Reliability Analysis of Accelerated Life-Test Data from a Repairable System

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Key Words — **Accelerated testing, Nonhomogeneous Poisson process, Regression model, Maximum likelihood estimation**

Summary & Conclusions — **This paper analyzes accelerating testing of a repairable item modeled by a nonhomogeneous Poisson process with covariates. We extensively analyze a single accelerating variable with 2 stress levels, and derive closed-form maximum likelihood (ML) solutions. These closed-form solutions provide: 1) an easier way to obtain point estimates of the unknown parameters under usual operating conditions, and 2) a way to obtain confidence intervals on the process parameters and function thereof which are more accurate than those based on asymptotic normality of ML estimates. We analyze the circumstances in which a drawback to closed-form estimation arises, and guide the extent that our procedures may equally apply. An example application drawn from a real situation of accelerated testing is presented, and numerical estimates based on our procedures are derived & discussed. Theoretical & simulation results show that estimation procedures based on the power-law process and regression methods can be a flexible, useful tool for reliability analysis of a repairable item.**

1. INTRODUCTION

Tests for reliability assessment often require an unacceptable amount of time when performed under usual operating conditions. Thus, it is common to subject the item under study to stresses which are more severe than usual, in order to shorten life and/or accelerate performance degradation. When quantitative reliability evaluation under usual conditions is of interest, methods are required to extrapolate from data obtained under higher stresses. An extensive literature exists for non-repairable items which addresses the problem above; for a recent, comprehensive treatment see Nelson [17].

However, we are not aware of any paper which specifically addresses accelerated testing of *repairable* items, although there are many papers, in engineering & biology, which use models pertinent to this problem in the context of heterogeneous data analysis, *eg*, Cox [6, 7], Prentice *et al* [18], Ascher [2], Dale [9], Lawless [16]. In that context, the basic approach is to model the failure pattern of an engineering or biological item by a stochastic point process and assume that the process intensity is not only a function of time (or more generally a function of the history up to that time), but is also affected by *concomitant* variables (possibly depending on history). Through these variables, differences in operating conditions are taken into account, thus enabling heterogeneous data to be analyzed on a common basis. Since concomitant (regressor) variables are involved, these are commonly referred to as *regression* models. In the framework of heterogeneous data analysis, regression models have been mainly considered as a tool with which to evaluate whether some factors affect system reliability. Consequently the interest has been primarily focused on the estimation of regression coefficients.

This paper assumes a nonhomogeneous Poisson process for the failure pattern, and uses regression models as a tool to analyze accelerated testing of a repairable system. Thus, attention is essentially focused on estimators of process parameters under usual conditions and some functions thereof useful for reliability evaluations. The statistical properties of these estimators are analyzed on both a theoretical basis and by Monte Carlo simulation.

Acronyms

AT	accelerated time (regression model)
PI	proportional intensity (regression model)
NHPP	nonhomogeneous Poisson process
PLP	power-law process
ML	maximum likelihood.

Notation

$\boldsymbol{x}$	covariate vector (of conditions)
$\boldsymbol{a}$	parameter vector
$g(\boldsymbol{x},\boldsymbol{a})$	a positive-valued function
t	operating time
β, α	[shape, scale] process parameters
$\alpha(\boldsymbol{x})$	scale parameter under $\boldsymbol{x}$
$\upsilon(t\|\boldsymbol{x})$	intensity function under $\boldsymbol{x}$
$\Upsilon(t\|\boldsymbol{x})$	cumulative intensity function under $\boldsymbol{x}$
$T^{(k)}$	failure-time k, r.v.
$Y^{(k)}$	$\ln(T^{(k)})$, r.v.
m	number of stress levels
N, n	number of failures [r.v., fixed or observed value]
$t_{i,j,l}$	failure-time l for system-copy j under stress-level i
$\tau_{i,j}$	fixed test time for system-copy j under stress-level i
$T_{i,j}$	$[t_{i,j,n_{i,j}}, \tau_{i,j}]$: for [failure, time] censored case
ξ	extrapolation factor
$\omega_\xi(\theta_i)$	$\theta_1^\xi \cdot \theta_2^{1-\xi}$
$e^{(k)}$, $e_p^{(k)}$	loggam$(\cdot;k)$ [r.v., p quantile]
$y_p^{(k)}$	p quantile of $Y^{(k)}$
$\hat{}$	implies a ML estimate/estimator
$_0$	implies a baseline quantity.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

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Assumptions

1. The system is repairable, *ie*, it can be restored to satisfactory performance after a failure, by the substitution of only a part of it. Satisfactory performance is bad-as-old, *viz*, a repaired failure-intensity equal to the failure intensity just before failure.
2. Reliability characteristics under usual use conditions are to be estimated on the basis of failure data obtained under higher stresses.
3. The failure process is a NHPP with a power-law intensity function with respect to operating time.
4. Intensity functions under various stress levels are proportional to one another.
5. The log(intensity) is a linear function of the stress.
6. $g(\boldsymbol{x};\boldsymbol{a}) = \exp(\boldsymbol{a}^T\cdot\boldsymbol{x})$.
7. Under stress x_i, the failure pattern can be modeled by a PLP with intensity function:

$$v(t|x_i) = v_0(t)\cdot\exp(\beta\cdot a\cdot x_i)$$

$$= (\beta/\alpha_0)\cdot(t/\alpha_0)^{\beta-1}\cdot\exp(\beta\cdot a\cdot x_i),\ i=0,1,..,m;$$

data are not available for baseline stress $\boldsymbol{x}_0=0$. ◀

2. THE MODEL

The usual simplifying approach to reliability analysis of a repairable system is to treat the item as a black-box, and model its failure pattern by a stochastic point process, $N(t)$ say, assuming that repair times are negligible or are measured on a different time scale than operating time.

When heterogeneous processes are involved, it is desirable to consider the relationship of the failure process to explanatory factors. One way to do this is through regression models in which the failure point-process has an intensity which is a function of both time and concomitant variables. Let,

$$v(t) = \lim_{\Delta\to 0}(\Pr\{N(t-\Delta,t] \geq 1\}/\Delta); \quad (1)$$

$$\Upsilon(t) = \int_0^t v(u)\ du; \quad (2)$$

(2) coincides with the mean number of failures in $(0,t]$ if simultaneous arrivals cannot occur.

Extending to a point process the well-established concepts developed for non-repairable items [6, 13, 15], two basic approaches, which are physically plausible and provide sufficient flexibility, can be conceived in formulating the dependence of the failure process on the covariates.

1. The PI approach assumes:

$$\Upsilon(t|\boldsymbol{x}) = \Upsilon_0(t)\cdot g(\boldsymbol{x};\boldsymbol{a}), \quad (3)$$

ie, the covariates act multiplicatively on the baseline cumulative intensity (or equivalently on the baseline intensity). This implies that the ratio of the intensity functions for two items with regression vectors $\boldsymbol{x}_1$ & $\boldsymbol{x}_2$ does not depend on t. Thus, different items have intensity functions which are proportional to one another.

2. The AT approach assumes that the covariates act multiplicatively on operating time t:

$$\Upsilon(t|\boldsymbol{x}) = \Upsilon_0(g(\boldsymbol{x};\boldsymbol{a})\cdot t), \quad (4)$$

ie, the mean number of failures for an item, under $\boldsymbol{x}$ in a time interval $(0,t]$ is the same as that for the item under baseline conditions in the time interval $(0,g(\boldsymbol{x};\boldsymbol{a})\cdot t]$. ◀

More-complex approaches which assume the process intensity depends on the history up to time t, and time-varying covariates could also be exploited [18], but they are not considered here.

In a fully parametric approach to regression analysis, the baseline intensity (or cumulative intensity) needs to be specified up to a vector of parameters. A useful mathematical model for describing the failure point process of a repairable system is the NHPP (a Poisson process with a time varying intensity function) [3, 8]. This model implies that a repair restores the initial value of the performance specifications, but, unlike the renewal process, a repair returns the reliability of the system to the level it was in just before the occurrence of the failure and not to the initial value.

The most popular form of the intensity of a NHPP is:

$$v(t) = (\beta/\alpha)\cdot(t/\alpha)^{\beta-1},\ \alpha,\beta > 0;\ t \geq 0. \quad (5)$$

In recent literature a NHPP with intensity (5) is usually referred to as a PLP [19].

Regression methods based on proportional intensity assumption and NHPP (in the special form of PLP, too) were analyzed in [16].

The PLP model has the useful property that, in formulating the dependence of the failure intensity on the covariates, the PI & AT approaches coincide. In terms of process parameters, either assumption implies that the effect of the $\boldsymbol{x}$ is to alter the scale parameter α, while leaving the shape parameter β unchanged. Thus, for a PLP,

$$\Upsilon(t|\boldsymbol{x}) = [t/\alpha(\boldsymbol{x})]^{\beta}, \quad (6)$$

$$\alpha(\boldsymbol{x}) = \alpha_0/g(\boldsymbol{x};\boldsymbol{a}). \quad (7)$$

The PLP model and PI (or AT) assumption also imply that:

$Y^{(k)} = \ln(T^{(k)})$, $(k=1,2,\dots)$, given $\boldsymbol{x}$, has a log-Gamma distribution [15], by a location parameter $\mu(\boldsymbol{x})$, a *constant* scale parameter σ, and by k:

$$Y^{(k)} = \mu(\boldsymbol{x}) + \sigma\cdot e^{(k)}, \quad (8)$$

$$\mu(\boldsymbol{x}) \equiv \ln\alpha(\boldsymbol{x}),\ \sigma \equiv 1/\beta,$$

Thus, the model is completely specified but for $g(\boldsymbol{x};\boldsymbol{a})$. In principle, many functions can be used for $g(\boldsymbol{x};\boldsymbol{a})$. However, the usual choice is to assume:

$$g(\boldsymbol{x};\boldsymbol{a}) = \exp(\boldsymbol{a}^T\cdot\boldsymbol{x}), \tag{9}$$

(see assumption #6) since this model is reasonable and sufficiently flexible for many purposes, and guarantees $\upsilon(t|\boldsymbol{x})$ to be automatically nonnegative for all $\boldsymbol{x}$ & $\boldsymbol{a}$ [1, 6, 13, 15, 16].

Although conceived for different purposes, the previous regression model can also be viewed as a tool to *extrapolate* reliability characteristics under conditions the item has not been subjected to. Thus, the model is potentially useful in the reliability analysis of a repairable system subjected to accelerated testing. The model enables the estimation of the intensity function under use conditions from failure data obtained under higher stresses, provided that the relationship (6) between the cumulative intensity and $\boldsymbol{x}$ of (possibly transformed) accelerating variables is valid in the range of both actual & baseline values of $\boldsymbol{x}$.

When no extrapolation is involved and there are sufficient observations for each $\boldsymbol{x}_i$, a check on model (6) can be made by computing $\hat{\Upsilon}(t|\boldsymbol{x}_i)$ from the data at each $\boldsymbol{x}_i$. Taking logarithms, (6) gives:

$$\ln(\Upsilon(t|\boldsymbol{x})) = \beta\cdot\ln(t) - \beta\cdot\ln(\alpha(\boldsymbol{x})), \tag{10}$$

so that plots of $\ln(\hat{\Upsilon}(t|\boldsymbol{x}_i))$ *vs* $\ln(t)$ should be roughly parallel straight lines if the model (6) is appropriate. When extrapolation is involved, however, only a partial check can be made, and some caution is needed when no experience exists on the appropriateness of the model. At a minimum, one should always estimate the statistical uncertainty in the results, given that the model is true (this will often be large).

3. MAXIMUM LIKELIHOOD ESTIMATION

In [16], ML estimation for a PLP in a multivariate regression framework and time-censored sampling was considered. Numerical methods for point estimation are usually necessary, and interval estimation is based on approximate s-normality of the ML estimator.

Here, we extensively analyze the case of a single (possibly transformed) accelerating variable, under both time- and failure-censored sampling.

At each constant stress level x_i ($i = 1,\ldots,m$), J_i repairable systems are tested, and failure time data $t_{i,j,l}$ ($j=1,\ldots,J_i$; $l=1,\ldots,n_{i,j}$) are observed, either under failure-censored sampling with fixed number of failures $n_{i,j}$ or under time-censored sampling with fixed $\tau_{i,j}$. Assumption #7 implies:

$$L = \prod_{i=1}^{m}\prod_{j=1}^{J_i}\left[\prod_{l=1}^{n_{i,j}}\upsilon(t_{i,j,l}|x_i)\right]\cdot\exp[-\Upsilon(T_{i,j}|x_i)]$$

$$= \beta^n\cdot\alpha_0^{-\beta\cdot n}\cdot\exp\left[-\sum_{i=1}^{m}\sum_{j=1}^{J_i}(T_{i,j}\cdot\exp(a\cdot x_i)/\alpha_0)^{\beta}\right]$$

$$\cdot\prod_{i=1}^{m}\exp(n_i\cdot\beta\cdot a\cdot x_i)\cdot\prod_{j=1}^{J_i}\prod_{l=1}^{n_{i,j}}t_{i,j,l}^{\beta-1};$$

$$n_i = \sum_{j=1}^{J_i} n_{i,j} = \text{total number of failures under stress level } x_i,$$

$$n = \sum_{i=1}^{m} n_i = \text{total number of failures.}$$

Closed-form solutions are not generally unavailable for $\hat{\alpha}_0$, $\hat{\beta}$, $\hat{a}$; thus numerical methods are required.

However, when $m=2$, closed form ML solutions can be derived. These closed form expressions provide not only an easier way to obtain point estimates of the unknown parameters, but they provide a way to obtain s-confidence intervals which are more accurate than those based on asymptotic s-normality of ML estimates. Sections 3.1 & 3.2 are devoted to this analysis.

3.1 Testing 1 System At Each Stress Level

In the accelerated life-testing framework, it is common to test under 2 levels of the accelerating variable, *viz*, x_1 (low stress) and x_2 (high stress). In this context, the simplest experimental situation arises when only 2 copies of a system are involved, one tested under x_1, and the other under x_2. A closed-form ML solution can be obtained (for both failure- and time-censored sampling) by parameterizing the likelihood function in terms of $\alpha_1 = \alpha(x_1)$, $\alpha_2 = \alpha(x_2)$, *and* β (which gives closed-form solutions for all these parameters) and by realizing that α_0 & a are related to α_1 & α_2 through (7). Because $j\equiv 1$ in this section, the index j is suppressed. Thus, due to the known property of ML estimation:

$$\hat{\beta} = n/\left[\sum_{i=1}^{2}\sum_{l=1}^{n_i}\ln(T_i/t_{i,l})\right], \tag{11}$$

$$\hat{\alpha}_0 = \omega_{\xi}(T_i/n_i^{1/\hat{\beta}}), \tag{12}$$

$$\hat{a} = \ln[(T_1/T_2)\cdot(n_2/n_1)^{1/\hat{\beta}}]/(x_2-x_1), \tag{13}$$

$$\xi = x_2/(x_2-x_1).$$

ξ is the *extrapolation factor* which measures how far the baseline-stress is from the highest-stress, as a multiple of test range.

3.1.1 Interval estimation for β

For failure-censored sampling,

$$2n\cdot(\beta/\hat{\beta}) \sim \chi^2[2(n-2)],$$

and under time censored sampling, conditionally on $N = N_1 + N_2 = n$,

$$2n \cdot (\beta/\hat{\beta}) \sim \chi^2[2n].$$

Thus, *s*-confidence intervals for β, based on χ^2 distribution, are readily available. Unbiased estimators of β are easily obtained.

3.1.2 Interval estimation for α_0

For failure-censored sampling, (11) & (12) show that the estimate of α_0 depends on stress levels only through ξ. Further, the statistical properties of $\hat{\alpha}_0$ depend on α_0 & β. However, for any fixed ξ,

$$W \equiv (\hat{\alpha}_0/\alpha_0)^{\beta} = \omega_{\xi}(U_i)^{n/V}/\omega_{\xi}(n_i), \tag{14}$$

$$V \equiv n \cdot \beta/\hat{\beta};$$

$$U_i \equiv [t_{i,n_i} \cdot \exp(a \cdot x_i)/\alpha_0]^{\beta}, \; (i = 1,2).$$

W is pivotal (its distribution does not depend on process parameters), since U_1, U_2, V are *s*-independent standard Gamma r.v. with parameters n_1, n_2, $(n-2)$, respectively.

Thus, *exact* *s*-confidence intervals on α_0 can be obtained, although they are calculated by multiple numerical integration or by simulation. This is not a strong limitation today since, with simple statistical software available, the computation by simulation of percentage points $w_{\gamma}(n_1,n_2,\xi)$ such that,

$$\Pr\{(\hat{\alpha}_0/\alpha_0)^{\beta} \le w_{\gamma}(n_1,n_2,\xi)\} = \gamma, \tag{15}$$

requires only a few seconds on a personal computer.

Alternatively, *s*-confidence intervals on α_0 based on *s*-normal approximation are given in appendix A.1, where their accuracy is also analyzed.

For time-censored sampling, a method for constructing *s*-confidence intervals on α_0 is not available which is completely free of nuisance parameters. For failure-censored sampling, (14) shows that,

$$Q \equiv [\omega_{\xi}(n_i) \cdot (\hat{\alpha}_0/\alpha_0)^{\beta}]^{1/n} = \omega_{\xi}(U_i)^{1/V}, \tag{16}$$

is a pivotal quantity.

For time-censored sampling, a quantity analogous to (16) is:

$$Q' \equiv [\omega_{\xi}(N_i) \cdot (\hat{\alpha}_0/\alpha_0)^{\beta}]^{1/N} = \omega_{\xi}(\Upsilon(\tau_i))^{1/V'}; \tag{17}$$

N_1 & N_2 are *s*-independent Poisson r.v. with parameters $\Upsilon(\tau_1)$ & $\Upsilon(\tau_2)$ respectively, and, conditionally on $N = N_1 + N_2 = n$, V' is a standard Gamma r.v. with parameter n. In this case, however, Q' is no longer pivotal, since its distribution depends on the unknown parameters through $\Upsilon(\tau_1)$ & $\Upsilon(\tau_2)$.

Nevertheless, by extending to the present context fiducial arguments developed in [4] for the simple case of a single NHPP, it can be assumed that, given $N_1 = n_1$ and $N_2 = n_2$, quantiles for (16) with n_1 & n_2 replaced by n_1+1 & n_2+1 are approximately correct for Q', so that:

$$\Pr\{Q' \le q_{\gamma}(n_1+1,n_2+1,\xi)\} \approx \gamma, \tag{18}$$

$$q_{\gamma}(n_1+1,n_2+1,\xi) \equiv [\omega_{\xi}(n_i+1) \cdot w_{\gamma}(n_1+1,n_2+1,\xi)]^{1/(n+2)},$$

$w_{\gamma}(\cdot)$ is defined in (15).

Thus, given $N_1 = n_1$ and $N_2 = n_2$,

$$\Pr\left\{\left(\frac{\hat{\alpha}_0}{\alpha_0}\right)^{\beta} \le \frac{[q_{\gamma}(n_1+1,n_2+1,\xi)]^n}{\omega_{\xi}(n_i)}\right\} \approx \gamma, \tag{19}$$

and approximate *s*-confidence intervals on α_0 follow.

The accuracy of the approximation (18) & (19) can be evaluated for specified values of $\Upsilon(\tau_1)$, $\Upsilon(\tau_2)$, ξ by:

$$\Pr\{Q' \le q_{\gamma}(N_1+1,N_2+1,\xi)\}$$

$$= \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} G\left(\frac{\ln[\omega_{\xi}(\Upsilon(\tau_i))]}{\ln[q_{\gamma}(n_1+1,n_2+1,\xi)]}; n_1+n_2\right) \cdot \prod_{i=1}^{2} \frac{\Upsilon(\tau_i)^{n_i} \cdot \exp(-\Upsilon(\tau_i))}{n_i! \cdot [1-\exp(-\Upsilon(\tau_i))]},$$

$$G(u;n) \equiv \begin{cases} \text{gamf}(u;n), & \text{for } q_{\gamma} < 1, \\ \text{gamfc}(u;n), & \text{for } q_{\gamma} \ge 1. \end{cases}$$

TABLE 1
Exact values of $\Pr\{Q' \le q_{\gamma}(N_1+1,N_2+1,\xi=2)\}$

	nominal γ values					
$\Upsilon(\tau_1),\Upsilon(\tau_2)$	0.025	0.050	0.100	0.900	0.950	0.975
5,5	0.022	0.046	0.094	0.905	0.955	0.980
5,10	0.030	0.059	0.116	0.927	0.969	0.989
5,15	0.034	0.065	0.127	0.938	0.975	0.993
10,10	0.023	0.047	0.096	0.896	0.948	0.974
10,15	0.025	0.050	0.101	0.902	0.952	0.976

Table 1 shows exact values of probability (18) corresponding to nominal γ values for $\xi = 2$ and some combinations of $\Upsilon(\tau_i)$. The true values are quite close to nominal ones except when $\Upsilon(\tau_1)$ is small and $\Upsilon(\tau_2)$ is moderate or large. Even in these cases, actual *s*-confidence levels of nominal $(1-\gamma)$ 2-sided symmetric intervals are close to $(1-\gamma)$.

3.1.3 Interval estimation for quantile p of failure-time k

Another useful estimate is quantile p (*eg*, median) of the distribution of $T^{(k)}$ under baseline conditions. This is equivalent to estimating $y_p^{(k)}$, say, of $Y^{(k)}$.

Under the baseline condition,

$$\hat{y}_p^{(k)} = \ln(\hat{\alpha}_0) + \hat{\beta}^{-1}\cdot e_p^{(k)}. \tag{20}$$

For failure censored sampling,

$$W' \equiv \hat{\beta}\cdot(\hat{y}_p^{(k)} - y_p^{(k)})$$
$$= (n/V)\cdot[\ln(\omega_\xi(U_i)) - e_p^{(k)}] + e_p^{(k)} - \ln(\omega_\xi(n_i)), \tag{21}$$

Notation

U_1, U_2, V *s*-independent standard Gamma r.v. with parameters $[n_1, n_2, n-2]$.

Hence, W' is a pivotal quantity with a known distribution, although it cannot be given in a simple form. Thus, exact *s*-confidence intervals on $y_p^{(k)}$ can be obtained, although computing γ quantiles, $w'_\gamma(n_1,n_2,\xi)$ such that,

$$\Pr\{\hat{\beta}\cdot(\hat{y}_p^{(k)} - y_p^{(k)}) \le w'_\gamma(n_1,n_2,\xi)\} = \gamma, \tag{22}$$

requires multiple numerical integration or simulation.

Alternatively, *s*-confidence intervals on $y_p^{(k)}$ based on *s*-normal approximation are given in appendix A.1, where their accuracy is also analyzed. Define,

$$s_\gamma(n_1,n_2,\xi) \equiv \frac{w'_\gamma(n_1,n_2,\xi) + \ln(\omega_\xi(n_i)) - e_p^{(k)}}{n_1+n_2}$$

as the γ quantile of S.

$$S \equiv \frac{\hat{\beta}\cdot(\hat{y}_p^{(k)} - y_p^{(k)}) + \ln(\omega_\xi(n_i)) - e_p^{(k)}}{n_1+n_2}$$
$$= [\ln(\omega_\xi(U_i)) - e_p^{(k)}]/V.$$

For time-censored sampling, an approach analogous to that used for α_0 leads one to assume that, given $N_1 = n_1$ and $N_2 = n_2$,

$$\Pr\{\hat{\beta}\cdot(\hat{y}_p^{(k)} - y_p^{(k)}) \le (n_1+n_2)\cdot s_\gamma(n_1+1,n_2+1,\xi)$$
$$+ e_p^{(k)} - \ln(\omega_\xi(n_i))\} \approx \gamma. \tag{23}$$

The accuracy of (23) can be evaluated for specific values of $\Upsilon(\tau_1)$, $\Upsilon(\tau_2)$, ξ, k by:

$$\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} G\left(\frac{\ln[\omega_\xi(\Upsilon(\tau_i))] - e_p^{(k)}}{s_\gamma(n_1+1,n_2+1,\xi)};\, n_1+n_2\right)$$
$$\cdot\prod_{i=1}^{2}\frac{\Upsilon(\tau_i)^{n_i}\cdot\exp(-\Upsilon(\tau_i))}{n_i!\cdot[1-\exp(-\Upsilon(\tau_i))]},$$

$$G(u;n) \equiv \begin{cases}\text{gamf}(u;n), & \text{for } s_\gamma < 0,\\ \text{gamfc}(u;n), & \text{for } s_\gamma \ge 0.\end{cases}$$

TABLE 2
Exact Values of Probability (23)

	nominal γ values					
$\Upsilon(\tau_1),\Upsilon(\tau_2)$	0.025	0.050	0.100	0.900	0.950	0.975
5,5	0.025	0.051	0.100	0.920	0.966	0.986
5,10	0.037	0.068	0.132	0.942	0.978	0.993
5,15	0.038	0.074	0.140	0.946	0.979	0.995
10,10	0.025	0.050	0.100	0.902	0.951	0.976
10,15	0.028	0.055	0.108	0.912	0.957	0.979

Table 2 shows exact *vs* nominal values of probability (23) for $\xi=2$, $k=3$, and some combinations of $\Upsilon(\tau_i)$ values. Results are similar to those obtained for α_0.

3.2 Testing More Than 1 System At Each Stress Level

J_i copies of a system are tested at x_i, for a common test time τ_i $(i=1,2)$. The following closed-form solutions result:

$$\hat{\beta} = n\Big/\left[\sum_{i=1}^{2}\sum_{j=1}^{J_i}\sum_{l=1}^{n_{i,j}}\ln(\tau_i/t_{i,j,l})\right], \tag{24}$$

$$\hat{\alpha}_0 = \omega_\xi(\tau_i/\bar{n}_i^{1/\hat{\beta}}), \tag{25}$$

$$\hat{a} = \ln[(\tau_1/\tau_2)\cdot(\bar{n}_2/\bar{n}_1)^{1/\hat{\beta}}]/(x_2-x_1), \tag{26}$$

$$\bar{n}_i = \left[\sum_{j=1}^{J_i} n_{i,j}\right]\Big/J_i,\ (i=1,2),$$

$$\xi = x_2/(x_2-x_1).$$

Conditionally on $N=N_1+N_2=n$,

$$2n\cdot(\beta/\hat{\beta}) \sim \chi^2[2n], \tag{27}$$

thus, *s*-confidence intervals for β easily follow. Further,

$$Q'' \equiv [\omega_\xi((N_i/J_i))\cdot(\hat{\alpha}_0/\alpha_0)^{\beta}]^{1/N}$$
$$= \omega_\xi(\Upsilon(\tau_i))^{1/V''} \tag{28}$$

Notation

N_1, N_2 *s*-independent Poisson r.v. with parameter $[J_1\cdot\Upsilon(\tau_1), J_2\cdot\Upsilon(\tau_2)]$

V'' a standard Gamma r.v. with parameter n (conditional on $N=n$)

G_1, G_2, V'' *s*-independent standard Gamma variates with parameters $[n_1+1,\ n_2+1,\ n]$.

Q'' is not a pivotal quantity, since its distribution depends on the unknown parameters through $\Upsilon(\tau_1)$ & $\Upsilon(\tau_2)$. Using fiducial arguments, Q'' is approximately distributed as

$$\omega_\xi(G_i/J_i)^{1/V''}. \quad (29)$$

Then, conditionally on $N_1 = n_1$ and $N_2 = n_2$, quantiles of (29), q''_γ say, can be used to obtain approximate s-confidence intervals for α_0 by:

$$\Pr\{(\hat{\alpha}_0/\alpha_0)^\beta \le (q''_\gamma)^n \cdot \omega_\xi(J_i/n_i)\} \approx \gamma. \quad (30)$$

The accuracy of the approximation can be evaluated for specific values of $\Upsilon(\tau_1)$, $\Upsilon(\tau_2)$, J_1, J_2, ξ by:

$$\Pr\{Q'' \le q''_\gamma(N_1+1, N_2+1, J_1, J_2, \xi)\}$$

$$= \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} G\left(\frac{\ln(\omega_\xi(\Upsilon(\tau_i))}{\ln[q''_\gamma(n_1+1, n_2+1, J_1, J_2, \xi)]}; n_1+n_2\right)$$

$$\cdot \prod_{i=1}^{2} \frac{[J_i \cdot \Upsilon(\tau_i)]^{n_i} \cdot \exp(-J_i \cdot \Upsilon(\tau_i))}{n_i! \cdot [1-\exp(-J_i \cdot \Upsilon(\tau_i))]},$$

$$G(u;n) \equiv \begin{cases} \text{gamf}(u;n), & \text{for } q''_\gamma < 1, \\ \text{gamfc}(u;n), & \text{for } q''_\gamma \ge 1. \end{cases}$$

TABLE 3
Exact Values of Probability (30)

	nominal γ values					
$J_1 \cdot \Upsilon(\tau_1)$, $J_2 \cdot \Upsilon(\tau_2)$	0.025	0.050	0.100	0.900	0.950	0.975
4×3,4×3	0.025	0.051	0.102	0.904	0.953	0.977
4×2,4×5	0.034	0.067	0.128	0.930	0.968	0.985
4×5,4×2	0.021	0.044	0.089	0.890	0.944	0.972
6×2,2×5	0.029	0.058	0.115	0.919	0.961	0.981

Table 3 shows exact *vs* nominal values of probability (30) for $\xi = 2$ and some combinations of J_i & $\Upsilon(\tau_i)$. When $J_1 = J_2$, true values are quite close to nominal ones except when $\Upsilon(\tau_1) \ll \Upsilon(\tau_2)$. In such a circumstance, satisfactory results can be obtained if one uses J_i values which assure a balanced mean number of failures under both stress levels. In all cases, however, actual s-confidence levels of nominal $(1-\gamma)$ 2-sided symmetric intervals are close to $(1-\gamma)$.

3.3 Drawback To Closed-Form Based Procedures

Closed-form solutions derive from the unconstrained maximization of the (log)likelihood function. Provided some trivial conditions on the minimum number of observations in failure- and time-truncated sampling are met, (11) - (13) and (24) - (26) show that:

- a unique solution exists;
- closed form estimates of α_0 & β are always inside the parameter space ($\alpha_0 > 0$; $\beta > 0$).

Under $g(x;a) = \exp(a \cdot x)$ and $x > 0$, an accelerated testing implies $a > 0$. Nevertheless, an estimate $\hat{a} < 0$ may come from (13) or (26) even underlying $a > 0$, especially when $x_1 \approx x_2$ independently of how far x_1 is from the baseline stress. Under such a circumstance, the closed-form estimate of α_0 is senseless. In fact, the rationale of the model is to turn an accelerated testing into a baseline one by scaling the observed failure times through the factor $\exp(\hat{a} \cdot x)$. When $\hat{a} < 0$, the effect of scaling is in the direction of shortening failure times instead of increasing them. Thus, instead of being consistent with the baseline stress, $\hat{\alpha}_0$ is paradoxically consistent with stress conditions more severe than those experienced by the items during testing.

Under accelerating testing, ML estimates are the values which maximize the (log)likelihood function under the constraint $a > 0$. For computational purposes, one can first estimate a from (13) or (26). Then,

- $\hat{a} > 0$: Closed-form estimation is a short-cut to obtain ML point estimates of the baseline parameters;
- $\hat{a} < 0$: ML estimates of the baseline parameters are obtained by (numerical) maximization of the (log)likelihood function with $\hat{a} = 0$. Searching for this boundary solution is the same as maximizing the (log)likelihood function for the case where multiple copies of an item are subjected to identical operating conditions [8].

In the latter circumstance, $\hat{\alpha}_0$ is often as senseless as the corresponding closed-form estimate. Due to the position $\hat{a} = 0$, no extrapolation is possible, and $\hat{\alpha}_0$ represents a value consistent with the stresses x_1 & x_2, instead of the baseline stress.

As a consequence, when $\hat{a} < 0$, there are often reasons to reject $\hat{\alpha}_0$, and caution is needed in exploiting point or interval estimation for this parameter. It might be better to account for the possibility of increasing the sample size and/or changing the stress levels.

Appendix A.2 obtains $\Pr\{\hat{a} \le 0\}$ as a function of process parameters and testing conditions. In many practical situations this probability appears to be small; so an $\hat{a} > 0$ is usually obtained. Even in such a circumstance, however, a theoretical drawback to using the procedures for interval estimation in sections 3.1 & 3.2 arises. Sampling distributions of the constrained & unconstrained estimators of α_0 & β obviously differ, unless $\Pr\{\hat{a} \le 0\} \to 0$. Hence, s-confidence intervals developed in those sections are valid for the constrained ML estimators in the limit $\Pr\{\hat{a} \le 0\} \to 0$, only.

If the experimenter is *a priori* inclined to reject point estimates whether or not $\hat{a} < 0$, s-confidence levels should in principle be derived from conditional ($\hat{a} > 0$) sampling distributions and an analogous theoretical problem arises.

Since no analytic tool seems available for constrained or conditional ML estimators, a large simulation study was carried out to ascertain to what extent s-confidence limits, derived in sections 3.1 & 3.2 under closed-form (unconstrained) estimation, can equally be applied when $\Pr\{\hat{a} \le 0\} \ne 0$. Here, for the sake of space, only relevant conclusions are reported.

- A $\Pr\{\hat{a} \le 0\} < 0.01$ does not practically affect the distribution of any pivotal quantity of interest.

- Both tails of the distribution of $2n\cdot(\beta/\hat{\beta})$ are only slightly affected by a $\Pr\{\hat{a} \le 0\} > 0.01$, even when values as high as 0.25 are involved. Thus, actual *s*-confidence levels of intervals based on the χ^2 distribution are very close to nominal values, even when $\Pr\{\hat{a} \le 0\} > 0.01$.
- The right tail of the distribution of $(\hat{\alpha}_0/\alpha_0)^{\hat{\beta}}$ is only slightly affected by a $\Pr\{\hat{a} \le 0\} > 0.01$, even when values as high as 0.25 are involved. The left tail, however, is dramatically affected even by a $\Pr\{\hat{a} \le 0\}$ as low as 0.05. Thus, when $\Pr\{\hat{a} \le 0\} > 0.01$, the proposed procedures for the interval estimation of α_0 should be used to obtain a lower limit, only.

4. APPLICATION EXAMPLE

The increasing market requirements ask for testing programs during the development phase of a complex repairable system such as a new car (or engine), in order to demonstrate the achievement of reliability targets before starting mass production and launching the product. During testing, when a failure occurs, the car (or the engine) is repaired, and the same unit undergoes testing until a fixed test time is accumulated or a fixed number of failures is observed. Thus, data consist of repeated events on the same item and methods of analysis pertaining to stochastic processes rather than to failure distributions are involved. Two basic types of tests are usually exploited: bench & road. Both tests are usually run at more severe than use conditions in order to save time & money. Since, however, reliability targets are usually referred to usual use conditions, analyses based on extrapolative methods are required.

Refs [5, 10 - 12, 14, 20] are a few papers which witness the interest paid by vehicle manufacturers such as FIAT, Renault, Volvo, Daimler in this problem. Case studies with detailed real failure data are lacking in the literature, however, mainly due to industrial reserve. In [10], a reliability demonstration study concerning a light-duty diesel engine was presented. Data collected under 1 bench- and 2 road-accelerated tests were pooled, scaling actual failure times by empirical acceleration factors. Then, a non-parametric estimate of the baseline cumulative intensity was obtained, and a polynomial function was fitted to these estimates. The failures were assumed to be a renewal process.

The experimental situation in [10] lends itself to an analysis in the formal setting of the our PLP regression model, which appears to be far more realistic since it incorporates the effect of reliability degradation during testing. Unfortunately, numerical calculations cannot be based on the real data in [10], since the associated engine-testing condition is not specified there. So, in the illustrative example, we have reproduced an experimental context similar to that in [10], but we have generated a set of pseudo-random data using stress levels and parameter estimates from [10] as inputs for the simulation.

In this hypothetical study, 8 *s*-identical copies of a diesel engine are mounted into 2 groups of 4 cars, each group undergoing a different road test. The cars in the group #1 run for 150000 km on a circuit at maximum speed, the others run for 130000 km on a mountain track at full load. On a normalized scale, stress level #1 is $x_1=0.75$ and stress level #2 is $x_2=1$. During testing the following engine failures are observed:

Group 1				Group 2			
Car 1	Car 2	Car 3	Car 4	Car 1	Car 2	Car 3	Car 4
74226	17650	65584	105182	64195	15265	56721	90968
143625	92046	109057	134829	124216	79607	94319	116608
	148702	137842	139298		128606	119215	120473
	149436				129241		

$n_1=n_2=12,\ n=24;$

since $J_1=J_2=4$, then $\bar{n}_1=\bar{n}_2=3$.

From (24) - (26),

$$\hat{\beta}=2.32,\ \hat{\alpha}_0=1.44\times10^5,\ \hat{a}=0.572.$$

From (20), point estimates of the median of failure-times 1 - 3 are:

$$\hat{T}^{(1)}_{0.50} = \hat{\alpha}_0\cdot 0.6931^{1/2.32} = 1.23\cdot10^5 \text{ km},$$

$$\hat{T}^{(2)}_{0.50} = \hat{\alpha}_0\cdot 1.678^{1/2.32} = 1.80\cdot10^5 \text{ km},$$

$$\hat{T}^{(3)}_{0.50} = \hat{\alpha}_0\cdot 2.674^{1/2.32} = 2.20\cdot10^5 \text{ km}.$$

Interval estimation for β & α_0 can be obtained from (27) & (30) respectively. The accuracy of the approximate procedure for α_0 should be good, since balanced mean numbers of failures under the 2 stress levels are observed.

However, before using interval estimation, a check on the $\Pr\{\hat{a} \le 0\}$ is necessary, although $\hat{a} > 0$. Based on point estimates, $[(\exp(\hat{a}\cdot x_1)/\exp(\hat{a}\cdot x_2)]^{\hat{\beta}} = 0.72$, and a high value of $\Pr\{\hat{a} \le 0\}$ is suspected (see appendix A.2). Thus, considerations in section 3.3 apply:

- *s*-confidence intervals for β can be obtained from (27), but they are approximate;
- only a lower (approximate) *s*-confidence limit for α_0 is available from (30).

Approximate 2-sided symmetric 0.90 *s*-confidence limits for β are:

$$\beta_{\text{lo}} = 33.10\cdot2.32/48 = 1.60,$$

$$\beta_{\text{hi}} = 65.17\cdot2.32/48 = 3.15.$$

An approximate lower 0.90 *s*-confidence limit for α_0 is:

$$\alpha_{0\text{lo}} = \hat{\alpha}_0\cdot[(12/4)^4\cdot(12/4)^{-3}/1.138^{24}]^{1/2.32} = 6.07\cdot10^4 \text{ km},$$

1.138 is the 0.90 quantile of (29) evaluated by simulation with $n_1+1=13,\ n_2+1=13,\ \xi=4$.

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APPENDIX

Notation

ψ, ψ' [digamma, trigamma] functions
$\zeta_{1,2}$ $\exp(a \cdot x_1)/\exp(a \cdot x_2)$
z_γ γ quantile of gauf(z).

A.1 *s*-Normal Approximations

A.1.1 $\ln(W)$

From (14), the mean & variance of $\ln(W)$ are available in explicit form. Thus, approximate *s*-confidence intervals on α_0 can be obtained by assuming:

$$\frac{\ln(W) - E\{\ln(W)\}}{\text{StdDev}\{\ln(W)\}} \sim N(0,1), \tag{A-1}$$

$$\text{E}\{\ln(W)\} = \frac{n}{n-3} \cdot g_1(n_1,n_2,\xi) - \ln(\omega_\xi(n_i)),$$

$$\text{StdDev}\{\ln(W)\}^2 = \frac{n^2}{(n-3)(n-4)}$$

$$\cdot \left[g_2(n_1,n_2,\xi) + \frac{[g_1(n_1,n_2,\xi)]^2}{n-3} \right],$$

$$g_1(n_1,n_2,\xi) = \xi \cdot \psi(n_1) + (1-\xi) \cdot \psi(n_2),$$

$$g_2(n_1,n_2,\xi) = \xi^2 \cdot \psi'(n_1) + (1-\xi)^2 \cdot \psi'(n_2).$$

For integers, the values of ψ & ψ' can be calculated by:

$$\psi(i) = -c + \sum_{j=1}^{i-1} 1/j, \; i=2,3,\ldots;$$

$$\psi'(i) = (\pi^2/6) - \sum_{j=1}^{i-1} 1/j^2, \; i=2,3,\ldots;$$

c=0.577215... is Euler's constant.

Table 4 shows the accuracy of approximation (A-1) for small n_1 & n_2, and $\xi=2$.

Even for small n_i, the actual *s*-confidence level of a nominal $(1-\gamma)$ 2-sided symmetric interval is close to nominal value (especially when this value increases), although *s*-normal approximation can result in appreciably non-symmetric intervals.

TABLE 4

Actual *vs* Nominal Values of $\Pr\left\{\frac{\ln(W) - \text{E}\{\ln(W)\}}{\text{StdDev}\{\ln(W)\}} \le z_\gamma\right\}$

	nominal γ values					
n_1,n_2	0.025	0.050	0.100	0.900	0.950	0.975
5,5	0.013	0.027	0.062	0.911	0.946	0.963
5,15	0.033	0.054	0.096	0.918	0.960	0.980
10,10	0.011	0.028	0.073	0.901	0.940	0.961
10,15	0.017	0.036	0.085	0.899	0.941	0.964

A.1.2 W'

From (21), the mean & variance of W' are available in explicit form. Thus, approximate *s*-confidence intervals on $y_p^{(k)}$ can be obtained by assuming:

$$\frac{W' - \text{E}\{W'\}}{\text{StdDev}\{W'\}} \sim N(0,1), \tag{A-2}$$

$$\text{E}\{W'\} = \text{E}\{\ln(W)\} - \frac{3}{n-3} \cdot e_p^{(k)}$$

$$\text{StdDev}\{W'\}^2 = \frac{n^2}{(n-3)(n-4)}$$

$$\cdot \left[g_2(n_1,n_2,\xi) + \frac{[g_1(n_1,n_2,\xi) - e_p^{(k)}]^2}{n-3} \right]$$

TABLE 5

Actual *vs* nominal values of $\Pr\left\{\frac{W' - \text{E}\{W'\}}{\text{StdDev}\{W'\}} \le z_\gamma\right\}$

	nominal γ values					
n_1,n_2	0.025	0.050	0.100	0.900	0.950	0.975
5,5	0.025	0.042	0.078	0.919	0.955	0.971
5,15	0.039	0.061	0.100	0.928	0.970	0.987
10,10	0.020	0.038	0.081	0.903	0.944	0.966
10,15	0.025	0.046	0.091	0.905	0.948	0.970

Table 5 shows the accuracy of approximation (A-2) for small n_1 & n_2, p=0.50, and k=3. Actual *s*-confidence levels of nominal $(1-\gamma)$ 2-sided symmetric intervals are close to $(1-\gamma)$, although appreciably non-central intervals can result when n_1 & n_2 are quite different.

A.2 Probability of a Negative $\hat{a}$

From (13), in order for $\hat{a}$ to be positive,

$$(T_1/T_2) \cdot (n_2/n_1)^{1/\hat{\beta}} > 1,$$

$$\Pr\{\hat{a} \le 0\} = \Pr\{T_1/T_2 \le (n_1/n_2)^{1/\beta}\}. \quad \text{(A-3)}$$

A.2.1 Failure-censored sampling

The T_i are r.v. and the n_i are known constants, and (A-3) can be rewritten as:

$$\Pr\{\hat{a} \le 0\} = \Pr\left\{\frac{U_1}{U_2}\cdot\left[\frac{n_2}{n_1}\right]^{V/n} \le \zeta_{1,2}^{\beta}\right\}, \quad \text{(A-4)}$$

U_1, U_2, V are s-independent standard Gamma r.v. with parameters n_1, n_2, $(n-2)$, respectively. Thus, the probability of interest depends on n_1, n_2, β, $\zeta_{1,2}$ under stress x_1 & x_2. Table 6 shows several values of this probability, obtained by simulation.

TABLE 6
$\Pr\{\hat{a} \le 0\}$, Failure-Censored Case

	$\zeta_{1,2}^{\beta}$				
n_1,n_2	0.1	0.2	0.3	0.5	0.8
5,5	$6\cdot10^{-4}$	$1\cdot10^{-2}$	$4\cdot10^{-2}$	$1.5\cdot10^{-1}$	$3.6\cdot10^{-1}$
5,15	$6\cdot10^{-4}$	$1\cdot10^{-2}$	$5\cdot10^{-2}$	$1.9\cdot10^{-1}$	$4.6\cdot10^{-1}$
10,10	$1\cdot10^{-5}$	$3\cdot10^{-4}$	$5\cdot10^{-3}$	$6.0\cdot10^{-2}$	$3.1\cdot10^{-1}$
10,15	$1\cdot10^{-5}$	$2\cdot10^{-4}$	$5\cdot10^{-3}$	$7.0\cdot10^{-2}$	$3.3\cdot10^{-1}$

A.2.2 Time-censored sampling

Eq (A-3) can be rewritten as:

$$\Pr\{\hat{a} \le 0\} = \Pr\left\{\left[\frac{N_2}{N_1}\right]^{V'/(N_1+N_2)} \le \zeta_{1,2}^{\beta}\cdot\frac{\Upsilon(\tau_2)}{\Upsilon(\tau_1)}\right\}, \quad \text{(A-5)}$$

N_1 & N_2 are s-independent Poisson r.v. with parameters $\Upsilon(\tau_1)$ & $\Upsilon(\tau_2)$ respectively; and V$'$, given $N=N_1+N_2=n$, is a standard Gamma r.v. with parameter n. The probability (A-5) depends on: a) the ratio of the acceleration factors, b) β, and c) the s-expected number of failures in τ_1 & τ_2. Table 7 shows simulation results for assigned values of $\Upsilon(\tau_1)$ & $\Upsilon(\tau_2)$.

TABLE 7
$\Pr\{\hat{a} \le 0\}$, Time-Censored Case

	$\zeta_{1,2}^{\beta}$				
$\Upsilon(\tau_1),\Upsilon(\tau_2)$	0.1	0.2	0.3	0.5	0.8
5,5	$6\cdot10^{-3}$	$2\cdot10^{-2}$	$5\cdot10^{-2}$	$1.4\cdot10^{-1}$	$3.5\cdot10^{-1}$
5,15	$1\cdot10^{-5}$	$7\cdot10^{-4}$	$6\cdot10^{-3}$	$9.0\cdot10^{-2}$	$3.6\cdot10^{-1}$
10,10	$4\cdot10^{-4}$	$4\cdot10^{-3}$	$1\cdot10^{-2}$	$7.0\cdot10^{-2}$	$3.0\cdot10^{-1}$
10,15	$1\cdot10^{-5}$	$4\cdot10^{-4}$	$3\cdot10^{-3}$	$5.0\cdot10^{-2}$	$3.0\cdot10^{-1}$

For multiple copies tested under the 2 stress levels, (26) shows:

$$\Pr\{\hat{a} \le 0\} = \Pr\left\{\left[\frac{N_2}{N_1}\cdot\frac{J_1}{J_2}\right]^{V'/(N_1+N_2)} \le \zeta_{1,2}^{\beta}\cdot\frac{\Upsilon(\tau_2)}{\Upsilon(\tau_1)}\right\}, \quad \text{(A-6)}$$

N_1 & N_2 are s-independent Poisson r.v. with parameter $J_1\cdot\Upsilon(\tau_1)$ & $J_2\cdot\Upsilon(\tau_2)$ respectively

V', given $N=N_1+N_2=n$, is a standard Gamma r.v. with parameter n.

The probability (A-6) depends on: a) the ratio of the acceleration factors, b) β, c) the s-expected number of failures for a single system in τ_1 & τ_2, and d) J_1 & J_2. Since in (A-6) only the ratios J_1/J_2 and $\Upsilon(\tau_2)/\Upsilon(\tau_1)$ are involved, when $J_1=J_2=J$, many values of $\Pr\{\hat{a} \le 0\}$ can be read in table 7, by using $J\cdot\Upsilon(\tau_i)$ in place of $\Upsilon(\tau_i)$.

A.2.3 Discussion

When $\zeta_{1,2}^{\beta} < 0.2$, then $\Pr\{\hat{a} < 0\}$ is small even for (observed or s-expected) as few as 5 failures.

When $\beta=1$, $\zeta_{1,2}^{\beta} = 0.2$ implies an acceleration factor, under x_2, 5 times greater than under x_1. Since, however, in the application of interest, generally $\beta > 1$, the same value, $\zeta_{1,2}^{\beta} = 0.2$, generally implies $\zeta_{1,2} > 0.2$. For example, when $\beta=1.4$, the value $\zeta_{1,2}=0.2$ corresponds to an acceleration factor under x_2 only 3 times greater than that under x_1. Thus, in many practical cases $\Pr\{\hat{a} \le 0\} < 10^{-2}$, for which, procedures based on closed-form solutions can be considered exact for practical purposes when extended to ML estimators.

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