \omega_m = p_L Q_L$$

or:

$$T_g = \frac{Q_L}{\omega_m} p_L$$

The quantity Q_L/ω_m has dimensions volume/radian. This quantity is a characteristic of a specific motor or pump and is called the displacement, D_m or D_p . Thus a simple, yet useful, expression relating the torque expected from an ideal motor is:

$$T_g = D_m p_L \quad (8.4)$$

Observe, however, that manufacturers usually will provide the information in $\text{in.}^3/\text{rev}$ or cm^3/rev rather than volume/radian, which is needed for application in Equation 8.4. Any real pump or motor will have friction and leakage so the expressions that should be used are:

$$T_g = \eta_m D_m p_L \quad 0.85 < \eta_m < 0.96 \quad \text{approximately} \quad (8.5)$$

and:

$$T_g = \frac{D_p p_L}{\eta_p} \quad 0.85 < \eta_p < 0.96 \quad \text{approximately} \quad (8.6)$$

8.2.1 Example: Drive for a Hoist

A hoist is required to lift a mass of 1000 kg. The cable is wound on a drum 0.15 m in diameter. The drum is driven by a hydraulic motor through a 5:1 reduction gearbox. The system pressure is 10 Mpa (1400 lb/in.²) and the motor discharges to atmospheric pressure. Find the ideal motor displacement required. Make a recommendation for the motor displacement that should actually be provided.

Torque on the drum shaft:

$$\begin{aligned} 1000 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2} \times \frac{0.15}{2} \text{ m} &= 735.75 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \\ &= 735.75 \text{ N} \cdot \text{m} \end{aligned}$$

Torque at the motor:

$$\frac{735.75}{5} = 147.15 \text{ N} \cdot \text{m}$$

Using:

$$T_g = D_m p_L$$

Then:

$$\begin{aligned} D_m &= \frac{T_m}{p_L} = \frac{147.15 \text{ N} \cdot \text{m}}{(10\text{E}+6 - 0) \text{ N/m}^2} \\ &= 14.72\text{E}-6 \text{ m}^3/\text{rad} \end{aligned}$$

Being conservative and selecting a worst case value for the motor efficiency of 85%, then the motor ought to have a displacement of:

$$\frac{14.72\text{E}-6}{0.85} = 17.32 \text{ m}^3/\text{rad}$$

Expressing this result in units that you would see in a manufacturer's catalogue:

$$\begin{aligned} 17.32\text{E}-6 \times 2\pi(100 \text{ cm/m})^3 &= 108 \text{ cm}^3/\text{rev} \\ &= \frac{108 \text{ cm}^3/\text{rev}}{(2.54 \text{ cm/in.})^3} \\ &= 6.64 \text{ in.}^3/\text{rev} \end{aligned}$$

8.3 LEAKAGE

Although seven different geometries (SPUR GEAR to SCREW) were listed earlier, we may analyze leakage with a generic model based on an axial piston geometry.

For any positive displacement pump or motor, there is an inlet chamber that is increasing in volume and a discharge chamber that is decreasing. There may be leakage directly from the high pressure discharge chamber to the low pressure inlet chamber. There may also be discharge from both chambers to the drain. The drain is some volume of the pump maintained at atmospheric pressure. Elastomeric seals on fluid power pumps are limited in application. Typically these seals are only used where the shaft passes through the case to the outside. Leakage elsewhere in the device is controlled by very fine machining tolerances. Although this approach minimizes sliding friction, it does mean that leakage is always present. It should be noted that leakage is not totally detrimental. Leakage provides oil for lubrication.

The term *drain* is used to indicate a volume at atmospheric pressure separate from the inlet and discharge volumes. Most pumps and motors have such volumes and these volumes are provided with a port so the fluid can be drained to a reservoir. The drain volume is ported to the volume in-board of the shaft seal because there should be minimum pressure difference across the shaft seal to limit leakage.

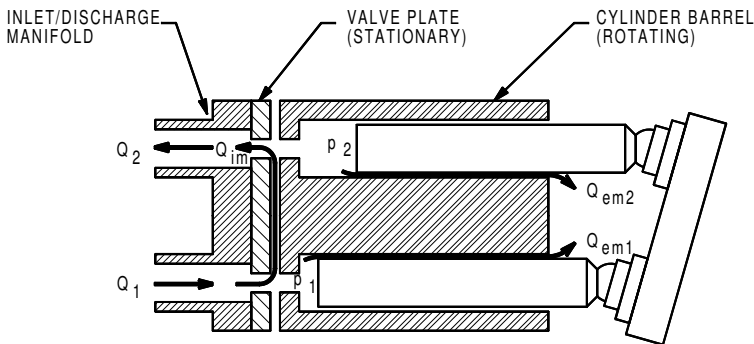


Figure 8.9: Basic leakage paths for a motor.

Figure 8.9 displays the basic leakage paths for a motor. You should observe that the paths shown are greatly exaggerated in size. Clearances

in a real device would be in the order of $5\text{ }\mu\text{m}$ (0.0002 in.). Because the clearances are so small, leakage flow is treated as laminar so:

$$Q \propto \Delta p$$

Thus for a motor we have:

Inlet to outlet leakage	$Q_{im} = C_{im}(p_1 - p_2)$
Inlet chamber to drain	$Q_{em1} = C_{em}(p_1 - p_0)$
Outlet chamber to drain	$Q_{em2} = C_{em}(p_2 - p_0)$

Note that p_0 is usually atmospheric pressure.

In an open circuit system, i.e., where a pump draws from a reservoir at atmospheric pressure and a motor discharges to a similar reservoir, p_2 may essentially be at atmospheric pressure. On the other hand, in a hydrostatic transmission the suction line will be maintained at 2 to 3 Mpa (280 to 430 lb/in.²). The elevated suction line pressure is provided by a charge pump (Section 10.3) and this pressure eliminates any possibility of suction line cavitation. Apply the principle of continuity to the inlet and outlet chambers:

$$\begin{aligned} Q_1 &= \text{Ideal motor rate of flow} \\ &\quad + \text{Leakage to outlet} \\ &\quad + \text{Leakage to drain} \end{aligned}$$

$$Q_1 = D_m \omega + C_{im}(p_1 - p_2) - C_{em}(p_1 - p_0) \quad (8.7)$$

Similarly for the suction chamber:

$$\begin{aligned} Q_2 &= \text{Ideal motor rate of flow} \\ &\quad + \text{Leakage from inlet} \\ &\quad + \text{Leakage to drain} \end{aligned}$$

$$Q_2 = D_m \omega + C_{im}(p_1 - p_2) - C_{em}(p_2 - p_0) \quad (8.8)$$

It is convenient to introduce a *load flow* variable defined as:

$$Q_L = (Q_1 + Q_2)/2 \quad (8.9)$$

Thus:

$$Q_L = D_m \omega + \left(C_{im} + \frac{C_{em}}{2} \right) (p_1 - p_2) \quad (8.10)$$

Now introduce a further variable *load pressure*:

$$p_L = p_1 - p_2 \quad (8.11)$$

Thus:

$$Q_L = D_m \omega + \left(C_{im} + \frac{C_{em}}{2} \right) p_L \quad (8.12)$$

This expression for Q_L is often useful when performing system simulation and a mean flow through the device is required.

The volumetric efficiency of a pump or motor can be specified under any set of operating conditions. When manufacturers quote performance characteristics in technical literature, they need to do so under some baseline condition. It is generally accepted that manufacturer's literature fixes the pump suction or motor discharge condition at zero gauge pressure.

Introduce a relationship between the leakage coefficients, viscosity, and motor displacement:

$$C_s \frac{D_m}{\mu} = C_{im} + C_{em} \quad (8.13)$$

You might wonder why $C_{im} + C_{em}$ and not $C_{im} + C_{em}/2$. When pump or motor volumetric efficiency is being specified, only the flow into a motor or out of a pump is of concern. Hence for a motor under baseline conditions with $p_2 = 0$, the flow into the motor is:

$$\begin{aligned} Q_1 &= D_m \omega + (C_{im} + C_{em}) p_1 \\ &= D_m \omega + C_s \frac{D_m}{\mu} p_1 \end{aligned} \quad (8.14)$$

The component $Q_s = (C_{im} + C_{em}) p_1$ is sometimes called slip flow. Examine the dimensions of C_s :

$$\begin{aligned} Q_s &= (C_{im} + C_{em}) p_1 \\ C_{im} + C_{em} &= \frac{Q_s}{p_1} = \frac{L^3}{T} \cdot \frac{LT^2}{M} = \frac{L^4 T}{M} \end{aligned}$$

Now examine:

$$\frac{D_m}{\mu} = L^3 \cdot \frac{LT}{M} = \frac{L^4 T}{M}$$

Thus C_s is dimensionless.

8.3.1 Example: Estimating Pump Performance Coefficient C_s

This example will examine the expected magnitude of C_s using data taken from the specification of a Sauer Sundstrand Series 20 pump. The specification sheet indicates that the volumetric efficiency is 96.5% at 3000 rpm when $p_L = 3000 \text{ lb/in.}^2$ (remember that $p_L = p_1$ and $p_2 = 0$). The nominal maximum displacement of this unit is $2.03 \text{ in.}^3/\text{rev}$. This example will be worked in inch-pound force-second units because this approach will give an opportunity to review conversions between various forms of viscosity units.

Start by defining the volumetric efficiency:

$$\begin{aligned}\text{VOLUMETRIC EFFICIENCY (PUMP)} &= \frac{\text{ACTUAL FLOWRATE}}{\text{NOMINAL FLOWRATE}} \\ &= \frac{\text{NOMINAL} - \text{SLIP}}{\text{NOMINAL}}\end{aligned}$$

Hence:

$$\begin{aligned}\text{SLIP} &= (1 - \eta_V) \times \text{NOMINAL} \\ \text{NOMINAL} &= \frac{3000 \text{ rpm} \times 20.3 \text{ in.}^3/\text{rev}}{60 \text{ s/min}} = 101.5 \text{ in.}^3/\text{s} \\ \text{SLIP} &= (1 - 0.965)101.5 = 3.553 \text{ in.}^3/\text{s}\end{aligned}$$

We now need to know the oil viscosity that was used. This was not given for the chosen pump, but it is quite common to reduce all data to a standard viscosity of 100 SUS. The full expression for conversion of SUS to kinematic viscosity is given in ASTM D2161 [3] requires a root finding procedure. When this is done for $\text{SUS} = 100$, the result is:

$$\nu = 21.52 \frac{\text{mm}^2}{\text{s}}$$

A simpler expression obtained from Merritt [2] may be used without much loss of accuracy:

$$\begin{aligned}\nu &= 0.216 \text{ SUS} - \frac{166}{\text{SUS}} \\ &= 0.216 \times 100 - \frac{166}{100} = 19.94 \frac{\text{mm}^2}{\text{s}}\end{aligned}$$

Using the conversion procedure presented in Chapter 2 that with $\nu = 199.94 \text{ mm}^2/\text{s}$ and a specific gravity of 0.85 then the absolute viscosity in inch-pound force-second units is:

$$\mu = 2.45\text{E}-6 \frac{\text{lb}_f \cdot \text{s}}{\text{in.}}$$

Noting:

$$C_s = \frac{\mu}{D_m} (C_{im} + C_{em})$$

Where:

$$\begin{aligned} (C_{im} + C_{em}) &= \frac{Q_s}{p_1} = \frac{3.553 \text{ in.}^3/\text{s}}{3000 \text{ lb}_f/\text{in.}^2} \\ &= 1.184\text{E}-3 \frac{\text{in.}^5}{\text{lb}_f \cdot \text{s}} \end{aligned}$$

The specification sheet value of D_m must be converted to $\text{in.}^3/\text{rad}$:

$$D_m = \frac{2.03 \text{ in.}^3/\text{rev}}{2\pi \text{ rad/rev}} = 0.323 \frac{\text{in.}^3}{\text{rad}}$$

$$\begin{aligned} C_s &= \frac{2.45\text{E}-6 \text{ lb}_f \cdot \text{s}/\text{in.}^2}{0.323 \text{ in.}^3/\text{rad}} 1.184\text{E}-3 \text{ in.}^5/\text{lb}_f \cdot \text{s} \\ &= 8.98\text{E}-9 \quad (\text{dimensionless}) \end{aligned}$$

8.4 FORM OF CHARACTERISTIC CURVES

8.4.1 Volumetric Efficiency

We shall first consider the form of volumetric efficiency vs. viscosity-speed/pressure (dimensionless speed). In practice, manufacturers will present volumetric efficiency vs. speed only and we shall show how this simplification is achieved. It was shown earlier that a real motor would have an inflow represented by:

$$Q_1 = D_m \omega + C_{im}(p_1 - p_2) - C_{em}(p_1 - p_0) \quad (8.7)$$

As indicated, manufacturers need a baseline condition and this is chosen as the condition where $p_1 = p_L$ and $p_2 = 0$. In this situation, the flow into a motor with leakage may be expressed as:

$$Q_1 = D_m \omega + C_s \frac{D_m}{\mu} p_1 \quad (8.14)$$

The volumetric efficiency for a motor is defined as:

$$\begin{aligned} \eta_{mV} &= \frac{\text{NOMINAL FLOW}}{\text{ACTUAL FLOW}} = \frac{D_m \omega_m}{D_m \omega_m + C_s \frac{D_m}{\mu} p_1} \\ &= \frac{1}{1 + C_s \left/ \left(\frac{\mu \omega_m}{p_1} \right) \right.} \end{aligned} \quad (8.15)$$

First examine $\mu \omega_m / p_1$. Because C_s was shown to be dimensionless, then $\mu \omega_m / p_1$ should also be dimensionless:

$$\frac{\mu \omega_m}{p_1} = \frac{\text{M}}{\text{LT T}} \frac{1 \text{ LT}^2}{\text{M}} = \text{M}^0 \text{L}^0 \text{T}^0$$

dimensionless as predicted. With some algebraic manipulation, it may be shown:

$$(\eta_{mV} - 1) = - \frac{C_s}{\left(\frac{\mu \omega_m}{p_1} \right) - (-C_s)} \quad (8.16)$$

If the common substitution x for the abscissa variable and y for the ordinate variable is made, then the equation of volumetric efficiency vs. dimensionless speed is:

$$(y - b) = \frac{-C_s}{(x - a)} \quad (8.17)$$

This function may be recognized as a rectangular hyperbola with the origin translated to (a, b) and mirrored in the x axis because of the minus sign in the numerator. Only the branch of the rectangular hyperbola above the x axis is relevant because η_V is necessarily positive. A generic volumetric efficiency curve is shown in [Figure 8.10](#).

Although the analysis just performed is quite useful and certainly produces curves that show the same trends as those found on manufacturers' specification sheets, it will be found that data taken at several different pressures cannot be forced to lie on one curve as would be suggested by the analysis. This situation is not surprising. If we were to consider the leakage of oil through the gap between the slipper and the swash plate in an axial piston pump, then it seems likely that this leakage would be a function of pressure and speed.

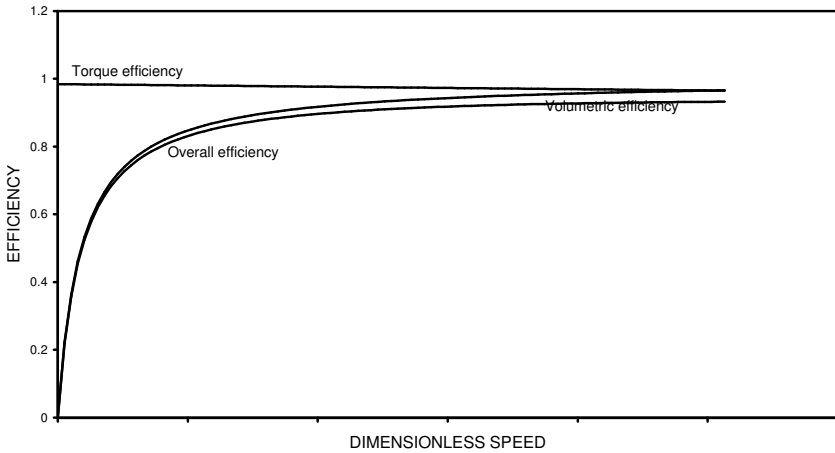


Figure 8.10: Volumetric, torque, and overall efficiencies vs. dimensionless speed $\mu\omega_m/p_1$.

Manufacturers use speed in revolution per minute as the abscissa rather than dimensionless speed. Because the volumetric efficiency will be affected by pressure and viscosity, the curve is presented at one standard viscosity. In the U.S. this would often be at 100 SUS. The effect of pressure is accounted for by presenting a family of curves at various pressures. Figure 8.11 shows the curves in the forms that might be presented by manufacturers. Here the abscissa values have been changed from dimensionless speed to speed in revolution per minute, the pressure is displayed as 1000, 2000, and 3000 lb_f/in^2 and the oil viscosity is displayed as 100 SUS. As expected, increasing pressure will decrease volumetric efficiency because there will be more leakage flow. On the other hand, increasing speed will increase the nominal flow term and leave the leakage terms (at least in the simple analysis) unchanged so volumetric efficiency will increase as speed increases.

8.4.2 Torque Efficiency

There are three resisting torques on the shaft of a pump or motor, viscous resistance, sliding friction resistance, and shaft sealing resistance.

Viscous resistance: The torque from viscous drag is a function of viscosity and angular velocity. As with the analysis for volumetric efficiency,

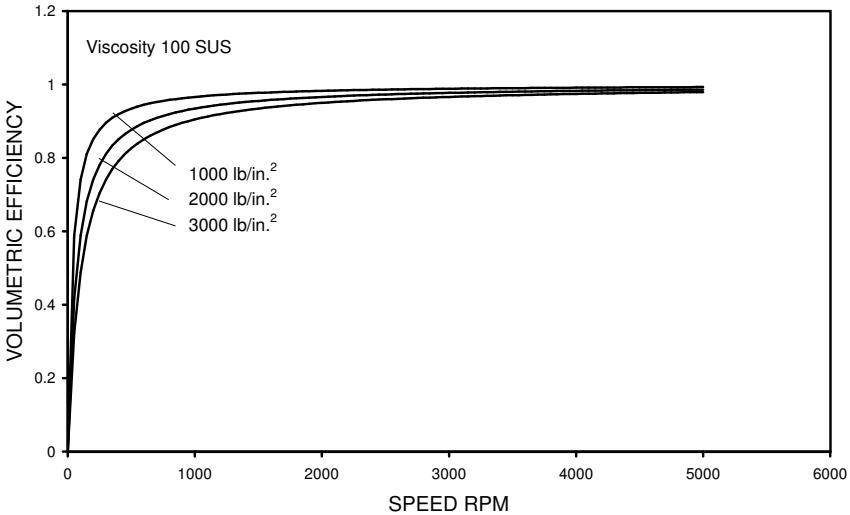


Figure 8.11: Manufacturers' method of presenting volumetric efficiency.

we shall not attempt to relate viscous drag to specific geometry. We shall consider a generic expression:

$$T_d = C_d \mu \omega_m \quad (8.18)$$

Checking the dimensions of C_d :

$$\begin{aligned} C_d &= \frac{T_d}{D_m \mu \omega_m} = \frac{\left(\frac{\text{ML}^2}{\text{T}^2} \right)}{\left(\text{L}^3 \frac{\text{M}}{\text{LT}} \frac{1}{\text{T}} \right)} \\ &= \text{M}^0 \text{L}^0 \text{T}^0 \quad \text{dimensionless} \end{aligned}$$

Sliding friction resistance: Although a pump or motor is well lubricated, there are still metal-to-metal contacts that contribute to Coulomb friction. It is generally assumed that the normal force between such rubbing parts will be directly proportional to pressure. Thus the expression for the drag torque due to Coulomb friction is:

$$T_f = \frac{\omega_m}{|\omega_m|} C_d D_m (p_1 + p_2) \quad (8.19)$$

The expression $\omega_m/|\omega_m|$ is a means of changing the sign of the friction torque to match the direction of rotation. If we examine:

$$\begin{aligned} C_f &= \frac{T_f}{pD_m} = \frac{\frac{ML^2}{T^2}}{\frac{ML^2}{LT^2}L^3} \\ &= M^0L^0T^0 \quad \text{dimensionless} \end{aligned}$$

Shaft sealing resistance: The shaft sealing torque is regarded as independent of pressure and velocity (it must, however, change sign as the direction of rotation changes).

$$T_c = \frac{\omega_m}{|\omega_m|} T_c \quad (8.20)$$

Note that the shaft sealing torque is usually small compared with the other two drag torques so it is often dropped from analyses.

Total expression for drag torque: The total drag torque on a pump or motor can be approximated by:

$$T_R = C_d D_m \mu \omega_m + C_f \frac{\omega_m}{|\omega_m|} D_m (p_1 + p_2) + \frac{\omega_m}{|\omega_m|} T_c \quad (8.21)$$

Hence using the usual baseline criterion that manufacturer's specifications are given for $p_2 = 0$, then the torque delivered by a motor will be:

$$T_L = D_m p_1 - \left(C_d D_m \mu \omega_m + C_f \frac{\omega_m}{|\omega_m|} D_m p_1 + \frac{\omega_m}{|\omega_m|} T_c \right) \quad (8.22)$$

For a motor rotating in one direction, we can write the torque efficiency as:

$$\begin{aligned} \eta_{Tm} &= \frac{T_L}{D_m p_1} \\ &= \frac{D_m p_1 - (C_d D_m \mu \omega_m + C_f D_m p_1 + T_c)}{D_m p_1} \end{aligned} \quad (8.23)$$

As indicated previously, the shaft sealing torque is usually sufficiently small that it can be ignored so:

$$\eta_{Tm} = (1 - C_f) - C_d \left(\frac{\mu \omega_m}{p_1} \right) \quad (8.24)$$

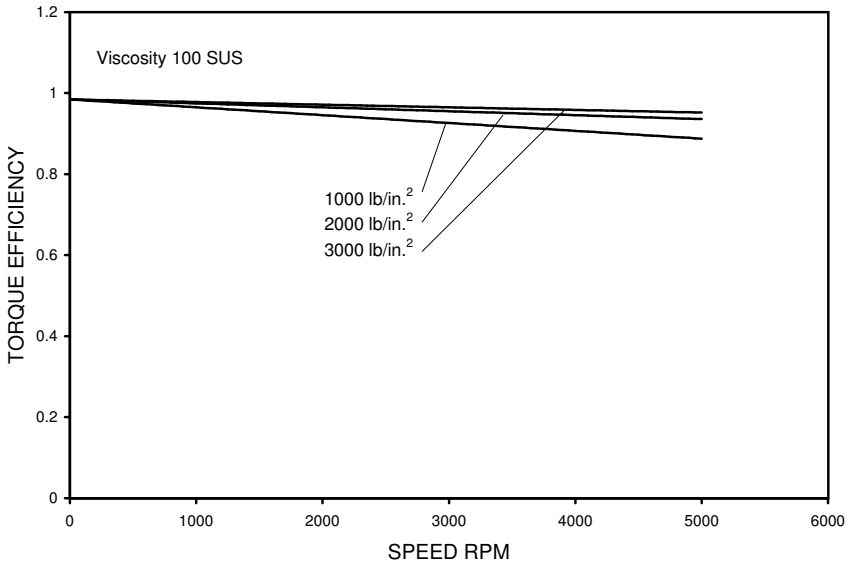


Figure 8.12: Manufacturers' method of presenting torque efficiency.

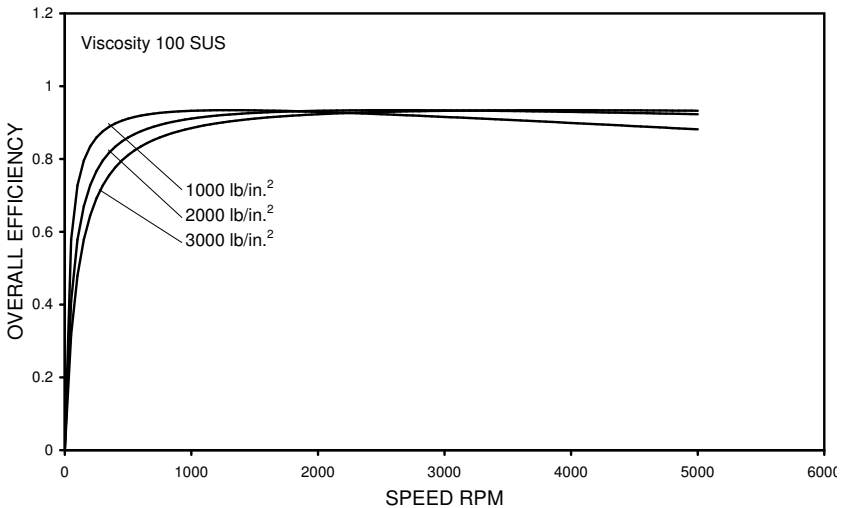


Figure 8.13: Manufacturers' method of presenting overall efficiency.

A graph of torque efficiency vs. speed is shown in [Figure 8.12](#). The form of this graph is not immediately obvious. Intuitively, we might expect the efficiency to drop as the load pressure increases because the Coulomb friction torque will increase with pressure. The reason for the trends shown is that the absolute power increases as the pressure increases. Likewise the absolute viscous torque is independent of the pressure. Thus the fraction of the absolute torque dissipated as viscous drag torque decreases as the load pressure increases. That is the trend shown in the figure.

8.4.3 Example: Estimating Motor Performance

Given manufacturer’s information on the torque efficiency of a specific motor and the information in Table 8.1, estimate the magnitude of C_d and C_f :

Table 8.1: Information for motor performance example

Characteristic	Value	Units
Motor displacement, D_m	0.323	in. ³ /rad
Speed, n	2400	rpm
Torque efficiency η_T	96.9	%
Pressure p_L	3000	lb _f /in. ²
Oil absolute viscosity, μ	2.4E−6	lb _f · s/in. ²
Partition between viscous and Coulomb friction	0.5:0.5	

If the motor were perfect, the torque delivered would be:

$$\begin{aligned} p_L D_m &= 3000 \frac{\text{lb}_f}{\text{in.}^2} \times 0.323 \frac{\text{in.}^3}{\text{rad}} \\ &= 969 \text{ in.} \cdot \text{lb}_f \end{aligned}$$

Hence the drag torque would be:

$$\begin{aligned} T_d &= p_L D_m (1 - \eta_{Tm}) \\ &= 3000 \frac{\text{lb}_f}{\text{in.}^2} \times 0.323 \frac{\text{in.}^3}{\text{rad}} \times (1 - 0.969) \\ &= 30.04 \text{ in.} \cdot \text{lb}_f \end{aligned}$$

Thus:

$$\begin{aligned}
 C_d &= \frac{0.5 \times 30.04 \text{ lb}_f \cdot \text{in.}}{0.323 \text{ in.}^3/\text{rad} \times 2.45\text{E}-6 \text{ lb}_f \cdot \text{s}/\text{in.}^2 \times 2\pi \text{ rad}/\text{rev} \times \frac{2400 \text{ rev}/\text{min}}{60 \text{ s}/\text{min}}} \\
 &= 75520
 \end{aligned}$$

and:

$$\begin{aligned}
 C_f &= \frac{T_f}{P_L D_m} = \frac{0.5 \times 30.04 \text{ lb}_f \cdot \text{in.}}{3000 \text{ lb}_f/\text{in.}^2 \times 0.323 \text{ in.}^3/\text{rad}} \\
 &= 0.0155
 \end{aligned}$$

8.4.4 Overall Efficiency

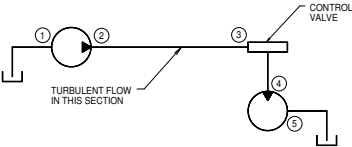
Overall efficiency plotted vs. speed at three load pressures is shown in [Figure 8.13](#). At high speeds, the volumetric efficiency increases and the torque efficiency drops ([Figure 8.10](#)), so it is to be expected that there will be a speed at which overall efficiency shows a maximum. It may be observed that the efficiency ranking with pressure is that lowest pressure leads to highest efficiency at low speed, this ranking is reversed at high speed. This result could have been predicted by examining the volumetric and torque efficiency curves.

$$\begin{aligned}
 \eta_{OA} &= \frac{\text{POWER OUT}}{\text{POWER IN}} \\
 &= \frac{T_L \omega_m}{p_1 Q_1} = \frac{T_m \omega_m}{p_1 \left(\frac{D_m \omega_m}{\eta_{Vm}} \right)} \\
 &= \frac{T_L}{p_1 D_m} \eta_{Vm} = \eta_{Tm} \eta_{Vm} \quad (8.25)
 \end{aligned}$$

If the theoretical curves presented here are compared to curves for real equipment, the trends are very similar. One feature that should be observed is that fluid power pumps and motors show quite dramatic drops in efficiency if they are operated at slow speeds. The theoretical model and real equipment agree on this trend. For a more recent discussion of pump efficiency, the reader is referred to [4].

PROBLEMS

8.1 A fluid power pump is used to drive a motor as shown in the figure. Pressure loss, Δp_V , across the valve at position 3 is 12 lb_f/in.².

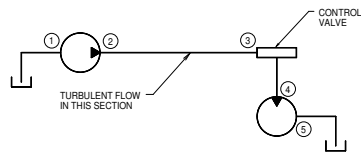


Determine the Reynolds number, Re , in the flow line between stations 2 and 3. Determine pressures, p_2 and p_3 , for the conditions given in the table.

Characteristics of a pump and motor system

Characteristic	Size	Units
Oil mass density, ρ	0.08E-3	lb _f · s ² /in. ⁴
Oil viscosity, μ	1.7E-6	lb _f · s/in. ²
Line length, 2 to 4, ℓ_L	63.0	in.
Line diameter, 2 to 4, d	0.4	in.
Pressure at 1, p_1	0.0	lb _f /in. ²
Pump speed, n	1075	rpm
Pump overall efficiency, η_{op}	92.0	%
Pump mechanical efficiency, η_{mp}	95.0	%
Valve loss factor, K	5.1	
Motor displacement, D_m	1.95	in. ³ /rev
Motor torque output, T_m	380	lb _f · in.
Motor discharge pressure, p_5	27.0	lb _f /in. ²
Motor mechanical efficiency, η_{mm}	97.0	%

8.2 A fluid power pump is used to drive a motor as shown in the figure. Pressure loss, Δp_V , across the valve at position 3 is 14 lb_f/in.².

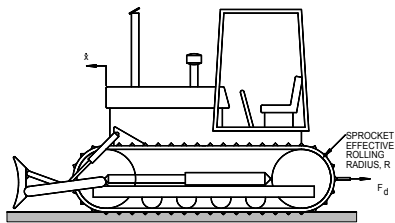


Determine the required speed, n_p for the pump. Determine the Reynolds number, Re , in the flow line between stations 2 and 3. Determine pressures, p_1 , p_3 , and p_4 , for the conditions given in the table.

Characteristics of a pump and motor system

Characteristic	Size	Units
Oil mass density, ρ	0.08E-3	$\text{lb}_f \cdot \text{s}^2/\text{in.}^4$
Oil viscosity, μ	1.7E-6	$\text{lb}_f \cdot \text{s}/\text{in.}^2$
Line length, 2 to 4, ℓ_L	65.0	in.
Line diameter, 2 to 4, d	0.4	in.
Pressure at 1, p_1	50.0	$\text{lb}_f/\text{in.}^2$
Pump overall efficiency, η_{op}	92.0	%
Pump volumetric efficiency, η_{vp}	95.0	%
Pump input torque, T_p	390	$\text{lb}_f \cdot \text{in.}$
Pump displacement, D_p	1.75	$\text{in.}^3/\text{rev}$
Valve loss factor, K	5.2	
Motor discharge pressure, p_5	27.0	$\text{lb}_f/\text{in.}^2$
Motor mechanical efficiency, η_{mm}	97.0	%

8.3 The two rear drive sprockets on a crawler tractor are powered with hydraulic motors through a reduction gear set. The motors are driven by a single variable displacement hydraulic pump with equal flow to each motor.



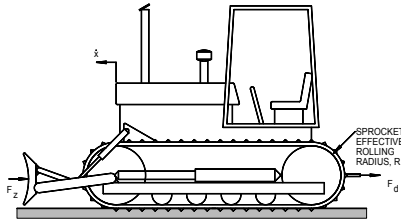
Determine the highest and lowest sprocket speeds, n_{sl} and n_{sh} and the lowest and highest tractor speeds, $\dot{x}$ and $\dot{X}$ that will occur as the pump displacement advances from D_{pl} to D_{ph} . Determine the tractor drawbar pull, F_d , and the tractor power, P , that will be produced with the values for pump displacement D_{pl} and D_{ph} , and the other values given in the table.

Characteristics of a hydrostatic transmission for a crawler tractor

Characteristic	Size	Units
Outlet pressure, p_s	23.5	MPa
Pump displacement, low, D_{pl}	25.0	mL/rev
Pump displacement, high, D_{ph}	55.0	mL/rev
Pump speed, n_p	1200	rpm
Pump volumetric efficiency, η_{vp}	97.0	%
Motor displacement, D_m	330	mL/rev
Motor volumetric efficiency, η_{vm}	97.0	%
Motor mechanical efficiency, η_{mm}	96.0	%
Motor discharge pressure, p_r	240	kPa
Gear ratio, $N = n_m/n_s$	4.5:1	
Sprocket effective rolling radius, r	375	mm

8.4 The two rear drive sprockets on a crawler tractor are powered with hydraulic motors through a reduction gear set. The motors are driven

by a single variable displacement hydraulic pump with equal flow to each motor.

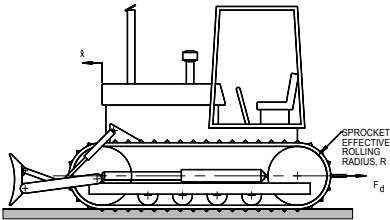


Determine the required motor displacement, D_m , to produce the given dozer force, F_z , and the drawbar force, F_d . Determine the required motor flow, Q , to produce the given tractor speed, $\dot{x}$.

Characteristics of a hydrostatic transmission for a crawler tractor

Characteristic	Size	Units
Drawbar force, F_d	12000	N
Dozer force, F_z	19500	N
Motor inlet pressure, p_s	30.0	MPa
Motor discharge pressure, p_2	350	kPa
Motor overall efficiency, η_{om}	94.0	%
Motor mechanical efficiency, η_{mm}	96.0	%
Gear ratio, $N = n_m/n_s$	3.9:1	
Tractor speed, $\dot{x}$	4.7	km/h
Sprocket effective rolling radius, r	377	mm

8.5 The two rear drive sprockets on a crawler tractor are powered with hydraulic motors through a reduction gear set. The motors are driven by a single variable displacement hydraulic pump with equal flow to each motor.

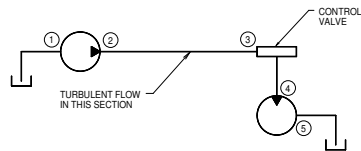


Determine the lowest and highest sprocket speeds, n_{sl} and n_{sh} , and the lowest and highest tractor speeds, $\dot{x}$ and $\dot{X}$, that will occur as the motor displacement changes from, D_{mh} to D_{ml} . Determine the tractor drawbar pull, F_d , and the tractor power, P , that will be produced with the values for motor displacement, D_{ml} and D_{mh} , for the values given in the table.

Characteristics of a hydrostatic transmission for a crawler tractor

Characteristic	Size	Units
Pump outlet pressure, p_s	23.5	MPa
Pump displacement, D_p	40.0	mL/rev
Pump speed, n_p	1200	rpm
Pump volumetric efficiency, η_{vp}	97.0	%
Motor low displacement, D_{ml}	220	mL/rev
Motor high displacement, D_{mh}	570	mL/rev
Motor volumetric efficiency, η_{vm}	97.0	%
Motor mechanical efficiency, η_{mm}	96.0	%
Motor discharge pressure, p_r	240	kPa
Gear ratio, $N = n_m/n_s$	4.5:1	
Sprocket effective rolling radius, r	375	mm

8.6 A fluid power pump is used to drive a motor as shown in the figure. Pressure loss, Δp_V , across the valve at position 3 is 14 lb_f/in.²



Determine the required displacement, D_p for the pump. Determine the Reynolds number, Re , in the flow line between stations 2 and 3. Determine pressures, p_2 and p_3 , for the conditions given in the table.

Characteristics of a pump and motor system

Characteristic	Size	Units
Oil mass density, ρ	0.08E-3	$\text{lb}_f \cdot \text{s}^2/\text{in.}^4$
Oil viscosity, μ	1.7E-6	$\text{lb}_f \cdot \text{s}/\text{in.}^2$
Line length, ℓ_L	65.0	in.
Line diameter, d	0.375	in.
Pressure at 1, p_1	0.0	$\text{lb}_f/\text{in.}^2$
Pump speed, n	1150	rpm
Pump overall efficiency, η_{op}	94.0	%
Pump mechanical efficiency, η_{mp}	95.0	%
Valve loss factor, K	5.7	
Motor displacement, D_m	2.1	$\text{in.}^3/\text{rev}$
Motor torque output, T_m	385	$\text{lb}_f \cdot \text{in.}$
Motor discharge pressure, p_5	33.0	$\text{lb}_f/\text{in.}^2$
Motor mechanical efficiency, η_{mm}	95.0	%

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