Power Buoy Generator Efficiency

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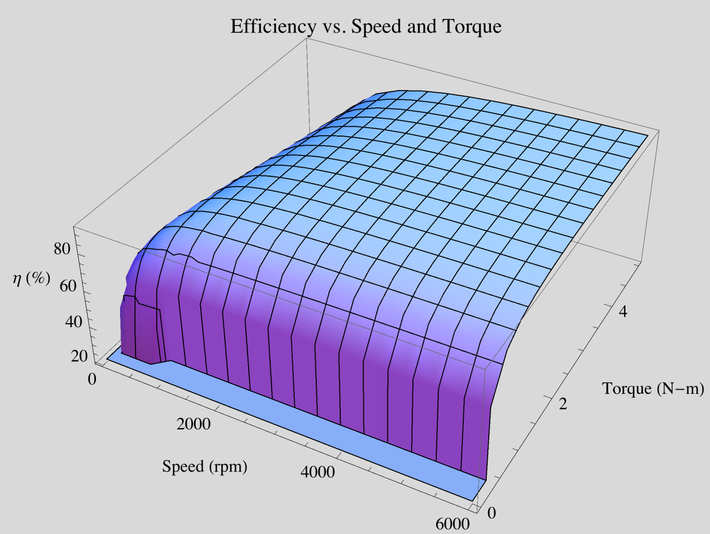
We want to plot generator output efficiency as a function of input speed ω (in rpm) and torque M (in N-m), given the generator speed constant Ke, torque constant Kt, and winding resistance Rg.

$$\eta[\omega\_, M\_, Ke\_, Kt\_, Rg\_, Mf\_] := \frac{100 \left(\frac{M-Mf}{Kt}\right)^2 \left(\frac{Ke\,\omega\,Kt}{M-Mf} - Rg\right)}{\omega \frac{2\,Pi}{60} M}$$

Now we can produce a 3D plot of efficiency for a Magmotor model BF34-500 motor with a “K” winding over its range of allowed speed and torque.

```
effpl = Plot3D[η[ω, M, 0.2, 1.91, 22.9, 0.141], {ω, 0, 6000}, {M, 0, 5}, {
   PlotLabel → "Efficiency vs. Speed and Torque",
   AxesLabel → {"Speed (rpm)", "Torque (N-m)", "η (%)"}}]
```




```
peaks =
  Map[Maximize[{η[#, M, 0.2, 1.91, 22.9, 0.141], 0 < M < 5}, M] &, Range[100, 6100, 100]];
```

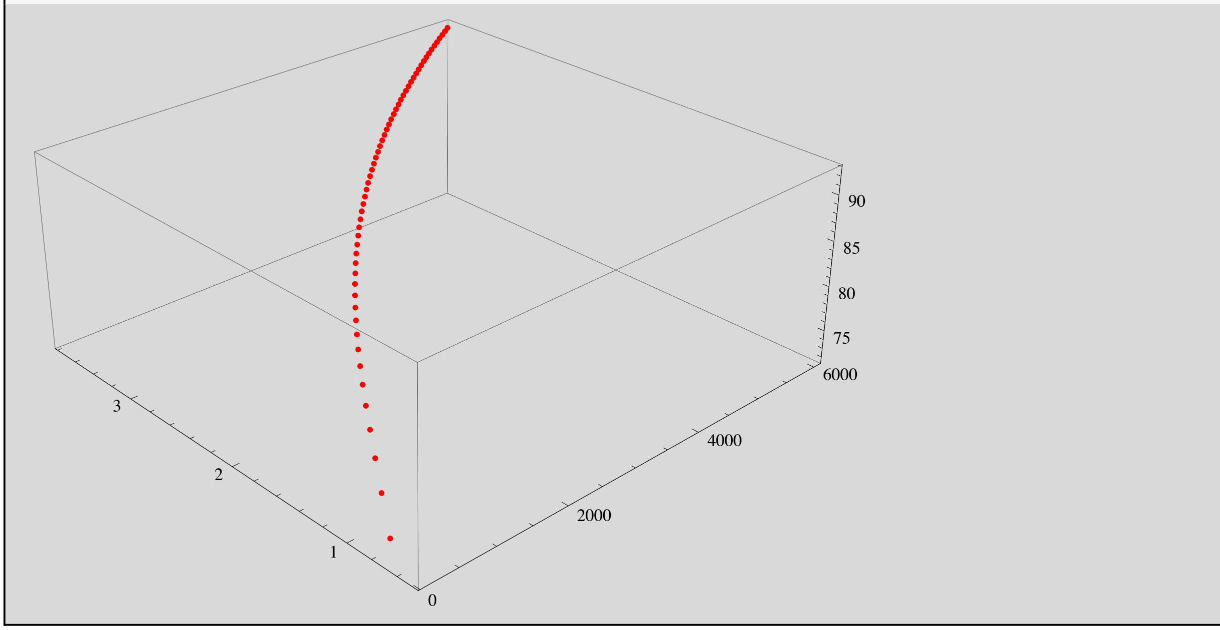
```
peaks2 = Transpose[peaks];
```

```
peaks3 = Riffle[Range[100, 6100, 100], M /. Transpose[peaks][[2]]];
```

```
peaks4 = Riffle[peaks3, peaks2[[1]], 3];
```

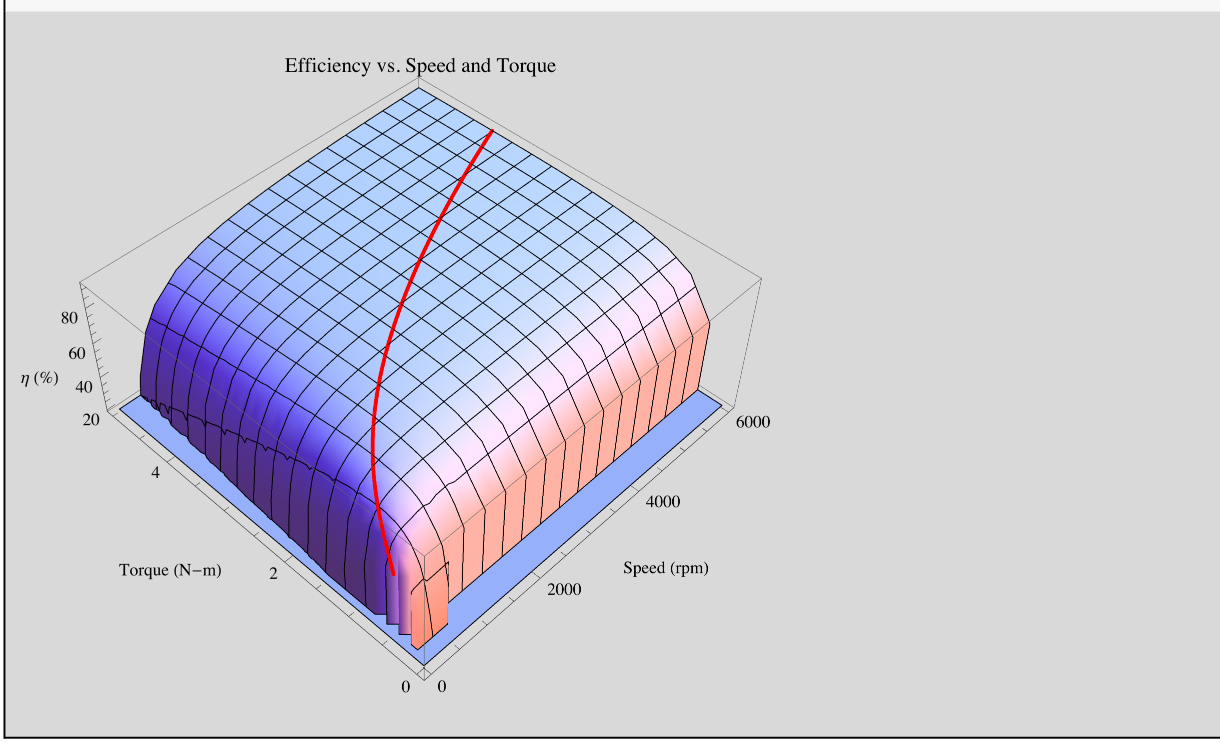
```
peaks5 = Partition[peaks4, 3];
```

```
peakpl = ListPointPlot3D[peaks5, {PlotStyle → Red}]
```




```
peakpl2 = Graphics3D[{Red, Thick, Line[peaks5]}];
```

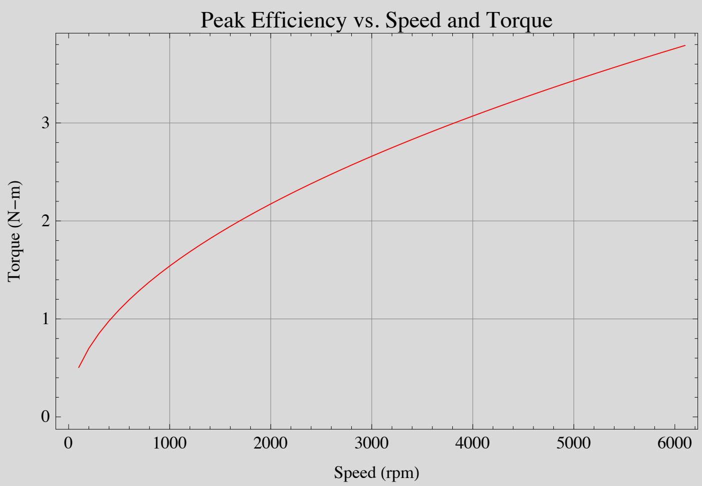
```
Show[effpl, peakpl2]
```



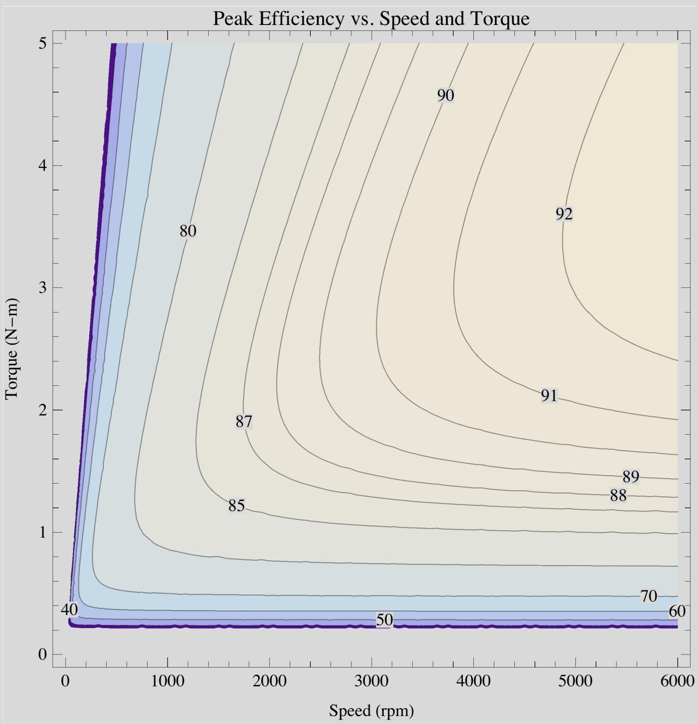
This is good for getting the big picture, but not for reading precise values. We can generate a 2D plot of the peak efficiency curve overlaid on a contour plot.

```
peaks6 = Partition[peaks3, 2];
```

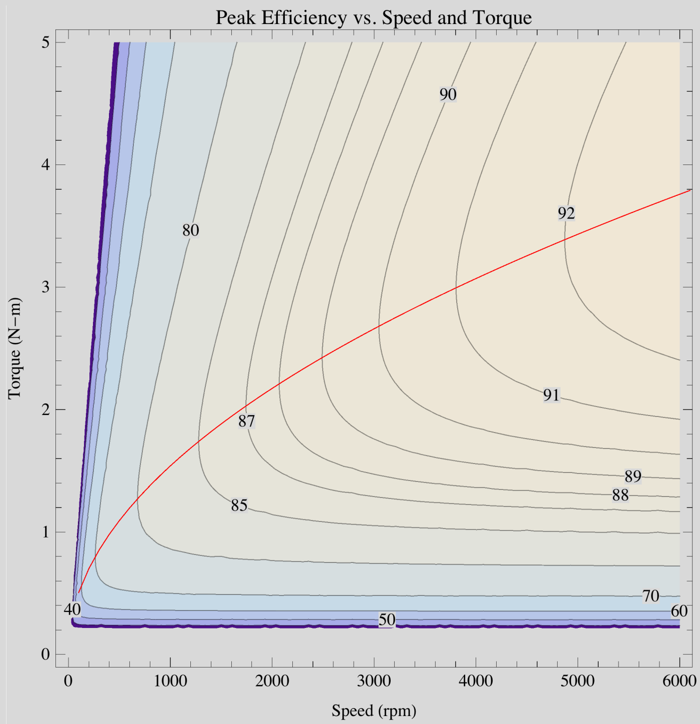
```
peakpl3 = ListPlot[peaks6,
  {Frame → True,
   Joined → True,
   GridLines → Automatic,
   PlotStyle → Red,
   PlotLabel → "Peak Efficiency vs. Speed and Torque",
   FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}]
```



```
conpl = ContourPlot[η[ω, M, 0.2, 1.91, 22.9, 0.141], {ω, 0, 6000}, {M, 0, 5},
  {Frame → True,
   Contours → {40, 50, 60, 70, 80, 85, 87, 88, 89, 90, 91, 92},
   ContourLabels → All,
   PlotLabel → "Peak Efficiency vs. Speed and Torque",
   FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}]
```



```
Show[conpl, peakpl3]
```



We'd also like to see how load voltage, current, and power varies with input speed and torque.

$$\text{Vl}[\omega\_, \text{M}\_, \text{Ke}\_, \text{Kt}\_, \text{Rg}\_, \text{Mf}\_] := (\omega\,\text{Ke}) - \frac{(\text{M} - \text{Mf})\,\text{Rg}}{\text{Kt}}$$

$$\text{A}[\text{M}\_, \text{Kt}\_, \text{Mf}\_] := \frac{\text{M} - \text{Mf}}{\text{Kt}}$$

$$\text{Pl}[\omega\_, \text{M}\_, \text{Ke}\_, \text{Kt}\_, \text{Rg}\_, \text{Mf}\_] := \left(\frac{\text{M} - \text{Mf}}{\text{Kt}}\right)^2 \left(\frac{\text{Ke}\,\omega\,\text{Kt}}{\text{M} - \text{Mf}} - \text{Rg}\right)$$

We also need to define some functions for two-axis plotting.

$$\text{defaultColor}[\text{n\_Integer}] := \text{Hue}\left[\text{N}\left[\text{FractionalPart}\left[.67 + \frac{2\,(\text{n} - 1)}{\text{GoldenRatio}}\right]\right], .6, .6\right]$$

```
SetAttributes[TwoAxisPlot, HoldAll]
```

```
TwoAxisPlot[{f_, g_}, {x_, x0_, x1_}, opts : OptionsPattern[]] :=
 Module[{f0, f1, g0, g1, gp, scale, gticks},
  {f0, f1} = Options[Plot[f, {x, x0, x1}, Frame → True, opts], PlotRange][[1, 2, 2]];
  {g0, g1} = Options[gp = Plot[g, {x, x0, x1}, Frame → True, opts], PlotRange][[1, 2, 2]];
  scale[y_] := f0 + ((f1 - f0) * (y - g0)) / (g1 - g0);
  gticks = Apply[{scale[#1], ##2} &, AbsoluteOptions[gp, FrameTicks][[1, 2, 2]], {1}];
  Plot[{f, scale[g]}, {x, x0, x1}, PlotRange → 1.001 * {f0, f1},
   Frame → True, FrameTicks → {{Automatic, gticks}, {Automatic, Automatic}},
   FrameStyle → {{defaultColor[1], defaultColor[2]}, {{}, {}}}, opts]]
```

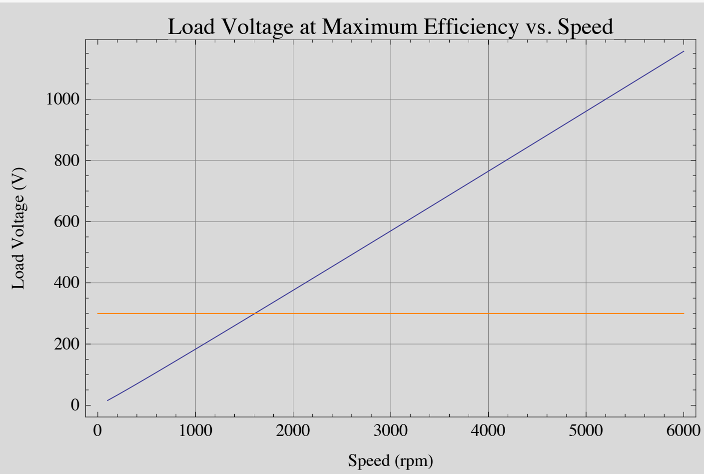
Now we can plot the load voltage, current, and power *at maximum efficiency*.

```
peaks6 = Partition[peaks3, 2];
```

```
loadvolts = Map[Vl[#[[1]], #[[2]], 0.2, 1.91, 22.9, 0.141] &, peaks6];
```

```
loadvolts2 = Partition[Riffle[Range[100, 6000, 100], loadvolts], 2];
```

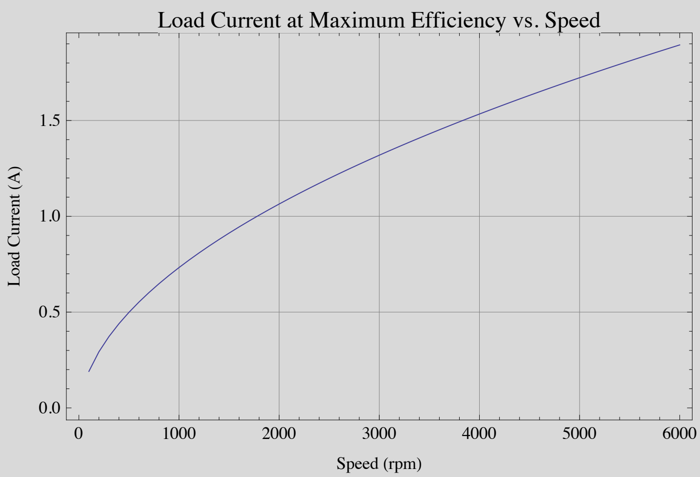
```
Show[ListPlot[loadvolts2,
  {Joined → True,
   Frame → True,
   GridLines → Automatic,
   PlotLabel → "Load Voltage at Maximum Efficiency vs. Speed"},
  FrameLabel → {"Speed (rpm)", "Load Voltage (V)"}],
 Graphics[{Orange, Line[{{0, 300}, {6000, 300}}]}]]
```




```
loadcurr = Map[A[#[[2]], 1.91, 0.141] &, peaks6];
```

```
loadcurr2 = Partition[Riffle[Range[100, 6000, 100], loadcurr], 2];
```

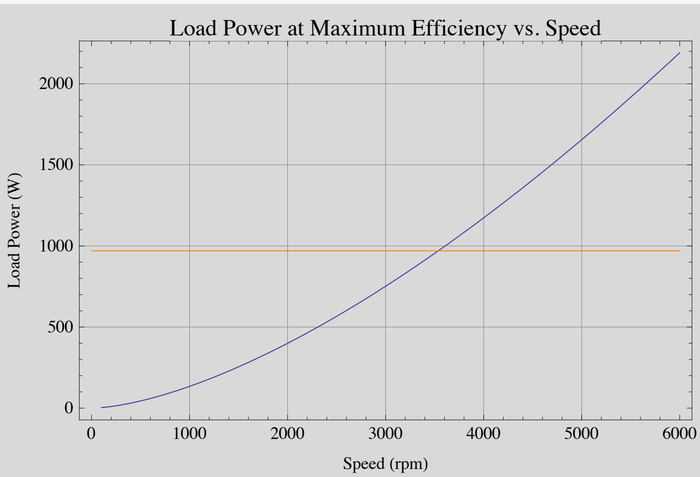
```
ListPlot[loadcurr2,
 {Joined → True,
  Frame → True,
  GridLines → Automatic,
  PlotLabel → "Load Current at Maximum Efficiency vs. Speed"},
 FrameLabel → {"Speed (rpm)", "Load Current (A)"}]
```




```
loadpow = Map[Pl[#[[1]], #[[2]], 0.2, 1.91, 22.9, 0.141] &, peaks6];
```

```
loadpow2 = Partition[Riffle[Range[100, 6000, 100], loadpow], 2];
```

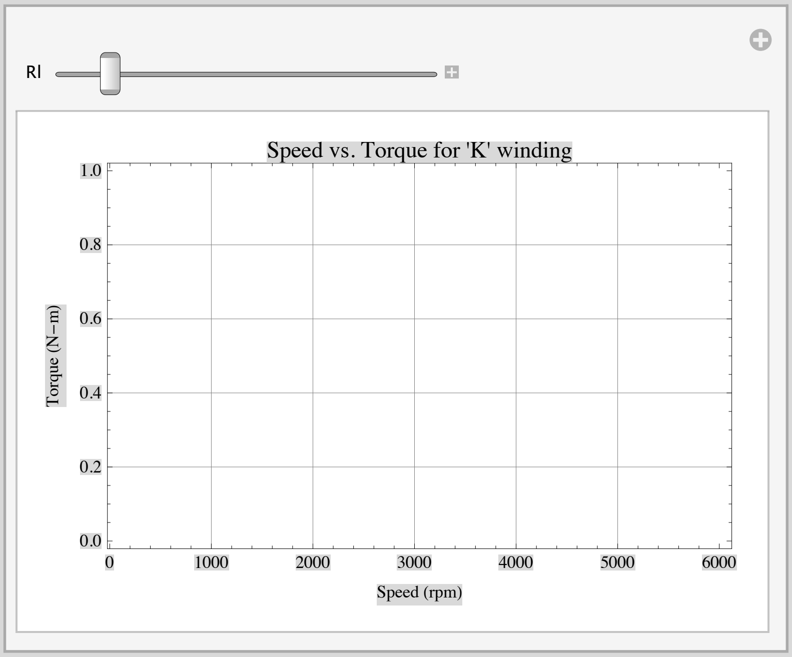
```
Show[ListPlot[loadpow2,
  {Joined → True,
   Frame → True,
   GridLines → Automatic,
   PlotLabel → "Load Power at Maximum Efficiency vs. Speed"},
  FrameLabel → {"Speed (rpm)", "Load Power (W)"}],
 Graphics[{Orange, Line[{{0, 970}, {6000, 970}}]}]]
```



How does the torque load we can present to the hydraulic motor vary as a function of the impedance that the power converter that presents to the generator (Magmotor model BF34-500 with a “K” winding)?

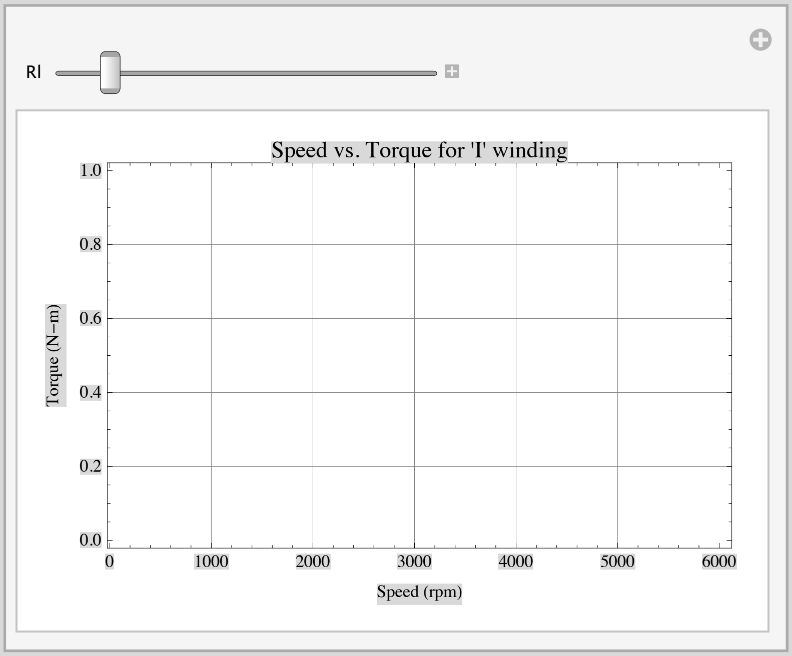
```
M[ω_, Ke_, Kt_, Rg_, Rl_, Mf_] := Kt (Ke ω)/(Rg + Rl) + Mf
```

```
Manipulate[
 Show[{Plot[M[ω, 0.2, 1.91, 22.9, Rl, 0.141], {ω, 100, 6000},
    {Frame → True,
     GridLines → Automatic,
     PlotLabel → "Speed vs. Torque for 'K' winding",
     FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}],
   Graphics[
    {Orange, Line[{{0, 4.24}, {6000, 4.24}}], Red, Line[{{0, 12.6}, {6000, 12.6}}]}]}],
 {Rl,
  0,
  100}]
```



How is this different for the same generator with an “E” winding?

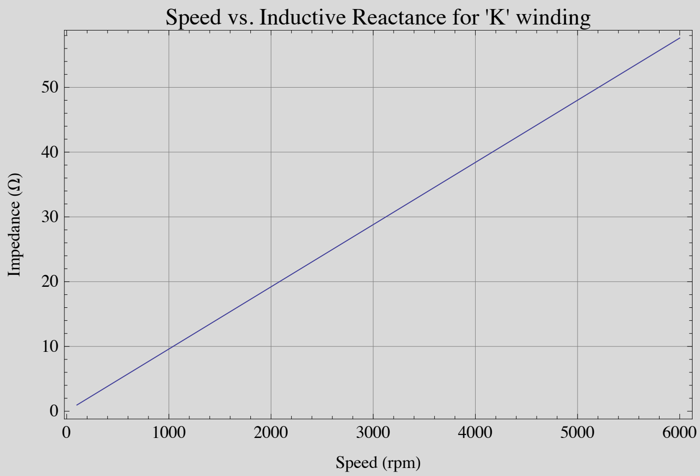
```
Manipulate[
 Show[{Plot[M[ω, 0.126, 0.501, 5.9, Rl, 0.141], {ω, 100, 6000},
    {Frame → True,
     GridLines → Automatic,
     PlotLabel → "Speed vs. Torque for 'I' winding",
     FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}],
   Graphics[
    {Orange, Line[{{0, 4.24}, {6000, 4.24}}], Red, Line[{{0, 12.6}, {6000, 12.6}}]}]}],
 {Rl,
  0,
  100}]
```



Does the inductance of the motor windings matter? Plot inductive reactance as a function of speed.

```
Xl[ω_, L_] := (2 Pi ω / 60) L
```

```
Plot[Xl[ω, 0.0917], {ω, 100, 6000},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel → "Speed vs. Inductive Reactance for 'K' winding",
  FrameLabel → {"Speed (rpm)", "Impedance (Ω)"}}]
```




```
Plot[Xl[ω, 0.0059], {ω, 100, 6000},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel → "Speed vs. Inductive Reactance for 'I' winding",
  FrameLabel → {"Speed (rpm)", "Impedance (Ω)"}}]
```