Analysis for Mission Reliability of a Combat Tank

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Key Words — **Mission reliability, Associated component set.**

Reader Aids —
Purpose: Describe some mathematical models
Special math needed for derivations: Probability
Special math needed to use results: None
Results useful to: System analysts, combat-tank analysts

Abstract — **Mission reliability is the probability that a system will perform its specified mission. In a reliability sense, some subsystems or components are not required to operate perfectly for a certain mission and sometimes it is not reasonable to assume that the states of all components are statistically independent. The purpose of this paper is to set up mathematical models for the moving and firing mission reliabilities of a combat tank, taking into account these two problems. It is discussed how the models can be used by the combat tank planner or the tank designer.**

I. INTRODUCTION

In reliability theory, mission reliability is defined as the probablity that a system will perform its specified mission. In the development of a system, mathematical modeling for the mission reliability is an important tool in predicting mission reliability, especially in military weapon systems.

But, in practice, it is sometimes not easy to set up a model for mission reliability. Two major difficulties are:

1. In a reliability sense, some components are not necessary for a certain mission, for example, the night vision telescope of a combat tank is not necessary for a day-time mission. This suggests that we have to consider appropriate components in the model for the given mission. A similar problem is discussed by Tillman et al. [4] who use relative performance factors for partially working subsystems or components. (In [2, 3], relative performance is called Mission Reliability Factor.)
2. It is usually assumed that the states of all the components are statistically independent. But in actual systems, the failures of some components are not statistically independent of those of other components.

This paper proposes mathematical models for the two main missions, moving and firing, of a combat tank taking into account the above two difficulties, and discusses how the models can be used by the combat tank planner and the tank system designer.

II. PRELIMINARIES

1. Basic Model

Let M and F be two component sets such that —

$M \equiv$ {components which are required to operate for the moving mission}

$F \equiv$ {components which are required to operate for the firing mission}.

Then —

$$\Pr\{M\} \equiv \Pr\{\text{all the components in } M \text{ are working}\}$$
$$\Pr\{F\} \equiv \Pr\{\text{all the components in } F \text{ are working}\}.$$

are the moving-mission reliability and the firing-mission reliability respectively. To devise a mathematical model of $\Pr\{M\}$ and $\Pr\{F\}$, we have to decide which components belong to M and F. The components in M can be chosen easily from the reliability block diagram. But for F, it is not obvious which components are required to operate for the firing mission since some firing missions are performed while the tank is moving and others are performed while the tank is stationary.

We can write —

$$\begin{aligned}\Pr\{F\} = {} & \Pr\{\text{tank is moving}\} \Pr\{[\text{all the components in } F \text{ are working}] \mid [\text{tank is moving}]\} \\ & + \Pr\{\text{tank is not moving}\} \Pr\{[\text{all the components in } F \text{ are working}] \mid [\text{tank is not moving}]\}\end{aligned} \tag{2-1}$$

Unfortunately, we cannot choose simply the subsystems or components for the set F since the required components depend on whether or not the tank is *required* to move rather than whether or not the tank *is* moving. If the tank is required to move a certain distance to destroy a given target, then all the components in M also have to belong to the set F. But if the firing mission can be performed in the stationary position, then some components such as transmission in M do not have to belongt to F.

Let Q be the event that the tank is required to move a certain distance. Then

$$\Pr\{F\} = \Pr\{Q\} \Pr\{F \mid Q\} + \Pr\{\bar{Q}\} \Pr\{F \mid \bar{Q}\}. \tag{2-2}$$

$\Pr\{Q\}$ can be obtained from combat records or a simulated combat scenario, and $\Pr\{F|Q\}$ and $\Pr\{F|\bar{Q}\}$ can be selected from the reliability block diagram, either assuming that the tank is required to move or that it is not.

2. *Associated Components Set*

Notation

C — $\{1,2, \ldots, n\}$, the system component set
E_j — event that component j is working
B, D — nonempty sets of components such that $B \subset C$, $D \subset C$ and $B \cap D = \phi$.

The component set $B \cup D$ is *associated*, if —

1. $\Pr\{\bigcap_{j\in B} E_j \mid \bigcap_{j\in D} E_j\} \geq \Pr\{\bigcap_{j\in B} E_j\}$
2. $\Pr\{\bigcap_{j\in B} E_j^c \mid \bigcap_{j\in D} E_j^c\} \geq \Pr\{\bigcap_{j\in B} E_j^c\}$

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Then if S is an *associated* components set, it is easily seen that —

$$\prod_{j \in S} \Pr\{E_j\} \le \Pr\{\bigcap_{j \in S} E_j\} \le \min_{j \in S}\{\Pr\{E_j\}\} \tag{2-3}$$

$$\prod_{j \in S} \Pr\{E_j^c\} \le \Pr\{\bigcap_{j \in S} E_j^c\} \le \min_{j \in S}\{\Pr\{E_j^c\}\} \tag{2-4}$$

The above definition and properties mimic those of *associated* random variables in [1, p 29].

III. MISSION RELIABILITY MODEL

Notation

- S_i moving mission reliability of subsystem i
- S_i' firing mission reliability of subsystem i under the condition that tank is required to move a certain distance
- S_i'' firing mission reliability of subsystem i under the condition that tank is not required to move
- R_{ij} reliability of component j in subsystem i

A combat tank [3] consists of seven major systems: Power Pack $(i=P)$, Suspension $(i=S)$, Hydraulic System $(i=H)$, Main Armament $(i=M)$, Communication System $(i=C)$, Electrical System $(i=E)$ and Fire Control System $(i=F)$.

Since the Main Armament and Fire Control System are not required for a moving mission and the other subsystems are statistically independent, then —

$$\Pr\{M\} \,.\, S_P\, S_S\, S_H\, S_E\, S_C. \tag{3-1}$$

Since we do not yet know which components are required for the firing mission under the conditions that [the tank is required to move] and [the tank is not required to move], we write $\Pr\{F \mid Q\}$ and $\Pr\{F \mid \overline{Q}\}$ as —

$$\Pr\{F \mid Q\} \,.\, S_P'\, S_S'\, S_H'\, S_E'\, S_C'\, S_M'\, S_F' \tag{3-2}$$

$$\Pr\{F \mid \overline{Q}\} = S_P''\, S_S''\, S_H''\, S_E''\, S_C''\, S_M''\, S_F'' \tag{3-3}$$

Now we have to choose appropriate components from each subsystem to calculate S_i, S_i', S_i''.

Notation

- F_Q the components which are required for a firing mission under the condition that the tank is required to move a certain distance
- $F_{\overline{Q}}$ the components which are required for a firing mission under the condition that the tank is not required to move

1. Power Pack System [3]

The Power Pack System consists of eight components: Engine (P_1), Cooling System (P_2), Air-In-Take System (P_3), Transmission (P_4), Transmission Control System (P_5), Fuel System (P_6), Final drive (P_7), Brake System (P_8).

$$\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\} \subset (M \cap F_Q)$$

$$\{P_1, P_2, P_3, P_6, P_7, P_8\} \subset F_{\overline{Q}}$$

and $\{P_1, P_2, P_3, P_4, P_5\}$ is an *associated* component set, since the failure of the Cooling System will increase the Engine's and the Transmission's failure probability, the failure of the Air-In-Take System will increase the Engine's failure probability and the failure of the Transmission Control System will increase the Transmission's failure probability. So we can write —

$$S_P = S_P' = R_{\{P_1,P_2,P_3,P_4,P_5\}} \prod_{j=6}^{8} R_{P_j}$$

and from (2-3), lower and upper bounds for $R_{\{P_1,P_2,P_3,P_4,P_5\}}$ are —

$$\prod_{j=1}^{5} R_{P_j} \le R_{\{P_1,P_2,P_3,P_4,P_5\}} \le \min_{1 \le j \le 5}\{R_{P_j}\}.$$

Thus —

$$\prod_{j=1}^{8} R_{P_j} \le S_P = S_P' \le (\min_{1 \le j \le 5}\{R_{P_j}\}) \prod_{j=6}^{8} R_{P_j}. \tag{3-4}$$

Further —

$$S_P'' = R_{\{P_1,P_2,P_3\}} \prod_{j=6}^{8} R_{P_j}$$

so that —

$$\prod_{j=1}^{3} R_{P_j} \prod_{j=6}^{8} R_{P_j} \le S_P'' \le (\min_{1 \le j \le 3}\{R_{P_j}\}) \prod_{j=6}^{8} R_{P_j}. \tag{3-5}$$

2. Suspension System [3]

The Suspension System has 20 components: Track System (S_1), Sprocket System (S_2), Torsion Bar No. 1 – No. 6 $(S_3, \ldots, S_8)$, Road Wheel Arm Assembly No. 1 – No. 6 $(S_9, \ldots, S_{14})$, Right Support Roller No. 1, No.2(S_{15}, S_{16}), Left Support Roller No. 1, No. 2 (S_{17}, S_{18}), Idler Wheel Arm Assembly (S_{19}), and Track Tension Adjuster (S_{20}).

$$\{S_1, \ldots, S_{20}\} \subset (M \cap F_Q)$$

$$\{S_1, S_3, \ldots, S_{14}\} \subset F_{\overline{Q}}$$

and $\{S_1, S_2\}$, $\{S_3, \ldots, S_{14}\}$, $\{S_{15}, S_{16}\}$, $\{S_{17}, S_{18}\}$ are four *associated* component sets. Thus S_S, S_S', S_S'' can be expressed as:

$$\prod_{j=1}^{20} R_{S_j} \le S_5 = S_5' \le (\min_{1 \le j \le 2}\{R_{S_j}\})(\min_{3 \le j \le 14}\{R_{S_j}\}) (\min_{15 \le j \le 16}\{R_{S_j}\})(\min_{17 \le j \le 18}\{R_{S_j}\}) \prod_{j=19}^{20} R_{S_j} \tag{3-6}$$

$$R_{S_1} \prod_{j=3}^{14} R_{S_j} \le S_S'' \le R_{S_1} (\min_{3 \le j \le 14}\{R_{S_j}\}) \tag{3-7}$$

3. Hydraulic, Electrical, Communication, Main Armament and Fire Control System [3]

In table 1, the symbol ✓ indicates that those components belong to the appropriate component set. There is no *associated* component set in any subsystem.

TABLE 1
Hydraulic, Electrical, Communication, Main Armament, and Fire Control System.

Subsystem	Symbol	Component	M	F_Q	$F_{\bar{Q}}$
Hydraulic System	H_1	Pump	✓	✓	✓
	H_2	Hull Dist. Manifold	✓	✓	✓
	H_3	Hydraulic Connector and Tube	✓	✓	✓
	H_4	Turret Dist. Manifold		✓	✓
Electrical System	E_1	Power Supply	✓	✓	✓
	E_2	Control Panel	✓	✓	✓
	E_3	Network Box	✓	✓	✓
	E_4	Slip Ring	✓	✓	✓
Communication System	C_1	Radio Set	✓	✓	✓
	C_2	Inter Comm.	✓	✓	✓
Main Armament	M_1	Gun		✓	✓
	M_2	Mount		✓	✓
Fire Control System	F_1	Fire Control Unit		✓	✓
	F_2	Ballistic Computer		✓	✓
	F_3	Gunner's Primary sight		✓	✓
	F_4	Commander's Primary sight		✓	✓
	F_5	Gun-Turret Driving System		✓	✓

Using the information in the table it follows that –

$$S_H = \prod_{j=1}^{3} R_{H_j} \tag{3-8}$$

$$S'_H = S''_H = \prod_{j=1}^{4} R_{H_j} \tag{3-9}$$

$$S_E = S'_E = S''_E = \prod_{j=1}^{4} R_{E_j} \tag{3-10}$$

$$S_C = S'_C = S''_C = \prod_{j=1}^{2} R_{C_j} \tag{3-11}$$

$$S'_M = S''_M = \prod_{j=1}^{2} R_{M_j} \tag{3-12}$$

$$S'_F = S''_F = \prod_{j=1}^{5} R_{F_j} \tag{3-13}$$

So combining (3-1) – (3-13) and (2-2) we see that:

$$\Pr\{M\}_L \le \Pr\{M\} \le \Pr\{M\}_U \tag{3-14}$$

$$\alpha \Pr\{F \mid Q\}_L + \bar{\alpha} \Pr\{F \mid \bar{Q}\}_L \le \Pr\{F\} \le \alpha \Pr\{F \mid Q\}_U + \bar{\alpha} \Pr\{F \mid \bar{Q}\}_U \tag{3-15}$$

$$\Pr\{M\}_L \equiv \prod_{j=1}^{8} R_{P_j} \cdot \prod_{j=1}^{20} R_{S_j} \cdot \prod_{j=1}^{3} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j}$$

$$\Pr\{M\}_U \equiv (\min_{1\le j\le 5} \{R_{P_j}\}) \cdot \prod_{j=6}^{8} R_{P_j} (\min_{1\le j\le 2} \{R_{S_j}\}) \cdot (\min_{3\le j\le 14} \{R_{S_j}\}) \cdot (\min_{15\le j\le 16} \{R_{S_j}\}) \cdot (\min_{17\le j\le 18} \{R_{S_j}\}) \cdot \prod_{j=19}^{20} R_{S_j} \cdot \prod_{j=1}^{3} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j}$$

$$\Pr\{F \mid Q\}_L \equiv \prod_{j=1}^{8} R_{P_j} \cdot \prod_{j=1}^{20} R_{S_j} \cdot \prod_{j=1}^{4} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j} \cdot \prod_{j=1}^{2} R_{M_j} \cdot \prod_{j=1}^{5} R_{F_j}$$

$$\Pr\{F \mid \bar{Q}\}_L \equiv \prod_{j=1}^{3} R_{P_j} \cdot \prod_{j=6}^{8} R_{P_j} \cdot R_{S_1} \cdot \prod_{j=3}^{14} R_{S_j} \cdot \prod_{j=1}^{4} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j} \cdot \prod_{j=1}^{2} R_{M_j} \cdot \prod_{j=1}^{5} R_{F_j}$$

$$\Pr\{F \mid Q\}_U \equiv (\min_{1\le j\le 5} \{R_{P_j}\}) \cdot \prod_{j=6}^{8} R_{P_j} \cdot (\min_{1\le j\le 2} \{R_{S_j}\})$$

$$\cdot \left(\min_{3\le j\le 14} \{R_{S_j}\}\right) \cdot \left(\min_{15\le j\le 16} \{R_{S_j}\}\right) \cdot \left(\min_{17\le j\le 18} \{R_{S_j}\}\right)$$

$$\cdot \prod_{j=19}^{20} R_{S_j} \cdot \prod_{j=1}^{4} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j}$$

$$\cdot \prod_{j=1}^{2} R_{M_j} \cdot \prod_{j=1}^{5} R_{F_j}$$

$$\Pr\{F \mid \bar{Q}\}_U \equiv \left(\min_{1\le j\le 3} \{R_{P_j}\}\right) \cdot \prod_{j=6}^{8} R_{P_j} \cdot R_{S_1}$$

$$\cdot \left(\min_{3\le j\le 14} \{R_{S_j}\}\right) \cdot \prod_{j=1}^{4} R_{H_j} \cdot \prod_{j=1}^{4} R_{E_j} \cdot \prod_{j=1}^{2} R_{C_j}$$

$$\cdot \prod_{j=1}^{2} R_{M_j} \cdot \prod_{j=1}^{5} R_{F_j}$$

$$\alpha \equiv \Pr\{Q\},\ \bar{\alpha} \equiv \Pr\{\bar{Q}\} = 1 - \alpha$$

IV. NUMERICAL EXAMPLE

Each component has a constant failure rate. Table 2 shows the component failure rates. Choose: $\alpha = 0.8$. The mission reliabilities for moving and firing under these assumptions are plotted in figure 1.

TABLE 2
Failure Rates

Component	Failure Rate (per 10^{12} hr)	Component	Failure Rate (per 10^{12} hr)
P_1	110	S_{19}	80
P_2	70	S_{20}	138
P_3	30	H_1	330
P_4	180	H_2	25
P_5	80	H_3	15
P_6	110	H_4	25
P_7	10	E_1	20
P_8	167	E_2	20
S_1	770	E_3	20
S_2	200	E_4	16.7
$S_3 - S_8$	620	C	10
$S_9 - S_{14}$	292	M	16.7
$S_{15} - S_{18}$	143	F	625

V. CONCLUDING REMARKS

These models may be used to specify the reliability requirement by the combat tank planner and to predict the mission reliabilities by the tank system designer before prototypes are developed. To use these models, the essential input data are the components in sets M and F, component *association*, the probability α, and the failure rates of the components. The components in F should be determined under the conditions that the tank is required to move a certain distance and that it is not.

In practice, if it is enough to predict lowest bounds for the mission reliabilities, then choose $\alpha = 1.0$, and all the

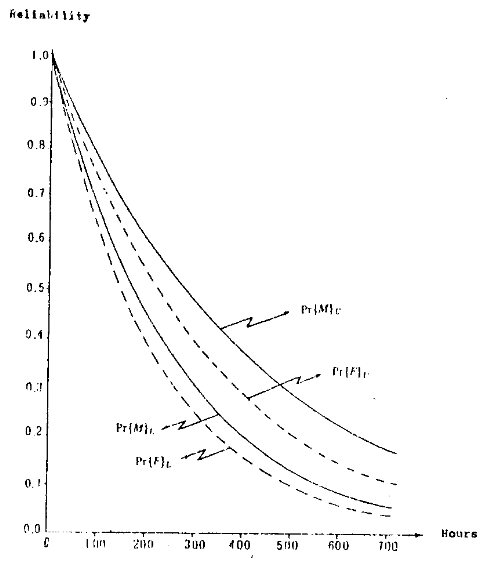

Fig. 1 Upper and Lower Bounds of $\Pr\{M\}$ and $\Pr\{F\}$.

components are statistically independent since in both cases the $\Pr\{M\}$ and $\Pr\{F\}$ can be replaced by lower bounds. Further, $\Pr\{F\} \le \Pr\{M\}$ since –

$\Pr\{F \mid Q\} = \Pr\{$all the components in F and M are working$\} \le \Pr\{M\}$

so it is enough to predict only $\Pr\{F\}$.

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