Power Buoy Generator Efficiency

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We want to plot generator output efficiency as a function of input speed ω (in rpm) and torque M (in N-m), given the generator speed constant Ke, torque constant Kt, and winding resistance Rg.

$$\eta[\omega\_, M\_, Ke\_, Kt\_, Rg\_, Mf\_] := \frac{100\left(\frac{M-Mf}{Kt}\right)^2\left(\frac{Ke\,\omega\,Kt}{M-Mf}-Rg\right)}{\omega\,\frac{2\,Pi}{60}\,M}$$

Now we can produce a 3D plot of efficiency for a Magmotor model BFA42-600 motor with a “5K” winding over its range of allowed speed and torque.

First convert the torque constant and friction torque in oz-in to N-m

```
Kt = 118.5 * 7.0615 * 10^-3
```

```
0.836788
```

```
Mf = 60 * 7.0615 * 10^-3
```

```
0.42369
```

```
Ke = 87.6 / 1000
```

```
0.0876
```

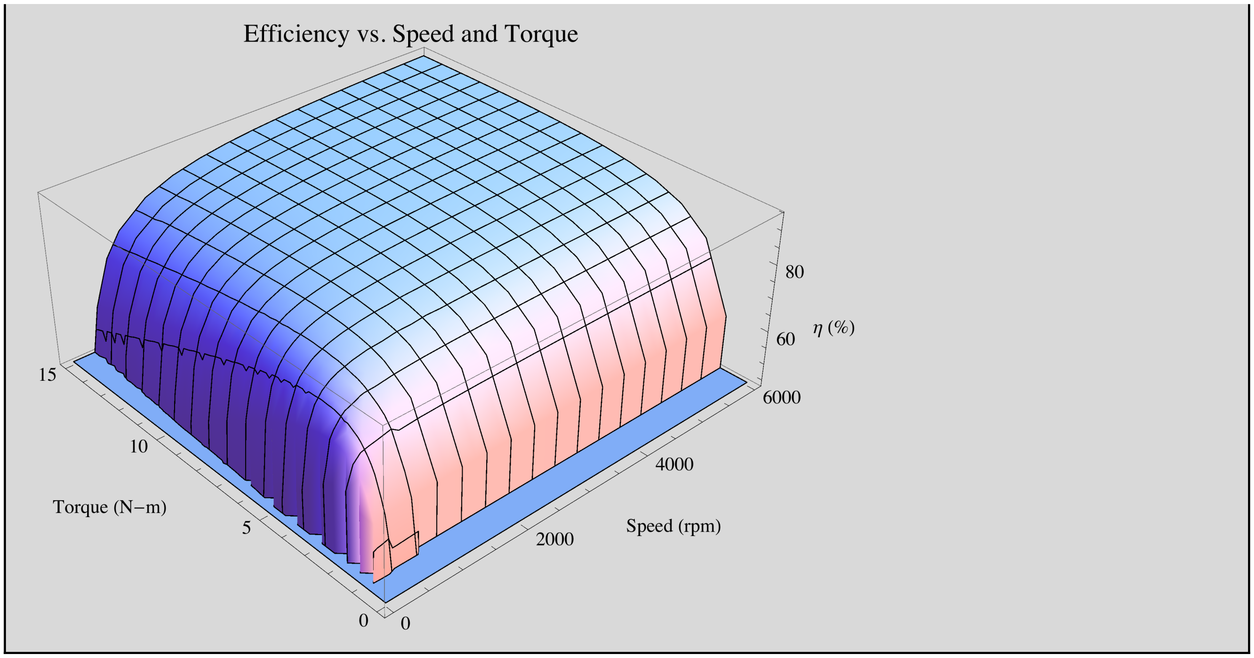
```
Rg = 0.88
```

```
0.88
```

```
1600 * 7.0615 * 10^-3
```

```
11.2984
```

```
effpl = Plot3D[η[ω, M, Ke, Kt, Rg, Mf], {ω, 0, 6000}, {M, 0, 15}, {
   PlotLabel → "Efficiency vs. Speed and Torque",
   AxesLabel → {"Speed (rpm)", "Torque (N-m)", "η (%)"}}]
```




```
peaks =
  Map[Maximize[{η[#, M, Ke, Kt, Rg, Mf], 0 < M < 15}, M] &, Range[100, 6100, 100]];
```

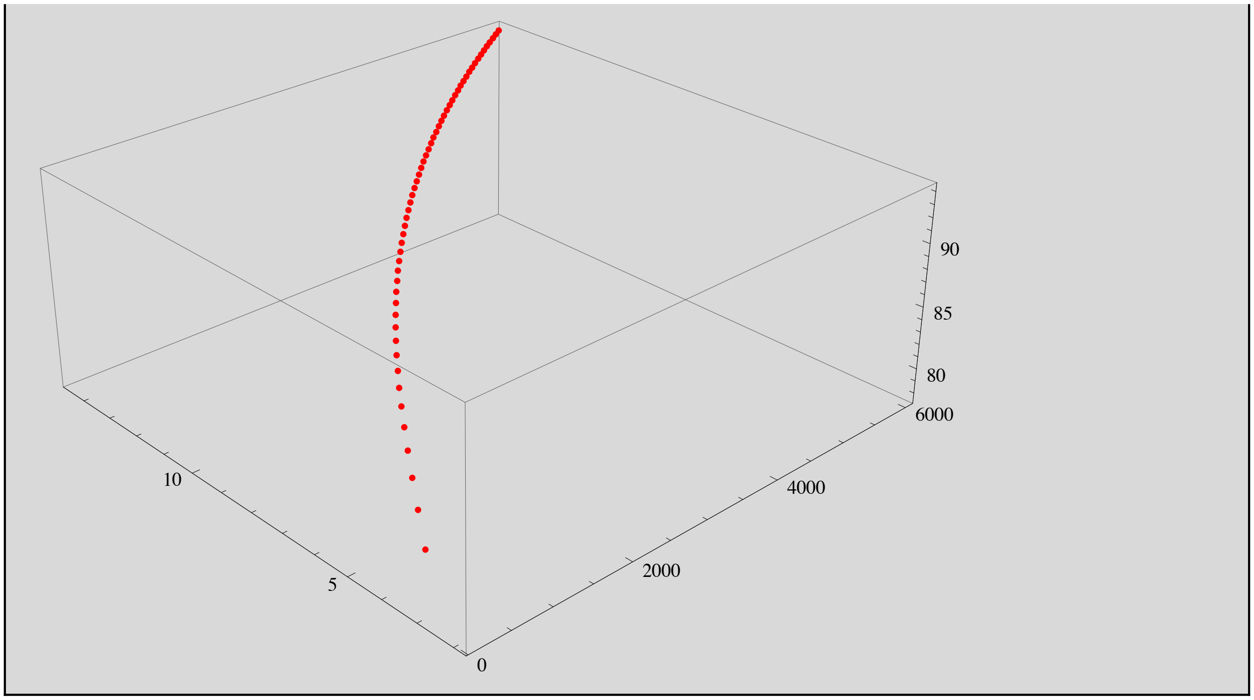
```
peaks2 = Transpose[peaks];
```

```
peaks3 = Riffle[Range[100, 6100, 100], M /. Transpose[peaks][[2]]];
```

```
peaks4 = Riffle[peaks3, peaks2[[1]], 3];
```

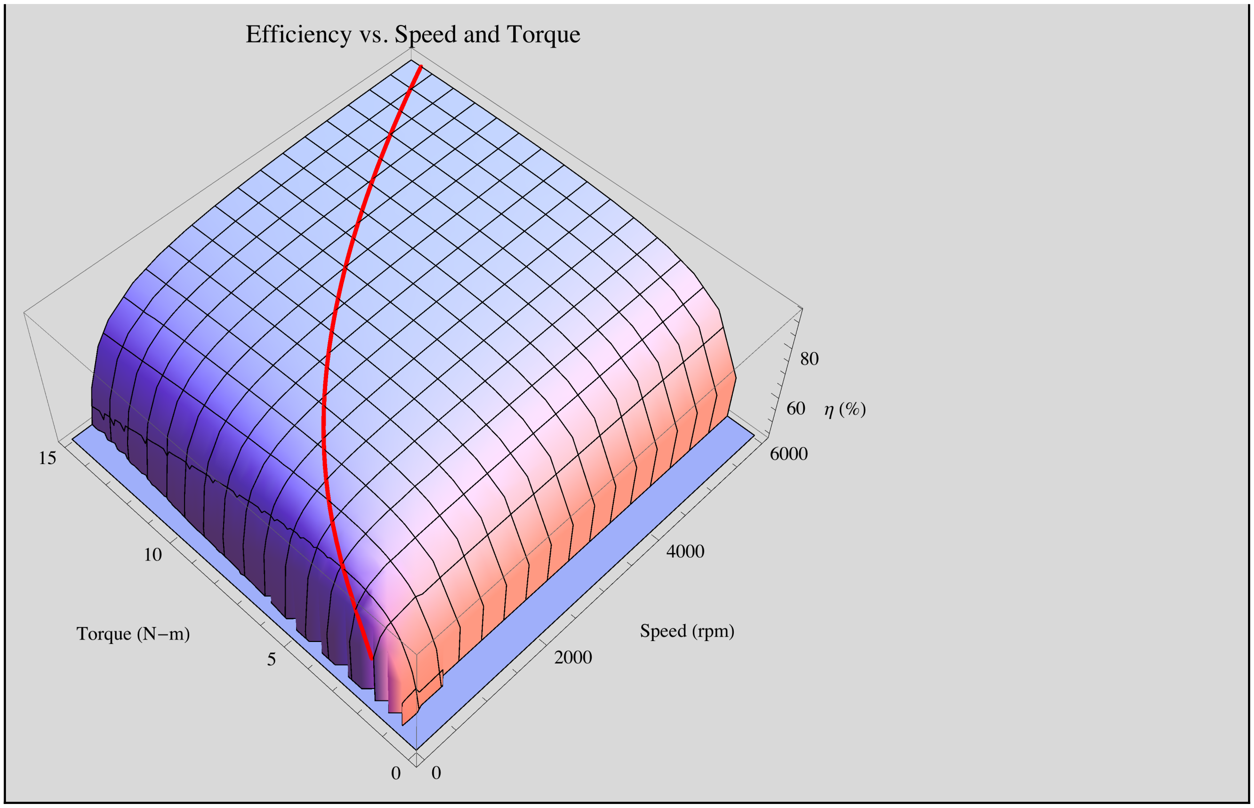
```
peaks5 = Partition[peaks4, 3];
```

```
peakpl = ListPointPlot3D[peaks5, {PlotStyle → Red}]
```




```
peakpl2 = Graphics3D[{Red, Thick, Line[peaks5]}];
```

```
Show[effpl, peakpl2]
```



This is good for getting the big picture, but not for reading precise values. We can generate a 2D plot of the peak efficiency curve overlaid on a contour plot.

```
peaks6 = Partition[peaks3, 2];
```

```
peakpl3 = ListPlot[peaks6,
  {Frame → True,
   Joined → True,
   GridLines → Automatic,
   PlotStyle → Red,
   PlotLabel → "Peak Efficiency vs. Speed and Torque",
   FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}]
```

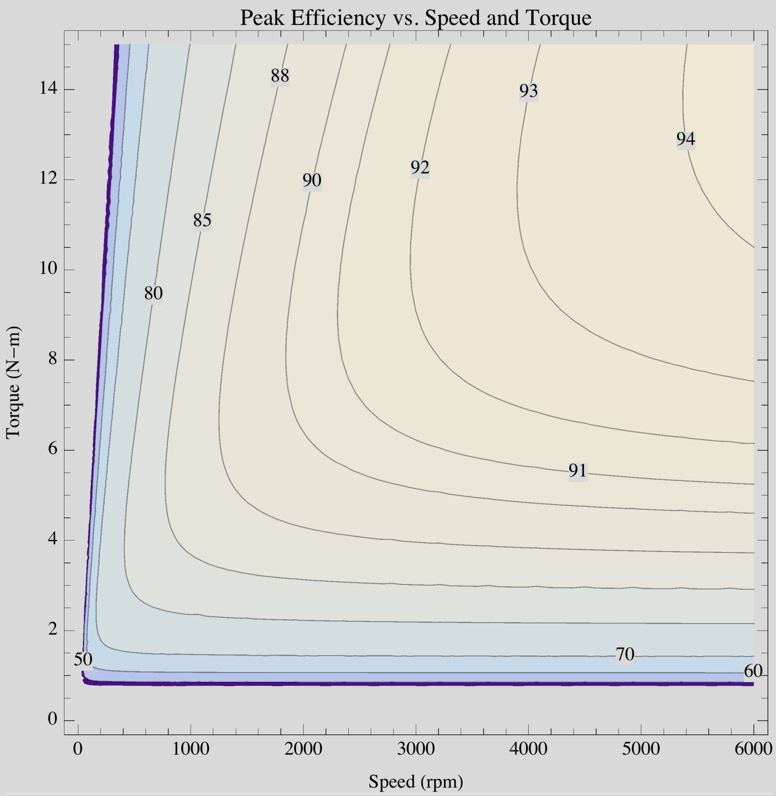

```
FindFit[peaks6, a + b x^c, {a, b, c}, x]
```

```
{a → 0.0648101, b → 0.183324, c → 0.50236}
```

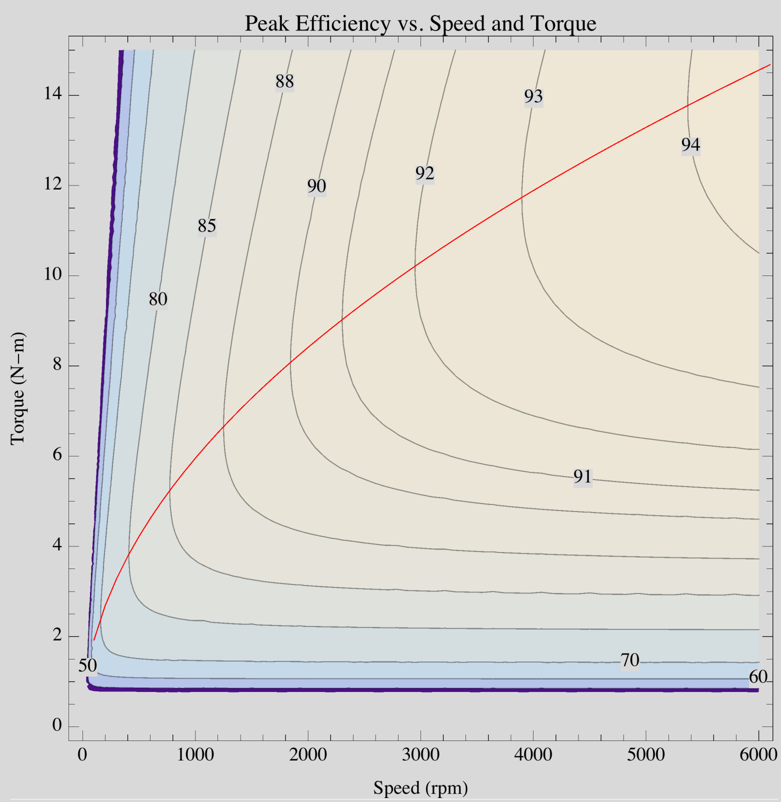
```
FindFit[peaks6, a x^b, {a, b}, x]
```

```
{a → 0.190367, b → 0.49849}
```

```
conpl = ContourPlot[η[ω, M, Ke, Kt, Rg, Mf], {ω, 0, 6000}, {M, 0, 15},
  {Frame → True,
   Contours → {50, 60, 70, 80, 85, 88, 90, 91, 92, 93, 94},
   ContourLabels → All,
   PlotLabel → "Peak Efficiency vs. Speed and Torque",
   FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}]
```



```
Show[conpl, peakpl3]
```



We'd also like to see how load voltage, current, and power varies with input speed and torque.

$$\text{Vl}[\omega\_,\ \text{M}\_,\ \text{Ke}\_,\ \text{Kt}\_,\ \text{Rg}\_,\ \text{Mf}\_] := (\omega\ \text{Ke}) - \frac{(\text{M} - \text{Mf})\ \text{Rg}}{\text{Kt}}$$

$$\text{A}[\text{M}\_,\ \text{Kt}\_,\ \text{Mf}\_] := \frac{\text{M} - \text{Mf}}{\text{Kt}}$$

$$\text{Pl}[\omega\_,\ \text{M}\_,\ \text{Ke}\_,\ \text{Kt}\_,\ \text{Rg}\_,\ \text{Mf}\_] := \left(\frac{\text{M} - \text{Mf}}{\text{Kt}}\right)^2 \left(\frac{\text{Ke}\ \omega\ \text{Kt}}{\text{M} - \text{Mf}} - \text{Rg}\right)$$

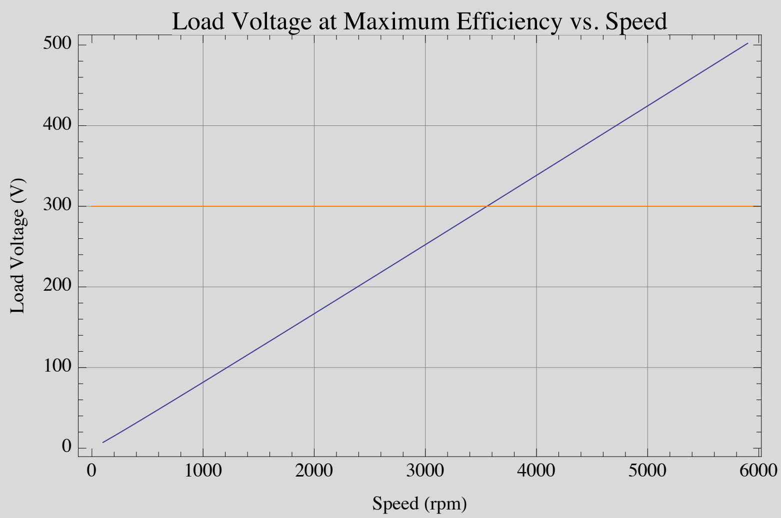
Now we can plot the load voltage, current, and power *at maximum efficiency*.

```
peaks6 = Partition[peaks3, 2];
```

```
loadvolts = Map[Vl[#[[1]], #[[2]], Ke, Kt, Rg, Mf] &, peaks6];
```

```
loadvolts2 = Partition[Riffle[Range[100, 6000, 100], loadvolts], 2];
```

```
Show[ListPlot[loadvolts2,
  {Joined → True,
   Frame → True,
   GridLines → Automatic,
   PlotLabel → "Load Voltage at Maximum Efficiency vs. Speed"},
  FrameLabel → {"Speed (rpm)", "Load Voltage (V)"}],
 Graphics[{Orange, Line[{{0, 300}, {6000, 300}}]}]]
```




```
loadcurr = Map[A[#[[2]], Kt, Mf] &, peaks6];
```

```
loadcurr2 = Partition[Riffle[Range[100, 6000, 100], loadcurr], 2];
```

```
ListPlot[loadcurr2,
 {Joined → True,
  Frame → True,
  GridLines → Automatic,
  PlotLabel → "Load Current at Maximum Efficiency vs. Speed"},
 FrameLabel → {"Speed (rpm)", "Load Current (A)"}]
```


```
loadpow = Map[Pl[#[[1]], #[[2]], Ke, Kt, Rg, Mf] &, peaks6];
```

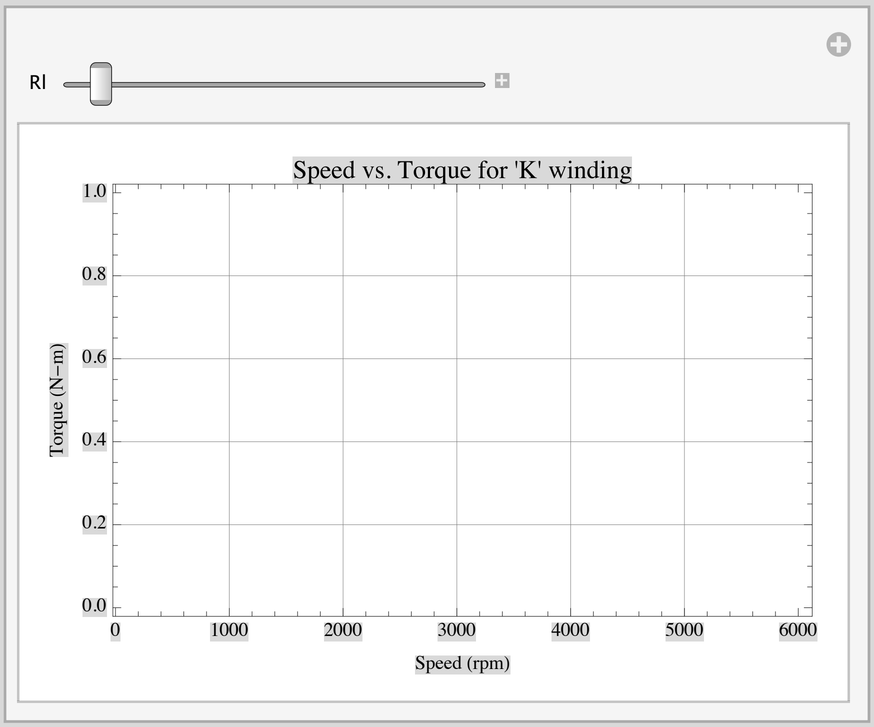
```
loadpow2 = Partition[Riffle[Range[100, 6000, 100], loadpow], 2];
```

```
Show[ListPlot[loadpow2,
  {Joined → True,
   Frame → True,
   GridLines → Automatic,
   PlotLabel → "Load Power at Maximum Efficiency vs. Speed"},
  FrameLabel → {"Speed (rpm)", "Load Power (W)"}],
 Graphics[{Orange, Line[{{0, 2800}, {6000, 2800}}]}]]
```

How does the torque load we can present to the hydraulic motor vary as a function of the impedance that the power converter presents to the generator (Magmotor model BF34-500 with a "K" winding)?

$$M[\omega\_, Ke\_, Kt\_, Rg\_, Rl\_, Mf\_] := Kt \frac{Ke\,\omega}{Rg + Rl} + Mf$$

```
Manipulate[
 Show[{Plot[M[ω, Ke, Kt, Rg, Rl, Mf], {ω, 100, 6000},
    {Frame → True,
     GridLines → Automatic,
     PlotLabel → "Speed vs. Torque for 'K' winding",
     FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}],
   Graphics[{Orange, Line[{{0, 11.3}, {6000, 11.3}}],
     Red, Line[{{0, 33.9}, {6000, 33.9}}]}]}],
 {Rl,
  0,
  100}]
```



How is this different for the same generator with an “5E” winding?

```
Kt5E = 29.6 * 7.0615 * 10^-3
```

```
0.20902
```

```
Mf = 60 * 7.0615 * 10^-3
```

```
0.42369
```

```
Ke5E = 21.9 / 1000
```

```
0.0219
```

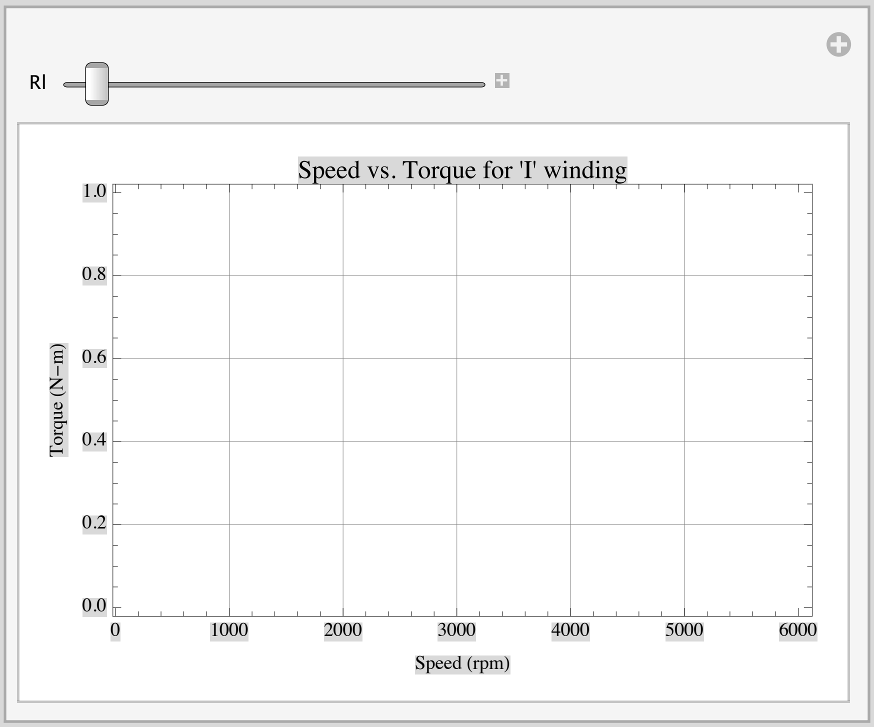
```
Rg5E = 0.15
```

```
0.15
```

```
1600 * 7.0615 * 10^-3
```

```
11.2984
```

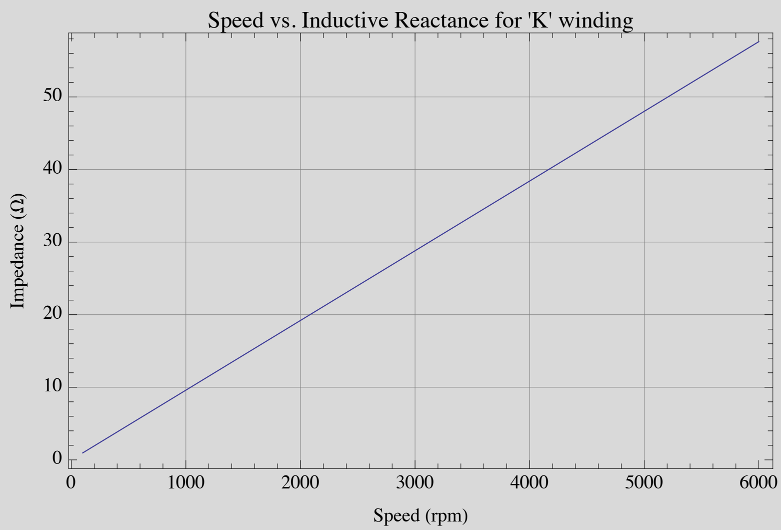
```
Manipulate[
 Show[{Plot[M[ω, Ke5E, Kt5E, Rg5E, Rl, Mf], {ω, 100, 6000},
    {Frame → True,
     GridLines → Automatic,
     PlotLabel → "Speed vs. Torque for 'I' winding",
     FrameLabel → {"Speed (rpm)", "Torque (N-m)"}}],
   Graphics[{Orange, Line[{{0, 11.3}, {6000, 11.3}}],
     Red, Line[{{0, 33.9}, {6000, 33.9}}]}]}],
 {Rl,
  0,
  100}]
```



Does the inductance of the motor windings matter? Plot inductive reactance as a function of speed.

```
Xl[ω_, L_] := (2 Pi ω / 60) L
```

```
Plot[Xl[ω, 0.0917], {ω, 100, 6000},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel → "Speed vs. Inductive Reactance for 'K' winding",
  FrameLabel → {"Speed (rpm)", "Impedance (Ω)"}}]
```




```
Plot[Xl[ω, 0.0059], {ω, 100, 6000},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel → "Speed vs. Inductive Reactance for 'I' winding",
  FrameLabel → {"Speed (rpm)", "Impedance (Ω)"}}]
```