

# Ocean Surface Wave Measurements Using a Small Autonomous Underwater Vehicle

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**Abstract-** Small Autonomous Underwater Vehicle (AUV) capabilities are advancing rapidly and small AUV's are being used increasingly to collect many types of oceanographic data. Techniques have been developed, tested by wave simulations, and preliminarily field tested to provide ocean surface wave information, including directional wave spectra, using small AUV's. The techniques are adaptable to several small AUV's being developed and operated by the U.S. Navy. Either one or two small pressure sensors attached to an AUV's hull are utilized. Using two sensors along an AUV's bow-stern axis constitutes a moving slope array for one of the data analysis methods. After corrections for wave-induced vehicle motions measured by small accelerometers and angular rate sensors, three different data processing methods can provide wave information. The first method provides key wave height, period, and direction parameters. The second method treats the corrected wave data as a moving slope array. Both of these methods correct for Doppler shifts caused by an AUV's velocity while it maintains a nearly constant depth. The third method treats the wave motion-corrected wave data as an array of many point measurements made by non-stationary wave sensors with their horizontal positions known as functions of time. Doppler shifts do not occur due to this method's mathematical formulation. The required sensors and a data acquisition/processing system have been integrated into a small AUV used as a test vehicle and the system is being used to collect preliminary field data. The small AUV wave measurement system can be used for coastal wave mapping to support military operations, coastal processes studies, and driving nearshore models such as surf models and sediment transport models.

## I. INTRODUCTION

The U.S. Navy is making substantial investments to develop and operate small Autonomous Underwater Vehicles (AUV's) for many reasons including cost effectiveness, covertness, automated measurement capabilities, and avoidance of endangering personnel. AUV operations as well as many applications and operations for which AUV's will collect data are affected significantly by ocean surface waves. Development of capabilities to measure waves from small AUV's will provide important additional data for naval operations. AUV's with wave measurement capability will also support research including coastal processes and beach erosion studies, model initializations and validations, and remote sensing algorithm validations. A mobile AUV-based wave measurement system could map waves over much larger areas than can be covered with conventional fixed in-situ instrumentation. Spatially dependent coastal processes models such as surf models could be initialized with system data.

A characteristic of small AUV's that enhances their

operational utility is their relatively small size. Typical small AUV's are approximately 2 m long and have diameters up to roughly 0.7 m. Their small size, combined with their design for various missions and incorporation of planned sensors, limits the size, weight, and power budget of new measurement systems that may be added. Also, there are limits on relaying large data volumes, especially in real-time. Thus, onboard data processing of voluminous data sets is useful so that relayed information is more compact. Because ocean waves need to be sampled fairly rapidly for reasonable time periods, onboard data analysis is desirable for wave data.

Using small, and low power, pressure sensors is an attractive approach for measuring waves with small AUV's. These sensors can be incorporated into vehicles that have little room for adding larger, or more power consuming, sensors. Basing wave data analysis on subsurface wave pressure data, rather than subsurface wave velocity data, also is less prone to errors caused by vehicle motion because pressure is a scalar while velocity is a vector. As for any data measured in a vehicle reference frame, the data should be corrected for wave-induced vehicle motions. These corrections are made by using measured vehicle vertical displacement, pitch, roll, and heading with the geometry of the pressure sensor placement. Many AUV's make these measurements for use with their control systems and these data can be utilized if they are time-tagged with the pressure data. A wave module consisting of angular rate sensors and accelerometers, along with a microprocessor, can also collect these data, make corrections, and perform wave analyses.

For operational missions with real-time data requirements, onboard data processing is needed. For research applications, post-processing after an AUV is recovered may be suitable. Estimating directional wave spectra is rather complicated and even estimating non-directional wave spectra is reasonably complicated. Three data analysis approaches have been developed and tested by simulations. The small AUV to be used as a test vehicle to evaluate these approaches with field data was delivered in July, 2002, and is providing initial field data. Descriptions of the analysis methods' theory, an overview of the test vehicle, and initial results from preliminary field data are provided (in the presentation).

## II. THEORY

### A. A Method for Estimating Wave Height, Period, and Direction

A direct method for estimating wave height, period, and direction parameters is based on the apparent zero up-crossing wave period sequence and the corresponding sequence of AUV

travel directions. Only one pressure sensor is needed. The vehicle's speed is assumed nearly constant and almost all travel directions are assumed to be included in the vehicle's path. The method works well with easily executed circular paths, but it should work well with any path, including irregular ones, that sample all directions.

Due to Doppler shifting, a wave's apparent frequency,  $f_a$ , is related to its actual frequency,  $f$ , by

$$f_a = f - \frac{kv}{2\pi} \cos(\phi - \theta) \quad (1)$$

where  $k$  is wave number,  $v$  is vehicle speed,  $\phi$  is vehicle travel direction, and  $\theta$  is wave direction. Apparent frequencies are obtained from the reciprocals of zero up-crossing periods. A graph of  $f_a$  values versus simultaneous  $\phi$  values approximates a cosine oscillation about  $f$  with minima and maxima at  $\theta$  and  $\theta \pm \pi$ . Differences between  $f_a$  values at minima and maxima estimate  $f$ .

A simple approach is to estimate  $f$  from the mean of these differences and to estimate  $\theta_0$  from the means of its values calculated at the minima and maxima. For a circular path, more than one circle can be executed to increase statistical confidence.

A better least-squares approach considers all the apparent wave periods and simultaneous vehicle headings. Equation (1) can be written as

$$f_a = f - \frac{kv}{2\pi} [\cos(\theta)\cos(\phi) + \sin(\theta)\sin(\phi)] \quad (2)$$

which is of the functional form

$$f_a = a \cos(\phi) + b \sin(\phi) + c \quad (3)$$

with

$$\begin{aligned} a &= -\frac{kv}{2\pi} \cos(\theta) \\ b &= -\frac{kv}{2\pi} \sin(\theta) \\ c &= f \end{aligned} \quad (4)$$

The least-squares solution for  $a$ ,  $b$ , and  $c$  is obtained in matrix notation by minimizing

$$Q = |f_a - X\beta|^2 \quad (5)$$

where

$$f_a = \begin{bmatrix} f_{a1} \\ f_{a2} \\ f_{a3} \\ \vdots \\ f_{aJ} \end{bmatrix} \quad (6)$$

$$X = \begin{bmatrix} 1 & \cos(\phi_1) & \sin(\phi_1) \\ 1 & \cos(\phi_2) & \sin(\phi_2) \\ 1 & \cos(\phi_3) & \sin(\phi_3) \\ \vdots & \vdots & \vdots \\ 1 & \cos(\phi_J) & \sin(\phi_J) \end{bmatrix} \quad (7)$$

$$\beta = \begin{bmatrix} c \\ a \\ b \end{bmatrix} \quad (8)$$

The vector  $\beta$  is obtained from the matrix least-square formula

$$\beta = (X^T X)^{-1} X^T f_a \quad (9)$$

where the superscript,  $T$ , denotes the matrix transpose. From the solution for  $\beta$

$$\begin{aligned} f_o &= c \\ \theta_o &= \arctan(b/a) \end{aligned} \quad (10)$$

where subscripts are used to indicate that these values are representative, or primary, values for all waves. Because  $f_o$  is estimated from zero up-crossings, it corresponds to the standard zero-crossing (often called mean) wave period computed from a non-directional wave spectrum's zero and second moments. By analogy,  $\theta_o$  is assumed to correspond to the standard mean wave direction estimated by vector averaging frequency dependent mean wave directions over a wave spectrum. Simulations show that these correspondences are approximately correct.

Wave numbers,  $k$ , that are needed to correct individual crest to trough wave heights (at vehicle depth) from the zero up-crossing analysis for pressure attenuation are estimated using the wave dispersion relationship given by

$$(2\pi f)^2 = gk \tanh(kd) \quad (11)$$

where  $g$  is the acceleration due to gravity and  $d$  is mean water depth during the measurement time period. Combining (1) and (11) provides the following function,  $H(k)$ , that is minimized, using a Newton's iteration, to estimate  $k$  for each apparent wave interval.

$$H(k) = [2\pi f_a + kv \cos(\phi - \theta_o)]^2 - gk \tanh(kd) \quad (12)$$

Crest to trough wave heights (at vehicle depth) are corrected to surface heights by dividing by the pressure attenuation factor,  $K_p$ , given by

$$K_p = \frac{\cosh[k(z + d)]}{\cosh(kd)} \quad (13)$$

in which  $z$  is mean vehicle depth below the surface, and  $d$  is mean bottom depth. Significant wave height is estimated by averaging the highest one-third waves.

Because wave zero-crossing analyses exhibit an artificially large number of very high frequency waves near zero-crossings, the lowest 90% of the apparent frequencies and corresponding vehicle directions and heights were utilized in applying this method.

The three parameters obtained from this method are probably the most widely used for practical applications. The calculations are simple and lend themselves to real-time onboard use. The primary assumption in using these parameters is that waves are relatively narrow-banded in frequency and direction.

#### B. A Moving Slope Array Method for Estimating Directional Wave Spectra

This method involves mounting pressure sensors near the bow and stern of a small AUV and treating the data as a moving one-dimensional wave slope array. Because of low signal to noise ratios across a small AUV's beam, sensors on the starboard and port sides are not used. The vehicle is operated along reasonably short (e.g. 128 s at a speed of about 1 m/s) straight line paths with planned headings and cross-spectral analysis is performed using Fast Fourier Transforms (FFT's) along each path. Assuming wave conditions are stationary during the total measurement time for multiple paths, cross-spectral results from the paths are combined to provide directional wave spectra similar to those from a pitch-roll-heave buoy.

Consider an AUV that transits an approximate square with sides parallel to horizontal orthogonal axes,  $x$  and  $y$ . Data are excluded from analysis while the vehicle is turning corners. Frequency dependent mean wave direction [e.g. 1] toward which waves travel measured counter-clockwise from the  $x$  axis is given by

$$\theta_1 = \tan^{-1} \left( \frac{Q_{ppy}}{Q_{ppx}} \right) \quad (14)$$

where  $Q_{ppx}$  and  $Q_{ppy}$  represent the quadrature spectra between one of the pressure sensors and the pressure slopes (computed from the pressure differences between the sensors),  $p_x$  and  $p_y$ , in the  $x$  and  $y$  directions. For a square, there are a pair of estimates for both  $Q_{ppx}$  and  $Q_{ppy}$  providing two estimate of  $\theta_1$ . There are four estimates of the pressure spectrum,  $C_{pp}$ . Using the mean of  $\theta_1(f)$  as an initial estimate of wave direction, these eight spectra are corrected for Doppler shifts. This is done by shifting the lower and upper frequencies of each frequency band to their corrected values using (1) and adjusting spectral densities to maintain constant energy. The corrected spectra with varying frequencies are interpolated to the original Fourier frequencies. This process is iterated several times (typically three).

An estimate of the directional spreading function,  $D(\theta)$ , for wave energy assuming a uni-modal directional distribution at each frequency is often assumed to have the functional form

$$D = \cos^{2s} \left( \frac{\theta - \theta_1}{2} \right) \quad (15)$$

where  $s$  is a spreading parameter given by

$$s = \frac{(Q_{ppx}^2 + Q_{ppy}^2)^{1/2}}{kC_{pp} - (Q_{ppx}^2 + Q_{ppy}^2)^{1/2}} \quad (16)$$

$D(\theta)$  is normalized to equal unity when integrated over 0 to  $2\pi$ . The non-directional wave spectrum computed from the mean of the four  $C_{pp}$  estimates is multiplied by the normalized  $D(\theta)$  to provide the directional wave spectrum. The frequency dependent non-directional and directional spectra (at depth) are divided by the frequency dependent  $K_p^2$  using (13) to provide spectra for surface elevations.

This method can also be applied to "figure eight" type paths where utilized approximately straight line segments constitute a cross. As long as the vehicle stays within a region within which wave conditions are homogeneous, there is no need for the utilized paths to form closed figures as long as they are appropriately perpendicular.

A more complicated generalization of this method has been developed for triangular type patterns to provide the first five directional Fourier coefficients that are commonly calculated for pitch-roll-heave buoys [e.g. 1], but the Doppler shift corrections are not as rigorous. This often used truncated Fourier series does not provide directional spreading functions that are significantly narrower than those from (15).

#### C. A General Method for Estimating Directional Wave Spectra from Non-Stationary Wave Sensors

A general method has been developed [2] for estimating directional wave spectra when the times and horizontal positions of an arbitrary number of wave elevation, or pressure, measurements are known. These data can be provided by an AUV executing various paths within a region in which wave conditions are assumed homogeneous. The mathematics is

extensive so that a summary is provided here. The measurements at  $I$  sensors (either one or two pressure sensors for a small AUV), each associated with a horizontal (x,y) position and time, for the sum of  $J$  long-crested waves with frequency,  $f_j$ , and direction,  $\theta_j$ , is given by the matrix equation

$$S = X \beta \quad (17)$$

where

$$X = \begin{bmatrix} T^c(f_1, \theta_1), & T^s(f_1, \theta_1) & \dots \\ T^c(f_J, \theta_J), & T^s(f_J, \theta_J) & \dots \end{bmatrix} \quad (18)$$

and

$$\beta = \begin{bmatrix} U_1 \\ V_1 \\ \vdots \\ U_J \\ V_J \end{bmatrix} \quad (19)$$

$S$  is a column vector of the  $I$  sensors' time series,  $X$  is a row vector that is a function of  $J$  wave frequencies and directions, and  $\beta$  is a column vector of unknown regression coefficients. For solution, (17) is written as

$$Y = X\beta + \epsilon \quad (20)$$

where  $\epsilon$  is an error term. A least-squares solution is found to minimize

$$Q = \epsilon^2 = |Y - X\beta|^2 \quad (21)$$

Modern regression theory techniques are needed to obtain solutions for the large matrices that occur [2]. A computational method based on searches over selected frequency and direction ranges, selecting and culling frequency and direction combinations, ridge regression, and stepwise regression works well with simulated data on today's personal computers. Because the method is fairly computationally intensive and involves analysis decisions, this method may be better suited for post-processing data after an AUV has been recovered or it relays its raw data..

Doppler shifts do not occur for this method and there may be potential for increased directional resolution when extensive data are collected over a region in which waves are homogeneous. As for the other methods, an implicit assumption is that wave conditions are stationary during the time that data are collected. Non-directional wave spectra and wave parameters (e.g. those in Method A) are easily calculated from the directional spectra. As for Method B, this method is applied to wave motion-corrected pressure sensor data at depth and final spectra are converted to surface elevation spectra by multiplying by the reciprocal of  $K_p^2$

where  $K_p$  is defined in (13).

### III. TESTS AND VERIFICATIONS

#### A. Simulations

The methods have been tested with realistic simulated data. Simulated data have been based on superimposing limited numbers of sinusoidal wave components with varying heights, periods, and directions as well as superimposing many sinusoidal wave components representing spectra (e.g. Pierson-Moskowitz spectra) and various directional spreading functions. Results are not provided here due to space limitations. Example simulations for Method C are provided in [2]. While more testing is needed, it appears that uncertainties of the methods are not greater than uncertainties associated with natural wave sampling variability.

#### B. Field Data

Simulations do not fully consider how well the methods work with actual field data and its various imprecisions including sensor errors, non-exact corrections of pressure sensor data for wave-induced vehicle motions, and other uncertainty sources (e.g. variations in vehicle speed and heading). Other possible factors include flow-induced pressure variations. Their affects can be minimized by operating the AUV at low speed while it is measuring waves and by modifying the pressure port orifices.

To test the methods with field data, a test vehicle has been instrumented to collect needed data including vehicle motion data and pressure data near the AUV's bow and stern. The test vehicle, Fetch II, was delivered by Sias Patterson, Inc. in July, 2002. This vehicle is 1.7 m long, has a diameter of 0.4 m, and weighs 80 kg. Its size and weight is similar to that of small AUV's to be procured by the U.S. Navy. Fig. 1 shows the test vehicle.



Fig. 1. Wave measurement test vehicle.

Vehicle motion and pressure data are acquired by a Wave Measurement Module (WMM). The small module includes a microprocessor (Onset Computer Tattletale 8), a tri-axial accelerometer (Crossbow Model CXL02TG3), and three mutually orthogonal angular rate sensors (Murata Model ENV-05DB-52). The WMM acquires data from the motion sensors and the two pressure sensors (Pressure Systems 35T Series) at a sampling rate of 4 Hz. These data are stored in compact flash memory and are

relayed via an RS-232 port to the vehicle's PC-based main data acquisition system that also stores vehicle navigation information. Another RS-232 port with a connector external to the AUV allows users to program the WMM and transfer data from it. The WMM is in a pressure housing near the vehicle's midpoint. Fig. 2 shows the internal components of the WMM.

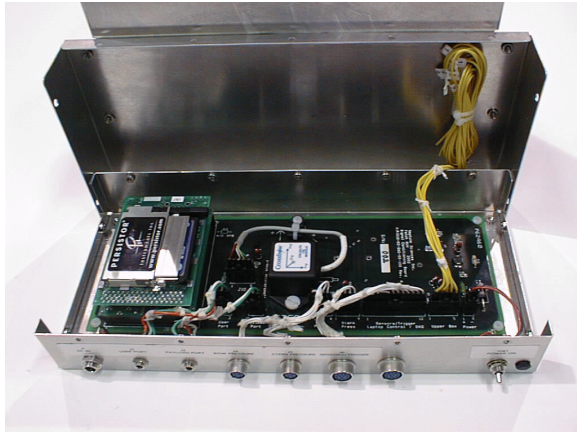


Fig. 2. Wave Measurement Module (WMM).

The WMM has been integrated into the test vehicle and preliminary field tests have been performed in Lake Ponchartrain near NSI's main office. Waves in this estuary are low, short, and choppy. These tests were made mainly to test the vehicle, including executing the paths described above, and the WMM.

Data more typical of open ocean coastal wave conditions should be collected during August - September, 2002, in the Gulf of Mexico. Preliminary results from analyzing these data will be provided at Oceans 2002.

#### IV. SUMMARY

Small AUV's will be used to support naval operations and research particularly in coastal waters. Ocean surface waves are one of the most important environmental phenomena affecting littoral naval operations and coastal processes. Due to the high spatial and temporal variability of nearshore waves, small AUV's are well-suited to serve as mobile wave measurement systems. But, their small size imposes various constraints (e.g. size, weight, and power) on the types of wave measurement systems that can be

integrated into them. Small pressure sensors can be fairly readily accommodated and measurement of pressure, a scalar, has advantages over more complicated measurements such as measurement of velocity, a vector.

Three analysis methods have been developed and preliminarily tested. Only Method B (moving slope array) requires using two pressure sensors. Method A (wave parameters) can clearly be implemented in real-time onboard small AUV's. Method B could also be implemented in this manner. Method C (complete non-stationary sensor approach) is likely best suited for post-processing data, but it provides the most complete directional wave spectra. Because a moving AUV is equivalent to many wave sensors, Method C may provide higher directional resolution than that obtained from existing fixed sensor analyses. These methods can also be applied using data from a narrow beam upward looking acoustic sensor (or sensors for Method B) in place of a pressure sensor (or sensors).

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