# Maximizing Power Extracted from the Power Buoy Plate

P. McGill
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```
Clear[p, ρ, Cd, v, w, Pd]
```

This analysis assumes that both the buoy and submerged plate are rising, as power generated when the system is descending comes only from the stored spring energy and not from the plate. We can find the pressure on the rising plate with a standard drag equation if we make the simplifying assumptions that the buoy, rod, and plate are in line, that the water is not moving at the depth of the plate (which is reasonable for small waves in deep water), and we ignore inertial effects:

$$p = \frac{1}{2} \rho \, Cd \, (v - w)^2$$

$$\frac{1}{2} Cd \, (v - w)^2 \rho$$

where p is the pressure on the plate, $\rho$ is the density of seawater, Cd is the plate's coefficient of drag, v is the velocity of the buoy (away from the plate is positive), and w is the extension velocity of the rod (out is positive). Note that (v - w) is the speed of the plate through the water, e.g. if the buoy is rising at 1 m/s and the rod is extending at 1/2 m/s, then the plate is rising at 1/2 m/s.

Now we can find the power density in $W/m^2$ as the product of the plate pressure and the extension rate:

$$Pd = p\,w$$

$$\frac{1}{2} Cd \, (v - w)^2 \, w \, \rho$$

We can get some real numbers if we define the constants. This is a typical density for seawater in $kg/m^3$:

```
ρ = 1027
```

```
1027
```

At-sea tests have shown a plate Cd of about 7:

```
Cd = 7
```

```
7
```

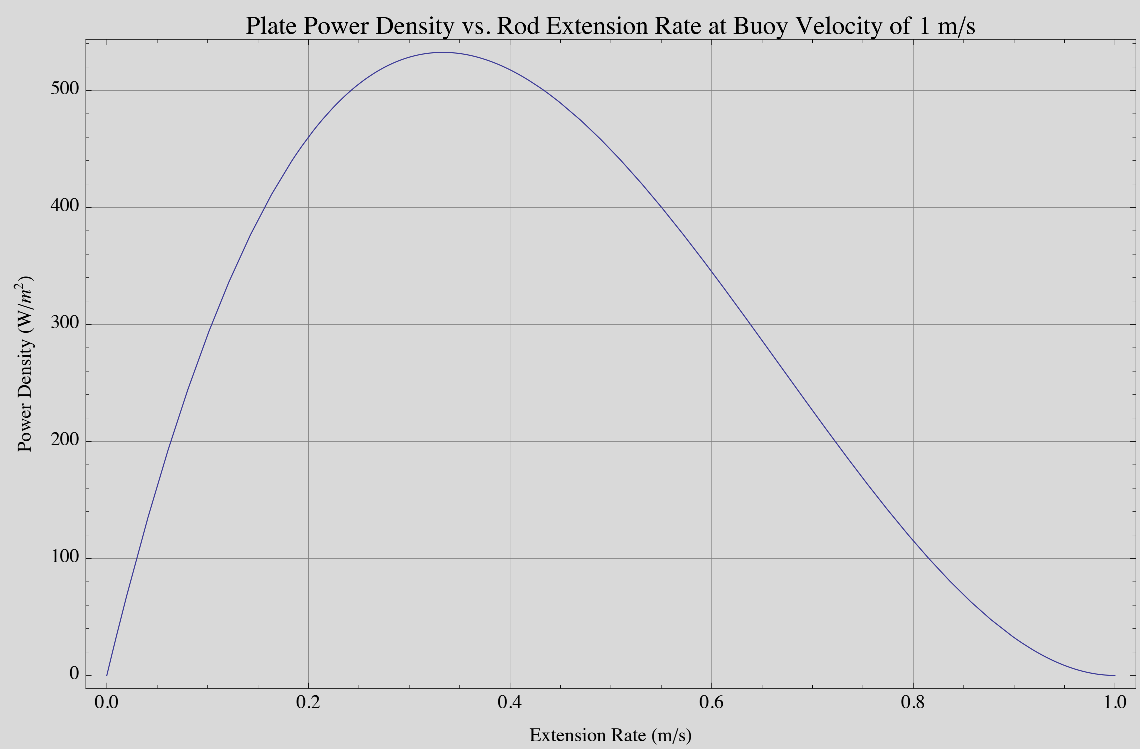
We'll assume a buoy velocity of 1 m/s:

```
v = 1
```

```
1
```

Now we can plot the power density as a function of extension rate:

```
Plot[Pd, {w, 0, 1},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel →
   "Plate Power Density vs. Rod Extension Rate at Buoy Velocity of 1 m/s",
  FrameLabel → {"Extension Rate (m/s)", "Power Density (W/m²)"}}]
```




There are at least two things to note here. First, the power density falls off to zero at the extrema where there is either no rod motion or no force. Second, the maximum values are skewed to lower extension rates, and so we probably want to operate with the rod moving somewhere between 1/5 to 1/2 of the buoy speed.

We can numerically find the maximum power density and its corresponding extension rate:

```
FindMaximum[Pd, {w, 0.1}]
```

```
{532.519, {w → 0.333333}}
```

A maximum of 532 $W/m^2$ is produced at an extension rate of $1/3\ m/s$ when the buoy is moving at $1\ m/s$.

We can also find an analytical solution by taking the derivative of Pd, setting it equal to zero, and solving for w:

```
Clear[v, ρ, Cd]
```

```
dPd = D[Pd, w]
```

$$\frac{1}{2}\,\mathrm{Cd}\,(v-w)^2\,\rho - \mathrm{Cd}\,(v-w)\,w\,\rho$$

```
Solve[dPd == 0, w]
```

$$\left\{\left\{w\to\frac{v}{3}\right\},\ \{w\to v\}\right\}$$

The maximum power density always occurs when the rod extension rate is one third of the buoy velocity, regardless of the values of $\rho$ and Cd. The second solution shown is the trivial case when there's no plate motion and does not represent a maximum.

This raises the question of what damping rate is required at a given buoy speed v to produce the desired $\frac{v}{3}$ extension rate. Let's define the maximum plate pressure, given that the plate is moving at 2/3 the velocity of the buoy, i.e. v - $\frac{v}{3}$:

$$\mathrm{pmax} = \frac{1}{2}\,\rho\,\mathrm{Cd}\left(\frac{2}{3}\,v\right)^2$$

$$\frac{2}{9}\,\mathrm{Cd}\,v^2\,\rho$$

Let's normalize to a 1 $m^2$ plate area so that the force in newtons is equal to the pressure in pascals :

```
f = pmax
```

$$\frac{2}{9}\,\mathrm{Cd}\,v^2\,\rho$$

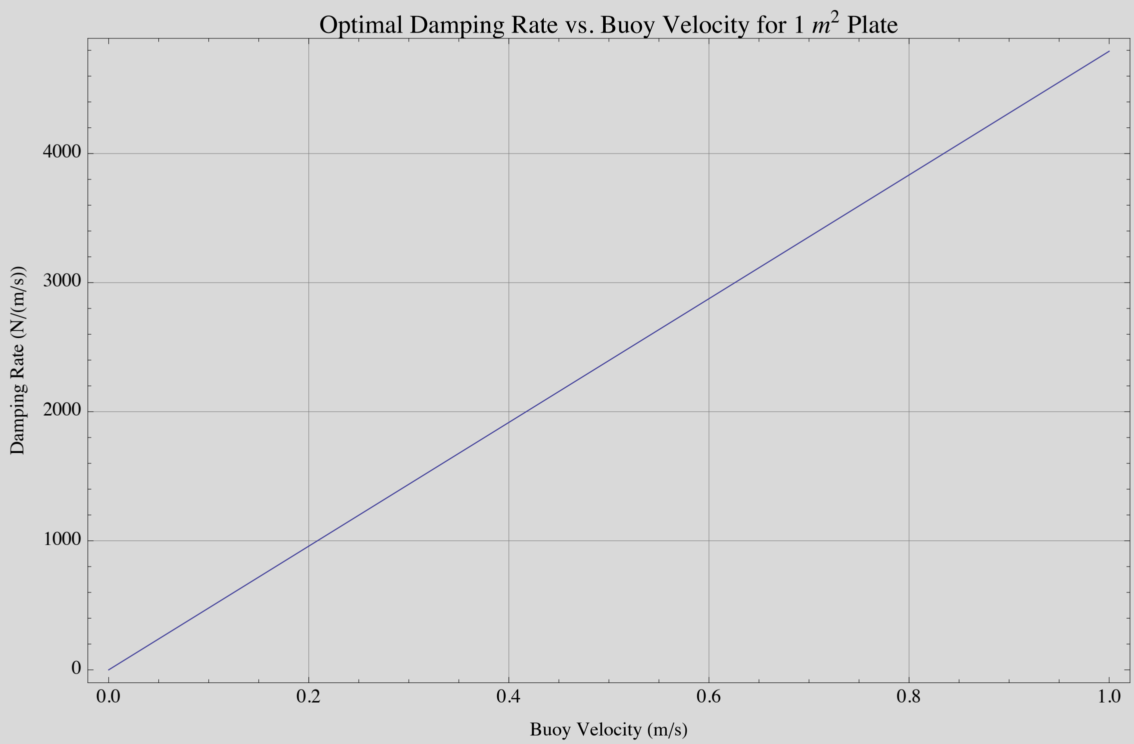
We can define the optimal damping rate as the force at maximum pressure divided by the optimal extension rate:

$$d = \frac{f}{\left(\frac{v}{3}\right)}$$

$$\frac{2\,\mathrm{Cd}\,v\,\rho}{3}$$

Now we can see how the optimal damping rate varies with buoy velocity, assuming the previous values of $\rho$ and Cd:

```
Plot[d /. {Cd -> 7, ρ -> 1027}, {v, 0, 1},
 {Frame → True,
  GridLines → Automatic,
  PlotLabel → "Optimal Damping Rate vs. Buoy Velocity for 1 m^2 Plate",
  FrameLabel → {"Buoy Velocity (m/s)", "Damping Rate (N/(m/s))"}}]
```




This is telling us that we want to increase the damping rate at higher buoy velocities in order to maximize the power extracted from the buoy/plate system. This damping rate will be controlled by the electrical load applied to the output of the PTO system. In practice, the value of the electrical load at any instant will be dictated by four things:

1) **The velocity of the buoy** according to the plot above.
2) **The position of the rod**, as increasing force will be applied by the pnuematic spring at greater extensions, requiring less force to be applied by the electrical load through the PTO. Most of the spring's stored energy will be recovered at a different force and velocity on the rod's return stroke.
3) **The shape of the efficiency curves** for not just the buoy/plate system, but the hydraulic piston/motor, the generator, and the power converter as well. It is the product of these four curves that will determine where we want to operate. We don't, for instance, want to extract the maximum amount of mechanical power from the buoy/plate system if it is produced at a force and velocity that is highly inefficient for the rest of the PTO systems.
4) **Inertial effects**. Plate acceleration forces will add or subtract from the forces applied by the electrical load and the spring, and will have to be estimated so that the total force achieves optimal damping. Fortunately, inertial forces will always increase the available power. When the system is rising, the plate's inertial force will add to the hydrodynamic force. This in turn requires a higher damping force to be applied by the electrical load, resulting in greater power recovery.