Reliability Prediction Using Nondestructive Accelerated-Degradation Data: Case Study on Power Supplies

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Key Words — **Accelerated testing, stochastic process, Birnbaum-Saunders distribution.**

Summary & Conclusions — This paper describes a conceptual framework for reliability evaluation from nondestructive accelerated degradation data (NADD). A numerical example of data sets from power supply units for electronic products is presented using this framework. We model NADD as a collection of stochastic processes for which the parameters depend on the stress levels. The relationship between these parameters and the associated stresses is explored using regression. The failure-time of power-supply units is modeled by the Birnbaum-Saunders distribution, for which the confidence bounds and tolerance limits can be easily obtained.

1. INTRODUCTION

To compete in today's global market, many electronic products are made with high reliability. This results in very few, if any, failures even under elevated stresses, so that conventional life testing is no longer effective in assessing their reliability. This problem is compounded by the short product-cycle in the fast-moving electronic industry where field-failure data were often collected too late to be useful.

Acronyms

NADD nondestructive accelerated degradation data
TC/BS[1] Tang-Chang Birnbaum-Saunders (method).

This paper formulates the TC/BS conceptual framework for evaluating the reliability of highly reliable electronic modules from NADD. This provides a channel for reliability assessment without the labor of conventional life testing. We model NADD as a collection of realizations of stochastic processes for which the parameters depend on the stress[2] levels. This is to account for the variability in the degradation path which has been overlooked in the random coefficient model of Nelson [9]. The relationship between these parameters and the associated stresses is explored using regression methods from which some linkage functions are established. The failure-time of the module, under some rather mild conditions, can be modeled by the Birnbaum-Saunders distribution for which the reliability bounds and tolerance limits can be obtained. TC/BS is applied to the testing of power-supply units.

The key element in using TC/BS is to identify some failure mechanisms of the module which degrade observably (with measurement error). For example, for a power-supply unit, failures are commonly due to low DC output, and most such units show downward drift in their DC outputs. The DC output can easily be measured & monitored. In other cases, resistance or other measures are a possible degradation measure.

After identifying a degradation measure, modules are then subject to a statistically designed accelerated experiment. In contrast with typical life testing, the experiment is conducted nondestructively to explore the degradation property. As in other types of accelerated testing, the statistics of interest can be obtained in a shorter time. For power-supply units, a simple 2-way layout with input voltage and temperature as factors has been used. Chang [3] established that a generalized Eyring model [9] is adequate for explaining degradation measure dependence on the accelerated stresses.

Using regression methods, it is frequently possible to establish linkage functions which relate the stress levels and the parameters of the stochastic process model of the degradation paths. The failure time is then given by the first passage time of the underlying stochastic process, under use condition, to a critical value. For power-supply units, some simple linear regression models using input voltage and temperature as independent variables provide very good fit. In other cases, nonlinear regression models might be needed.

A comprehensive account of probabilistic failure modeling was given by Lemoine & Wenocur [7]. Other references dealing with statistical models are in [9] where some degradation models were briefly reviewed. Lu & Meeker [8] treated a general degradation model and estimated the time to failure distribution via simulation.

Notation

AC, DC	[AC, DC] voltage output
ϵ	error for generalized Eyring model, a r.v.
ζ_i	coefficient for generalized Eyring model, $i=1,2,3,4$
T	absolute temperature
$L_r(\cdot)$	linkage function for parameter r
θ_i	p-dimensional parameter vector governing NADD at stress level i
Θ	set of θ_i
α, β	parameters of Birnbaum-Saunders distribution
μ_i, σ_i	[mean, standard deviation] of incremental degradation at stress level i

[1]*Editors' note:* We have assigned this acronym for simple, clear, unique reference to the concept.
[2]The term *stress* can imply a set of stresses at a particular *level*.

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μ, σ	[mean, standard deviation] of incremental degradation at use condition
W	total tolerable degradation level
W_o	total tolerable degradation level at use condition
$X(t)$	stochastic process describing the degradation path
t_k	sampling-epoch k
R^2	coefficient of determination
a_i	coefficients for regression model of μ, $i=1,2,3$
b_i	coefficients for regression model of σ, $i=1,2,3,4$
γ_i	level of s-significance, $i=1,2$
$Y_{i,j,k}$	amount of (transformed) degradation over time-interval k for item j at stress-level i
DC_0, DC_T	[initial, lowest tolerable] DC output.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. MODELING & ANALYSIS OF NADD

2.1 Incremental Degradation

Assumptions

1. There is a family of stochastic processes which are parameterized by:

$$\Theta = \{\theta\text{: } \theta \text{ is uniquely defined by some stress levels}\}.$$

Thus at any fixed stress-level i, there exists a $\theta_i \in \Theta$ and a collection of stochastic processes $\{X(t;\theta_i)\}$ which suitably describe the degradation paths (or their transformations) of the selected degradation measure for each item tested under the stress level.

2. There exist some linkage functions $L_r(\cdot)$, $r=1,\ldots,p$, such that:

$$\theta_i = L_r(\text{stress-level } i);$$

eg, a linear function of stresses.

3. The sampling interval, $\Delta t = t_{k+1} - t_k$, is constant.

4. $\{Y_{i,j,k}\}$ is a sequence of i.i.d. r.v. with the same μ_i and σ_i for all j,k. ◀

The $X(t)$ models the reality that if an item were tested repeatedly under the same condition, the degradation paths would be different. The linkage function gives the parameters for degradation measure under use conditions.

The purpose of transforming some variables is to make these incremental degradations i.i.d. r.v. This can be done using Box-Cox transformation as in regression analysis [5] or the analogous technique applied to time-series data.

For item j sampled at time epoch t_k, the $Y_{i,j,k} = X_j(t_{k+1};\theta_i) - X_j(t_k;\theta_i)$. From assumption #3, $\theta_i = (\mu_i, \sigma_i)$. Without loss of generality, let the constant sampling interval,

$$\Delta t = t_{k+1} - t_k = 1.$$

Then at some sufficiently large time, t,

$$X_j(t;\ \mu_i,\sigma_i) = \sum_{k=0}^{t} Y_{i,j,k} \sim N(\mu_i\cdot t,\ \sigma_i^2\cdot t)$$

by the central limit theorem.

2.2 Failure-Time Distribution

Let the failure time, τ, be the first passage time of $X(t;\mu_i,\sigma_i)$ to a level $W \gg \mu_i$. Then

$$F(t;\mu_i,\sigma_i,W) = \text{gauf}\left[\frac{1}{\sigma_i}\cdot\left(\mu_i\cdot\sqrt{t} - \frac{W}{\sqrt{t}}\right)\right], \tag{1a}$$

since $\{\tau \le t\} \equiv \{X(t) \ge W\}$.

$$F(t;\alpha_i,\beta_i) = \text{gauf}\left[\frac{1}{\alpha_i}\cdot\xi(t/\beta_i)\right],$$

the Birnbaum-Saunders distribution with parameters α_i, β_i;

$$\xi(t) \equiv \sqrt{t} - 1/\sqrt{t},\ t > 0$$

$$\alpha_i \equiv \sigma_i/\sqrt{\mu_i\cdot W},\ \beta_i \equiv W/\mu_i.$$

In showing that the failure time is a Birnbaum-Saunders r.v., there is no need for s-normality assumption in $Y_{i,j,k}$. We need $W \gg \mu_i$ such that the central limit theorem can be invoked. As the degradation is sampled at a constant time interval, a reasonable time-unit is the sampling interval. For power-supply units, it has the added advantage that the cyclical current and temperature stresses are analogous to mechanical fatigue which is the genesis of the Birnbaum-Saunders distribution [2]. Other virtues of the Birnbaum-Saunders distribution relative to those of inverse Gaussian are in Desmond [4] and Bhattacharyya & Fries [1]. The Birnbaum-Saunders distribution is also simpler than the generalized inverse Gaussian distribution, which is the first passage time of a diffusion process [6].

To establish the failure distribution at use condition, we estimate the corresponding parameters using the linkage functions:

$$\mu_i = f(\text{stress-level } i),$$

$$\sigma_i = g(\text{stress-level } i).$$

We can obtain these estimates at use condition by regressing the linkage function followed by extrapolation. s-Confidence intervals for μ, σ can be obtained from the linkage function as in regression analysis.

Substitute μ, σ, W_o into (1a) to get the Cdf for the failure time at use condition:

$$F(t;\mu,\sigma,W_o) = \text{gauf}\left[\frac{1}{\sigma}\cdot\left(\mu\cdot\sqrt{t} - \frac{W_o}{\sqrt{t}}\right)\right]. \tag{1b}$$

$$\frac{\partial F}{\partial \mu} = \frac{\sqrt{t}}{\sigma}\cdot \text{gaud}\left[\frac{1}{\sigma}\cdot\left(\mu\cdot\sqrt{t} - \frac{W_o}{\sqrt{t}}\right)\right] > 0 \text{ for all } t > 0;$$

$$\frac{\partial F}{\partial \sigma} = -\frac{1}{\sigma}\cdot\left(\mu\cdot\sqrt{t} - \frac{W_o}{\sqrt{t}}\right)\text{gaud}\left[\frac{1}{\sigma}\cdot\left(\mu\cdot\sqrt{t} - \frac{W_o}{\sqrt{t}}\right)\right]$$

> 0 for $t \le W_o/\mu$,

< 0 otherwise,

since gaud$(t) > 0$ for all t. These monotone properties of the Cdf provide the necessary condition to establish a s-confidence bound for $F(t)$.

2.3 s-Confidence Bounds and Tolerance Limits

Notation

$[\underline{\mu},\bar{\mu}]$ $(1-\gamma_1)$ s-confidence interval for μ
$[\underline{\sigma},\bar{\sigma}]$ $(1-\gamma_2)$ s-confidence interval for σ.

By definition, we only need to show that,

$$\Pr\{F(t;\underline{\mu},\underline{\sigma}) \le F(t;\mu,\sigma) \le F(t;\bar{\mu},\bar{\sigma})\} \ge 1-\gamma_1-\gamma_2$$

for $t \le W_o/\mu$;

$$\Pr\{F(t;\underline{\mu},\bar{\sigma}) \le F(t;\mu,\sigma) \le F(t;\bar{\mu},\underline{\sigma})\} \ge 1-\gamma_1-\gamma_2$$

otherwise.

From these monotone properties, for $\underline{\mu} \le \mu \le \bar{\mu}$, $\underline{\sigma} \le \sigma \le \bar{\sigma}$,

$$F(t;\underline{\mu},\underline{\sigma}) \le F(t;\mu,\sigma) \le F(t;\bar{\mu},\bar{\sigma}) \text{ for } t \le W_o/\mu$$

$$F(t;\underline{\mu},\bar{\sigma}) \le F(t;\mu,\sigma) \le F(t;\bar{\mu},\underline{\sigma}) \text{ otherwise.}$$

Hence, for $t \le W_o/\mu$,

$$\Pr\{F(t;\underline{\mu},\underline{\sigma}) \le F(t;\mu,\sigma) \le F(t;\bar{\mu},\bar{\sigma})\}$$

$$\ge \Pr\{\underline{\mu} \le \mu \le \bar{\mu},\ \bar{\sigma} \le \sigma \le \bar{\sigma}\}$$

$$\ge 1-\Pr\{(\underline{\mu} \le \mu \le \bar{\mu})^c\} - \Pr\{(\underline{\sigma} \le \sigma \le \bar{\sigma})^c\}$$

by the Bonferroni inequality

$$= 1-\gamma_1-\gamma_2.$$

A similar inequality holds for $t > W_o/\mu$ which gives a $(1-\gamma_1-\gamma_2)$ level s-confidence interval for $F(t)$ as follows:

$$F(t;\underline{\mu},\underline{\sigma}),F(t;\bar{\mu},\bar{\sigma}) \text{ for } t \le W_o/\mu, \tag{2a}$$

$$F(t;\underline{\mu},\bar{\sigma}),F(t;\bar{\mu},\underline{\sigma}) \text{ otherwise.} \tag{2b}$$

A 1-sided $(1-q)$ lower tolerance limit at $(1-\gamma_1-\gamma_2)$ s-confidence level is given by the $(1-\gamma_1-\gamma_2)$ s-confidence intervals of percentile q [10]; which, in turn, can be obtained by the corresponding upper s-confidence bound of the Cdf:

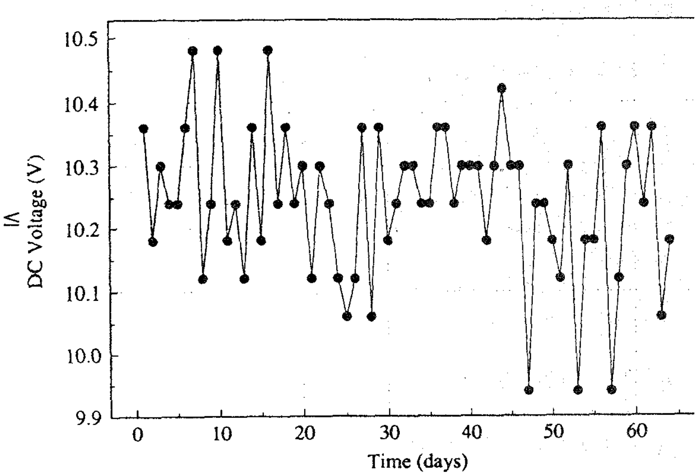
$$F(t;\bar{\mu},\bar{\sigma}) = q \text{ for } q \le 0.5,\ ie,\ t \le W_o/\mu;$$

$$F(t;\bar{\mu},\underline{\sigma}) = q \text{ otherwise;}$$

since W_o/μ is the median time to failure.

3. APPLICATION

For the power-supply unit, the DC output is used as the degradation measure because, besides thermal breakdown, low DC output is the most common cause of failure. A 2-way design using 3 temperatures (50 °C, 60 °C, 70 °C); and 2 input AC voltage levels (108 V, 116 V) was conducted. The response, DC output, was collected over fixed sampling interval by an automatic monitoring system. There were 5 replications in each setting. Figure 1 shows a typical data set. There is a downward drift in the DC output.



[T=333 K (60 °C), AC = 108 V]

Figure 1. A Typical DC Voltage Output

The following generalized Eyring dependence fits reasonably well [3].

$$\log[\mathrm{DC}_{i_1,i_2,k}(t)] = \zeta_0\cdot T_{i_1}\cdot \mathrm{AC}_{i_2}$$

$$- t\cdot[\zeta_1\cdot T_{i_1}\cdot\exp(-\zeta_2/T_{i_1})\cdot\exp[\log(\mathrm{AC}_{i_2})\cdot(\zeta_3+\zeta_4/T_{i_1})]]$$

$$+ \epsilon_{i_1,i_2,k}, \tag{3}$$

$i_1 = 1,2,3$ and $i_2 = 1,2$;

ϵ = measurement error (i.i.d. s-normal with zero mean).

From (3) the transformed incremental degradation $\log[\mathrm{DC}(t+\Delta t)]-\log[\mathrm{DC}(t)]$ can be modeled as i.i.d. s-normal r.v. for fixed sampling interval Δt, and given settings of AC & T.

Table 1 gives estimated means and standard deviations of the transformed increments for 6 combinations of stress levels with 5 realizations each. The sampling interval is fixed at 1-day. To check that autocorrelation does not exist, standard diagnostics on the residuals in [3] did not show any statistical evidence that this assumption is violated. Alternatively, standard techniques in time-series analysis using the sample autocorrelation function can be used because the data were sampled at a constant time-interval. Figure 2 shows a check on sample plot for the ACF.

TABLE 1
Estimates for Means and Standard Deviations of the Increments
[Multiply the numbers in the body of the table by 10^{-4}]

	AC Voltage			
	116 V		108 V	
Temperature	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
	11.63	7.132	6.275	6.374
70 °C	14.31	7.580	5.682	7.205
(343 K)	13.49	7.490	6.823	6.950
	13.85	7.599	6.828	6.756
	14.22	7.115	5.302	6.301
	16.12	6.441	6.298	7.107
60 °C	16.43	6.658	6.040	7.351
(333 K)	17.91	6.589	4.718	7.392
	17.16	6.471	7.040	7.117
	16.05	6.202	7.249	8.370
	15.46	7.369	9.564	7.485
50 °C	16.46	7.160	6.961	7.757
(323 K)	15.89	6.938	5.516	6.983
	16.00	7.352	6.760	7.181
	15.84	6.990	8.832	7.654

Under the usual assumption of s-normally distributed error with zero mean and constant variance in regression, several linkage functions relating the estimators to the stress levels have been explored.

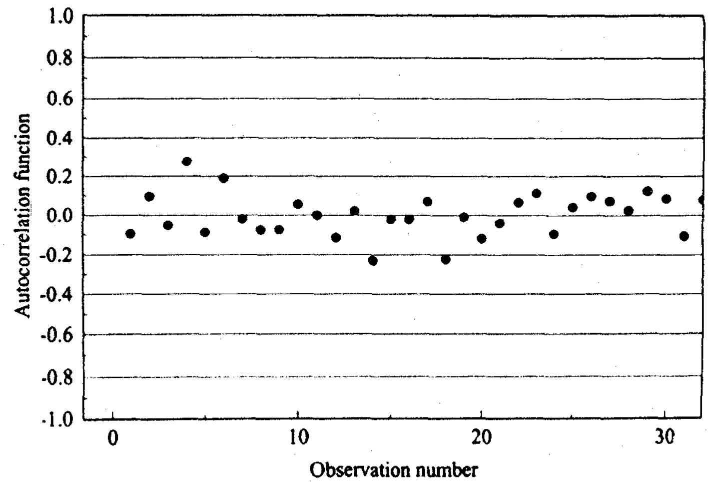
For μ_i, one of the best fit is the following with adjusted $R^2 = 0.9891$.

$$\mu_i = a_1 \cdot \mathrm{AC}_{i_2} + a_2 \cdot T_{i_1} + a_3 \cdot \mathrm{AC}_{i_2} \cdot T_{i_1}, \tag{4}$$

$$(\hat{a}_1, \hat{a}_2, \hat{a}_3) = (3.83\mathrm{E}{-5}, -3.33\mathrm{E}{-5}, 2.12\mathrm{E}{-7}),$$

all the P-values[3] are smaller than 0.0001 indicating that these parameters are highly s-significant.

[3] P-value is the smallest s-significance level that would lead to rejecting the hypothesis: parameters = 0.



[T=333 K (60 °C), AC = 108 V]

Figure 2. Typical Autocorrelation Function

For σ_i, a best fit with adjusted $R^2 = 0.9952$ is:

$$\sigma_i = b_1 \cdot \mathrm{AC}_{i_2} + b_2 \cdot T_{i_1} + b_3 \cdot \mathrm{AC}_{i_2} \cdot T_{i_1}, \tag{5}$$

$$(\hat{b}_1, \hat{b}_2, \hat{b}_3) = (9.60\mathrm{E}{-5}, 2.90\mathrm{E}{-5}, -3.57\mathrm{E}{-7}),$$

their P-values are 0.0064, 0.0006, 0.0055 respectively; showing that these parameters are s-significant.

From (4) & (5) we can compute $\hat{\mu}$ & $\hat{\sigma}$, at the use condition T= 293 K (20 °C) and AC = 120 V as:

$$\hat{\mu} = 2.29 \cdot 10^{-3},\ \hat{\sigma} = 7.46 \cdot 10^{-3}.$$

Their 95% s-confidence intervals, obtainable from standard multiple regression analysis, are:

$$(\underline{\mu}, \bar{\mu}) = (2.04 \cdot 10^{-3}, 2.54 \cdot 10^{-3}) \tag{6a}$$

$$(\underline{\sigma}, \bar{\sigma}) = (6.43 \cdot 10^{-3}, 8.49 \cdot 10^{-3}) \tag{6b}$$

When DC_0 = 12V and DC_T = 8V, such that $\mathrm{DC}_T < \mathrm{DC}_0$,

$$W_o = \log(\mathrm{DC}_0) - \log(\mathrm{DC}_T) = \log(12) - \log(8) = 0.4055;$$

substitute into the Cdf (1b):

$$\hat{F}(t) = \mathrm{gauf}[0.307\sqrt{t} - 54.4/\sqrt{t}].$$

A 90% s-confidence bounds for the Cdf (see figure 3) can be obtained by using (2) with the 95% s-confidence limits of μ, σ in (6) with $\gamma_1 = \gamma_2 = 0.05$. A 1-sided 80% lower tolerance limit at 90% s-confidence level can be obtained by solving:

$$F(t;\ \bar{\mu} = 2.54 \cdot 10^{-3},\ \bar{\sigma} = 8.49 \cdot 10^{-3}) = \mathrm{gauf}[0.299\sqrt{t} - 47.76/\sqrt{t}] = 0.2$$

which is 127.9. Alternatively, it can be read from figure 3 as 128.

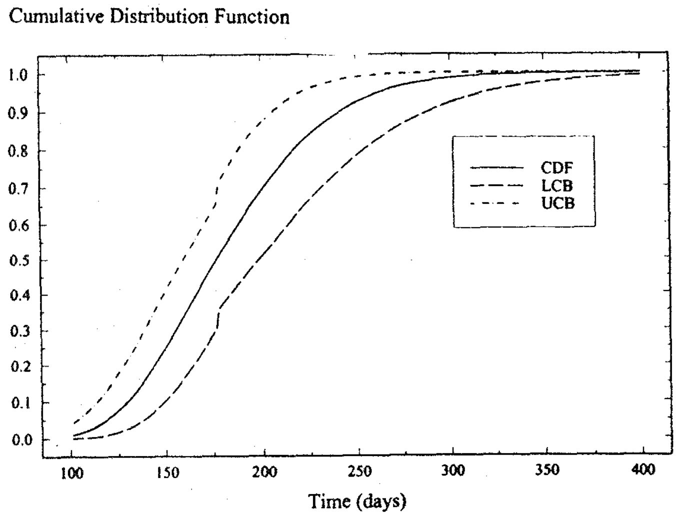


Figure 3. Cdf of the Power-Supply Unit and Its 90% *s*-Confidence Bounds

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