Fuzzy Reliability in Conceptual Design

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SUMMARY & CONCLUSIONS

Conceptual design's intrinsic uncertainty and influence on product life cycle cost make reliability engineering tools during early design phases important. However, typically poor knowledge bases make the prediction of component failure and system performance challenging. Fuzzy techniques are used here during initial conceptual design stages to quantify imprecision and uncertainty in reliability and risk analysis. More specifically, a fuzzy approach is compared to the probabilistic approach presented in *Reliability prediction models to support conceptual design* by Ormon, et al. (Ref. 2). Triangular fuzzy numbers instead of triangular probability distributions represent unknown failure rates here.

Two examples of fuzzy reliability analysis in conceptual design are presented where system reliability is evaluated at the subsystem level: The first is to familiarize the reader with fuzzy reliability subsystem level analysis. The second demonstrates fuzzy reliability prediction models for conceptual design tradeoffs. Examples include subsystems operating with active and standby redundancy. In the first example, defuzzified system reliability is a more conservative prediction than that found probabilistically. Recommendations in Example 2 do not differ from those based on the probabilistic models of Ormon, et al. (Ref. 2).

2. INTRODUCTION

Uncertainty is a condition where the possibility of error exists as a result of having less than total information about some existing environment. Inherent uncertainty exists within the early conceptual and preliminary phases of the design process creating risk where the majority of a product’s life cycle cost is determined. "Rigor and discipline during the later phases of system design cannot compensate for an ill-conceived concept and for premature commitments made during the conceptual design phase (Ref. 1)."

The importance of utilizing system reliability tools during early design phases has been recognized (Ref. 2), but a lack of system knowledge and representative data makes predicting component failure rates and system reliability performance challenging. Dingra (Ref. 3) wrote, "Fuzzy optimization techniques can be useful during initial stages of the conceptual design of engineering systems where the design goals and design constraints have not been clearly identified or stated." Fuzzy methodology has attained numerous reliability and risk analysis applications, creating a growing area (Ref. 4). This was highlighted by Bowles' (Ref. 5) tutorial describing basic fuzzy concepts and their reliability engineering design applications at last year's Symposium.

Fuzzy set theory was introduced by Lotfi Zadeh in 1965 to deal quantitatively with imprecision and uncertainty. It can be described as a body of concepts and techniques including fuzzy sets, fuzzy logic and fuzzy numbers that gives a form of mathematical precision to processes that are imprecise and ambiguous by the standards of classical methods. Fuzzy set theory is often utilized to quantify imprecision and uncertainty when insufficient data prohibits probabilistic risk analysis.

A fuzzy membership function $\mu(x)$ consists of real numbers in the interval [0, 1] that represent the degree of membership to which an object belongs within the fuzzy set. The higher the degree of membership (closer to 1), the stronger the object belongs to that set. Degrees of membership equal to zero and one correspond to no membership and full membership, respectively. A fuzzy number is a normal and convex fuzzy set with membership function $\mu(x)$ which satisfies both normality and convexity. Normality implies that there is at least one x that strictly matches the predicate of the fuzzy number. Convexity indicates the convex shape of a fuzzy set. A triangular fuzzy number is a special type of fuzzy number that is defined by a triplet corresponding to the smallest possible, most promising and largest possible values.

In the fuzzy reliability and risk analysis literature, Tanaka, et al. (Ref. 6) presents fuzzy fault-tree analysis to estimate the failure probability of some event. The authors replace exact component failure probability with the possibility of failure to account for system environment changes and components that have not failed before. Possibility distributions of events have been approximated for reliability and safety analysis of "complex and dangerous technological systems" (Ref. 7) such as those where persons in the vicinity of grinding machines can have their eyes injured (Ref. 8). Chen's (Ref. 9) fuzzy arithmetic where a triangular fuzzy number represents component reliability simplified the fuzzy number operations of "extended algebraic operations" (Ref. 7) and interval arithmetic (Ref. 8). Kai-Yuan, et al. (Ref. 10) explicitly introduce fuzzy (or *posbist*) reliability theory and studies it for k-out-of-n, parallel, parallel-series, series and series-parallel systems (Ref. 11). Kim, et al. (Ref. 12) applies fuzzy fault-tree analysis to calculate pessimistic and optimistic possibilities of human and equipment errors.

Park’s (Ref. 13) introduction to fuzzy-system theory has for its setting nonlinear optimization of reliability apportionment

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where the objective system reliability and constraints are flexible. A less complex analysis of two components in series constrained by budget precedes a generalized example problem of more components and multiple resource constraints that extends beyond the man-machine reliability application. Dhingra (Ref. 3) presents a specific example of several design constraints. Cai (Ref. 14) presents reliability-engineering illustrations of δ-equality, the extent to which two fuzzy sets are equal. Verma and Knezevic (Ref. 1) address feasibility in analysis and evaluation of conceptual design by presenting compliance analysis of "fuzzy required and predicted system reliability values… Feasibility assessment of potential concepts involves a comparative analysis of the compliance between anticipation and preference profiles… the greater this compliance the better." Overlapping profiles represent some degree of compliance, just as overlapping confidence intervals do not compel statisticians to reject hypotheses of equality.

Research presented here compares a fuzzy approach to handling uncertainty inherent in predicting system reliability in early design phases to the probabilistic approach presented by Ormon, et al (Ref. 2). In particular, unknown failure rates represented by triangular fuzzy numbers are compared to analogous failure rates approximated using triangular probability distributions.

3. NOTATION

m	mission length
n	number of subsystems
a_i	number of active redundant components in subsystem i
s_i	number of standby redundant components in subsystem i
λ_I	failure rate for components in subsystem i
λ_{SPi}	smallest possible (optimistic) fuzzy failure rate for components in subsystem i
λ_{MPi}	most promising fuzzy failure rate for components in subsystem i
λ_{LPi}	largest possible (pessimistic) fuzzy failure rate for components in subsystem i
R_i	reliability of subsystem i
R_{SPi}	smallest possible (optimistic) fuzzy reliability of subsystem i
R_{MPi}	most promising fuzzy reliability of subsystem i
R_{LPi}	largest possible (pessimistic) fuzzy reliability of subsystem i
R	mission (system) reliability
R_{SP}	smallest possible (optimistic) fuzzy system reliability
R_{MP}	most promising fuzzy system reliability
R_{LP}	largest possible (pessimistic) fuzzy system reliability
R_{DF}	defuzzified system reliability
R_{OCG}	system reliability from (Ormon, Cassady and Greenwood, 2002)
C	average cost of system operation
c_i	acquistion cost per component for subsystem i
c_f	cost of mission (system) failure
C_{SP}	smallest possible (optimistic) fuzzy average cost of system operation
C_{MP}	most promising fuzzy average cost of system operation
C_{LP}	largest possible (pessimistic) fuzzy average cost of system operation
C_{DF}	defuzzified average cost of system operation
C_{OCG}	average cost of system operation from (Ormon, Cassady and Greenwood, 2002)

4. SYSTEM DEFINITIONS AND PERFORMANCE

As defined by Ormon, et al. (Ref. 2), the system addressed in this research adheres to the following assumptions:

1. Due to the fact that the system is in its conceptual design phase, limited data is available to determine component failure rate, and simple reliability models are appropriate.
2. The system must perform just one mission of finite duration (m).
3. The system consists of n s-independent subsystems connected in series.
4. Each subsystem i operates in one of two ways:
   a. Active redundancy: a_i iid components connected in parallel, or
   b. Standby redundancy: s_i iid components connected via passive redundancy.
5. Failure detection and switching among redundant components are perfect.
6. All components, subsystems, and the system have binary status (operational or failed).
7. All components have exponential life-distributions.
8. Two measures of system performance are utilized:
   a. Mission reliability (R): the probability that the mission is successfully completed, and
   b. Average cost of system operation (C): the average costs associated with component acquisition and potential failure.

5. FUZZY SUBSYSTEM LEVEL ANALYSIS

The reliability of the system is analyzed at the subsystem level. Fuzzy reliability equations are derived from the traditional, crisp reliability equations employed by Ormon, et al. (Ref. 2) to accommodate component failure rates represented by triangular fuzzy numbers. In order to enforce data integrity in the form of a triangular fuzzy number from input to result, the exponential and n^{th} power approximations (Ref. 15) are employed while deriving the fuzzy equations. Both sets of equations are presented for comparative purposes.

The fuzzy subsystem level analysis consists of four steps:

1) **Determine the failure rate for all components in the system.** If the failure rate is known, a single crisp failure rate is used. If the failure rate is not known, system experts determine a triangular fuzzy failure rate. A triangular fuzzy number is a special type of fuzzy number that is defined by a triplet; for example, (λ_{SPi}, λ_{MPi}, λ_{MPi}) corresponds to the smallest possible (optimistic), most promising, and largest possible (pessimistic) component failure rate. The membership function for component failure rate is defined as:

$$\mu_\lambda(x)=\begin{pmatrix} 0 & where\ x<\lambda_{SP} \\ \left(\dfrac{x-\lambda_{SP}}{\lambda_{MP}-\lambda_{SP}}\right) & where\ \lambda_{SP}\le x\le\lambda_{MP} \\ \left(\dfrac{\lambda_{LP}-x}{\lambda_{LP}-\lambda_{MP}}\right) & where\ \lambda_{MP}\le x\le\lambda_{LP} \\ 0 & where\ x>\lambda_{LP} \end{pmatrix} \quad (1)$$

2) **Determine the subsystem reliability for all subsystems in the system**. Equation (2) describes the reliability of subsystems with active redundancy and crisp input data,

$$R_i=1-\left[1-e^{-\lambda_i m}\right]^{a_i}. \quad (2)$$

The triangular fuzzy reliability equation for subsystems with active redundancy is

$$\left(R_{SPi},R_{MPi},R_{LPi}\right) =\begin{pmatrix} 1-\left[1-e^{-\lambda_{SPi}m}\right]^{a_i}, \\ 1-\left[1-e^{-\lambda_{MPi}m}\right]^{a_i}, \\ 1-\left[1-e^{-\lambda_{LPi}m}\right]^{a_i} \end{pmatrix}. \quad (3)$$

Equation (4) is the traditional, crisp equation used to determine reliability of subsystems with standby redundancy,

$$R_i=e^{-\lambda_i m}\sum_{k=0}^{s_i-1}\frac{(\lambda_i m)^k}{k!}. \quad (4)$$

The triangular fuzzy reliability equation for subsystems with standby redundancy is

$$\left(R_{SPi},R_{MPi},R_{LPi}\right) =\begin{pmatrix} e^{-\lambda_{SPi}m}\displaystyle\sum_{k=0}^{s_i-1}\frac{(\lambda_{SPi}m)^k}{k!}, \\ e^{-\lambda_{MPi}m}\displaystyle\sum_{k=0}^{s_i-1}\frac{(\lambda_{MPi}m)^k}{k!}, \\ e^{-\lambda_{LPi}m}\displaystyle\sum_{k=0}^{s_i-1}\frac{(\lambda_{LPi}m)^k}{k!} \end{pmatrix} \quad (5)$$

3) **Determine system reliability.** The traditional, crisp reliability of the system is calculated from Equation (6),

$$R=\prod_{i=1}^{n}R_i. \quad (6)$$

The expanded triangular fuzzy system reliability is calculated as

$$\left(R_{SP},R_{MP},R_{LP}\right) =\left(\prod_{i=1}^{n}R_{SPi},\prod_{i=1}^{n}R_{MPi},\prod_{i=1}^{n}R_{LPi}\right). \quad (7)$$

The resulting system reliability $\left(R_{SP},R_{MP},R_{LP}\right)$ is in the form of a triangular fuzzy number. It may be desirable to defuzzify this result to a single, crisp value. The defuzzification method employed in this analysis is the Centroid method (Ross, 1995), which is mathematically efficient. The Centriod defuzzification equation for triangular fuzzy system reliability is

$$R_{DF}=\left(\frac{R_{SP}+R_{MP}+R_{LP}}{3}\right). \quad (8)$$

4) **Calculate the average cost of system operation.** The average cost of crisp system operation is calculated as

$$C=\sum_{i=1}^{n}c_i\cdot(a_i+s_i)+c_f\cdot(1-R). \quad (9)$$

The triangular fuzzy average cost of system operation can be found using

$$\left(C_{SP},C_{MP},C_{LP}\right) =\begin{pmatrix} \displaystyle\sum_{i=1}^{n}c_i\cdot(a_i+s_i)+c_f\cdot(1-R_{SP}), \\ \displaystyle\sum_{i=1}^{n}c_i\cdot(a_i+s_i)+c_f\cdot(1-R_{MP}), \\ \displaystyle\sum_{i=1}^{n}c_i\cdot(a_i+s_i)+c_f\cdot(1-R_{LP}) \end{pmatrix}. \quad (10)$$

6. EXAMPLES

Two examples of fuzzy reliability analysis in conceptual design are presented in this section. Example 1 is a simple example presented to familiarize the reader with the fuzzy reliability subsystem level analysis process. Example 2 presents a more complex system, demonstrating the use of fuzzy reliability prediction models for conceptual design tradeoffs. Both example systems consist of a combination of subsystems operating under active or standby redundancy.

6.1 *Example 1*

The system in Example 1 is required to perform a mission length of m = 10,000. The four-step fuzzy reliability subsystem level analysis for this example is presented next.

1) **Determine the failure rate for all components in the system.** The input data for Example 1 is presented in Table 1. All component failure rates are in the form of a triangular fuzzy number.

Table 1. Example 1 Input Data

Subsystem	a_i	s_i	λ_{SPi}	λ_{MPi}	λ_{Lpi}
1	3		7E-6	8E-6	9E-6
2		2	5E-6	1E-5	1.5E-5
3	4		1.5E-6	2E-5	2.5E-5

2) **Determine the subsystem reliability for all subsystems in the system.** Equation 3 is used to calculate the reliability for active Subsystems 1 and 3. Equation 5 is used to calculate the reliability for standby Subsystem 2. The fuzzy subsystem reliability results are presented in Table 2. The flaw in fuzzy notation should be noted. The higher reliability values appear as R_{SPi}, i.e. smallest possible. While as defined, this is the optimistic reliability outcome; it is not an ideal notation. The reverse is true for the R_{LPi} results. A similar phenomenon occurs when dealing with triangular fuzzy profit values.

Table 2. Example 1 Subsystem reliability

Subsystem (i)	R_{SPi}	R_{MPi}	R_{LPi}
1	0.999691	0.999546	0.999362
2	0.998791	0.995321	0.989814
3	0.999624	0.998920	0.997606

3) **Determine system reliability.** The system reliability was calculated from the subsystem reliabilities presented in Table 2 and Equation 7. Table 3 contains the smallest possible, most promising, and largest possible system reliabilities in addition to the defuzzified system reliability value and the value obtained by Ormon, et al. (2002). As derived, the R_{MP} and R_{OCG} are equivalent. The defuzzified system reliability is lower than that found by Ormon, et al. indicating that more conservative system reliability may be appropriate in this case. In addition to the result obtained by Ormon, et al., additional information is provided by the fuzzy system. This system provides the analyst with optimistic and pessimistic estimates of the system outcomes.

Table 3. Example 1 System Reliability

	System
R_{SP}	0.998106
R_{MP}	0.993795
R_{LP}	0.986815
R_{DF}	0.992905
R_{OCG}	0.993795

4) **Calculate the average cost of system operation.** Cost is not considered in Example 1. To calculate the average cost of system operation, the result of Step 3 would be an input into Equation 10 along with the required cost parameters. The average cost of system operation is calculated and used in the design tradeoff analysis in Example 2.

6.2 *Example 2*

The system in Example 2 has a mission length of m = 5000 and a mission failure cost of c_f = \$200,000. Table 4 contains the configuration, component failure rate, and acquisition cost for a preliminary system design (baseline). The total baseline system acquisition cost (AC) is \$311,000. Using these input data and the fuzzy reliability prediction models, a tradeoff analysis is performed by investigating various system configurations where one additional component is added to select subsystems and comparing the new configurations' average costs of operation to the baseline average cost of operation (\$323,153). Any new configuration resulting in a lower average cost of operation would be preferable to the original baseline design. Tables 5 and 6, respectively, contain the resulting mission reliabilities and average cost of operation for each configuration.

Table 4. Example 2 Input Data

Sub	a_i	s_i	λ_{SPi}	λ_{MPi}	λ_{LPi}	c_i	AC
1	2			2E-5		\$15k	\$30k
2		2		4E-5		\$10k	\$20k
3		2	1E-5	2E-5	3E-5	\$20k	\$40k
4	3		2E-5	4E-5	5E-5	\$7k	\$21k
5		3		5E-5		\$10k	\$30k
6		2	5E-6	7E-6	1E-5	\$15k	\$30k
7		3		6E-5		\$12k	\$36k
8	3		3E-5	4E-5	5E-5	\$8k	\$24k
9	4		6E-5	8E-5	1E-4	\$11k	\$44k
10		4		9E-5		\$9k	\$36k
System							\$311k

Table 5. Example 2 Mission Reliability

Subsystem	R_{SP}	R_{MP}	R_{LP}
Baseline	0.957576	0.939073	0.912822
2	0.973536	0.954725	0.928036
4	0.958324	0.943680	0.920601
5	0.959523	0.940982	0.914678
7	0.960780	0.942215	0.915876
8	0.959810	0.943680	0.920601
9	0.960792	0.946599	0.926418
10	0.958621	0.940098	0.913818
Subsystem	R_{DF}	R_{OCG}	
Baseline	0.936491	0.939236	
2	0.952099	0.954890	
4	0.940868	0.943116	
5	0.938394	0.941145	
7	0.939624	0.942378	
8	0.941364	0.943923	
9	0.944603	0.946929	
10	0.937512	0.940260	

Table 6. Example 2 Average Cost of Operation

Subsystem	C_{SP}	C_{MP}	C_{LP}
Baseline	$319,485	$323,185	$328,436
2	$326,293	$330,055	$335,393
4	$326,335	$329,264	$333,880
5	$329,095	$332,804	$338,064
7	$330,844	$334,557	$339,825
8	$327,038	$330,264	$334,880
9	$329,842	$332,680	$336,716
10	$328,276	$331,980	$337,236

Subsystem	C_{DF}	C_{OCG}
Baseline	$323,702	$323,153
2	$330,580	$330,022
4	$329,826	$329,377
5	$333,321	$332,771
7	$335,075	$334,524
8	$330,727	$330,215
9	$333,079	$332,614
10	$332,498	$331,948

Similar to Ormon, et al., the results indicate that none of the new system configurations are preferable to the baseline configuration in terms of average cost of operation. Even in the best-case scenario (C_{SP}), the baseline average cost of operation is less than any of the new configurations. It should be noted that the C_{MP} and C_{OCG} results differ slightly because Ormon, et al. analytically find the reliability of the subsystems with unknown failure rates using different equations and the law of total probability. All of the C_{OCG} results do fall within the C_{SP} and C_{LP} bounds.

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