Prioritizing System-Reliability Prediction Improvements

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***Abstract*—This method prioritizes system-reliability prediction activities once a preliminary reliability-prediction has been made. System-reliability predictions often use data and models from a variety of sources, each with differing degrees of estimation uncertainty. Since time and budgetary constraints limit the extent of analyzes and testing needed to estimate component reliability, it is necessary to allocate limited resources intelligently. A Reliability-Prediction Prioritization Index (RPPI) is defined to provide a relative ranking of components based on their potential for improving the accuracy of a system-level reliability prediction by decreasing the variance of the system-reliability estimate. If a component has a high RPPI, then additional testing or analysis should be considered to decrease the variance of the component reliability estimate. RPPI is based on a decomposition of the variance of the system-reliability or on a mean-time-to-failure estimate. Using these indexes, the effect of individual components within the system can be compared, ranked, and assigned to priority groups. The ranking is based on whether a decrease of the component-reliability estimate variance meaningfully decreases the system-reliability estimate variance. The procedure is demonstrated with two examples.**

***Index Terms*—Estimation variance, reliability prediction, variance decomposition.**

ACRONYMS[1]

LB	lower bound
MTTF	mean time to failure
RPP	reliability-prediction prioritization
RPPI	RPP index
SD	standard deviation
Var	variance

I. INTRODUCTION

A. Background

SYSTEM-LEVEL reliability predictions are generally developed based on a system model and component-level reliability predictions. Component-level reliability can be determined from a variety of sources. It is not unusual to use distinct sources of component-reliability data or models when predicting system reliability. A new system will likely have some components that have been used previously so there are plentiful data available. Alternatively, the same system can use newer-technology components with very little reliability data. For these components, reliability estimation can be based, e.g., on extrapolated accelerated life testing or an analytic physics-of-failure model. There are uncertainties within each of these sources. These uncertainties must be understood to decide how limited resources should be allocated to improve reliability predictions.

System-reliability prediction is often conducted iteratively as the design evolves. For some systems, a preliminary reliability prediction is determined using previous data to initially compute reliability predictions at the assembly or subsystem-level. Then, as more detailed design information becomes available, the assembly-level predictions are updated or replaced with reliability predictions based on the components within the assemblies. For other system-reliability predictions, a parts-count reliability prediction is initially determined, once the parts within the system have been identified, but prior to determining the specific stresses acting on the part. Then, when more detailed design information has been determined, the system-reliability prediction is revised, and hopefully improved, based on a parts-stress reliability prediction and part-level models.

When reliability prediction is conducted in stages, it is advisable to prioritize reliability-prediction improvement activities. The uncertainty of reliability predictions for some components might have negligible effect relative to the system-reliability prediction. In these cases, it might not be efficient to devote any additional effort to improving the prediction by decreasing the estimation variance. This requires additional and more detailed analyzes or testing that might yield unimportant system-level improvements. Alternatively, the reliability predictions for some components have a relatively large effect on the variance associated with the system-reliability estimate. Then, additional analyzes or testing is warranted.

This paper develops RPP strategies to prioritize or rank components based on their relative impact at the system-level. Different strategies are presented corresponding to when the performance measure is system reliability or system MTTF. The recommended method is based on system-reliability estimates. It can be used for all popular time-to-failure distributions (e.g., Weibull), but requires the selection of a mission time to provide a basis for reliability estimation. If there is no meaningful mission time, then an alternative approach is presented based on system MTTF. It requires the assumption of constant failure rates.

System-design schedules are becoming increasingly compressed, and the time and effort available for reliability analyzes has decreased. There are also more sophisticated analytic tools to study component failure mechanisms. However, use of these

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[1]The singular and plural of an acronym are always spelled the same.

models requires engineering effort and automated analytic tools. It becomes imperative to prioritize reliability-prediction activities based on their ability to reduce the variance of the system-reliability prediction. This paper presents an analytic method to identify those components that require additional analyzes or dedicated testing, or alternatively, where a crude estimate might be sufficient.

B. Sensitivity Measures

Various methods have been proposed to prioritize the most important or critical components within a system. The methods can generally be classified as:

- differential analysis methods,
- variance-decomposition methods.

For differential analysis, a sensitivity measure is defined as the partial derivative with respect to component reliability. Variance-based decomposition methods partition the system-reliability estimate variance among effects due to the component-reliability estimate variances. The partition of the model-output variance can then be used to rank the relative importance of the input variables. In this manner, the critical components are identified and additional test resources can be allocated to improve overall system-reliability estimation. In general, differential methods indicate which component's reliability levels should be increased to increase system reliability most efficiently. Variance decomposition methods indicate which components contribute most to the variance of the system-reliability estimate, and thus, which components might require additional testing or analysis to improve the component-reliability estimate.

Birnbaum [1] proposed the earliest differential importance measures for 2-state coherent system-reliability analysis. His measure characterizes the rate at which the system-reliability changes with respect to changes in the reliability of a given component. This measure is sometimes called first-order differential analysis [2]. Many variations of Birnbaum importance have been suggested including

- Lambert's critical importance [3],
- Gandini's upgrading importance [4],
- Xie and Shen [5] ranking of improvement actions,
- Barlow and Proschan's importance for k-out-n: F systems [6].

A common characteristic of differential-importance measures is that component parameters are treated as deterministic without considering the estimation uncertainty. The differential-sensitivity measures are useful to rank components based on their potential to improve system reliability. However, they are only marginally useful to rank component-reliability estimates based on their potential to minimize the variance of the system-reliability estimate.

Marseguerra, *et al.* [2], [7] report that Birnbaum importance and its extensions are local sensitivity measures in the sense that the importance, with respect to component reliability, r_i, is evaluated at the estimated value, $\hat{r}_i$. To remove the local restriction, a general differential sensitivity model is created by computing the s-expectation of a local sensitivity index over the range of r_i values. This alleviates the drawback that first-order differential sensitivity analysis applies only locally. The difficulty associated with this model is that the joint distribution for all r_i needs to be explicitly specified before computing the s-expectation. For systems with complex structures and few failure data, it is difficult to derive the joint distribution.

Nakashima and Yamato [8] developed a variance-importance measure based on a Taylor series expansion of the system-unreliability function around the mean of the component-unreliability estimate. Their model decomposes the system-level variance into a linear sum of individual component-variances by using a logarithm transformation. Then, the component-variance importance is defined as the partial derivative with respect to an individual component variance.

Easterling [9] and Rosenblatt [10] also use variance decomposition methods to approximate the variance of the system-reliability estimate as the linear combination of the variance of component-reliability estimates. When the component reliability is high and the associated estimation variance is small, the approximation yields very accurate estimates. Based on the variance of the system-reliability estimate, they further derived s-confidence intervals for system-reliability estimate using a binomial distribution assumption and the asymptotic normality for the system-reliability estimate respectively.

Cox [11] used conditional s-expectation to partition the variance of the system-reliability estimate among contributing causes using orthogonal decomposition. This approach applies for any nonlinear reliability function. However, as reliability-model complexity increases, this approach becomes cumbersome and difficult to apply. This method is not effective when there are s-correlated model parameters.

Coit [12] proposed a system-variance decomposition method. This model applies for any system design that can be decomposed into series and/or parallel connections as long as component reliability estimates are mutually s-independent. Jin and Coit [13] extended this variance decomposition to systems with arbitrarily repeated components. Subsystem-reliability estimates might be s-dependent due to the repeated use of the same type of component. This extension is valid for systems with series, parallel, and series-parallel structures.

Hora and Iman [14] and McKay [15] analyze the sensitivity problem based on the decomposition of the variance of a system-performance index. They use the s-correlation ratio of each component as the importance measure. The variance-decomposed method measures the proportion of the unconditional variance of the system reliability that can be attributed to the variance of the s-expected value of the system reliability, conditioned on the specific component parameter.

Variance is not always a robust measure of uncertainty when the model parameters experience highly skewed or fat-tailed distributions. To address this case, Iman and Hora [16] use the log-regression approach to transform the uncertainty sensitivity calculation into a logarithmic scale for the system-performance index. The log scale produces a reliable ordering of the variance sensitivity measures for each parameter. However, the approach fails to provide an absolute measure of importance in the ordinary scale of the analysis.

Notation

r_i	reliability of component i at time t
$\mathbf{r}$	$(r_1, \ldots, r_s)$
$\hat{}$	implies an estimate
$\hat{\mathbf{r}}$	$(\hat{r}_1, \ldots, \hat{r}_s)$
$R(t; \mathbf{r})$	system reliability at time t, as a function of $\mathbf{r}$
$\hat{R}(t)$	$R(t, \hat{\mathbf{r}})$
λ_i	exponential distribution parameter for component i (failure rate)
$\boldsymbol{\lambda}$	$(\lambda_1, \ldots, \lambda_s)$
μ_i	$1/\lambda_i$: MTTF for component i
$\hat{\lambda}_j$	empirical estimate j of failure rate
$\overline{\lambda}$	estimate of weighted average failure rate
n_j	number of failures used to estimate $\hat{\lambda}_j$
σ_i^2	variance of the component reliability, or of the MTTF estimate, for component i $\mathrm{Var}\{\hat{r}_i\}$, or $\mathrm{Var}\{\hat{\mu}\}_i$
δ_i	$[(\partial/\partial r_i)R(t;\ \mathbf{r})\vert_{\mathbf{r}=\hat{\mathbf{r}}}]^2$, prioritization based on system reliability, or $[(\partial/\partial \boldsymbol{\mu}_i)\int_0^\infty R(t;\boldsymbol{\lambda})\,dt\vert_{\boldsymbol{\lambda}=\hat{\boldsymbol{\lambda}}}]^2$, prioritization based on system MTTF
α_i	RPPI: $\delta_i \cdot \sigma_i^2 / \sum_j \delta_j \cdot \sigma_j^2$.

C. Assumptions

1) Components within a system operate and fail s-independently.
2) Components and the system are in 1 of 2 states: operating or failed.

II. RPP

RPP strategy requires that a preliminary reliability prediction (e.g., parts-count or assembly-level) has been determined. Once a preliminary reliability prediction has been calculated, it is necessary to decide how to improve the reliability prediction by decreasing the estimation variance. It would be desirable to conduct detailed engineering analyzes of each component and to perform dedicated life testing of each component; however, this generally is not possible, given the time and budget constraints.

A method is presented to rank system RPP activities according to a RPPI. The RPPI provides an approximate measure of the proportion of the system-level reliability estimate variance that can be attributed to a particular component. The components can then be ranked in accordance with the RPPI. This can serve as a quantitative guide to allocate additional engineering effort and testing.

Two RPPI methods are presented depending on whether system reliability or the system MTTF is the appropriate performance measure. RPPI based on system reliability is preferred. It is less complex (involves fewer calculations) and it is appropriate for any component time-to-failure distributions or a mix of distributions. If there is no meaningful mission time to provide a basis for reliability estimation, then RPPI indexes based on MTTF are useful. They are appropriate only for systems with components that have exponential time-to-failure. If a system has components whose time-to-failure is distributed according to some other distribution, we recommend that a compromise mission time be selected (e.g., average operating time in a year) and the reliability-based RPPI approach be used.

A. Variance Decomposition

It is necessary to estimate the variance of the system-reliability estimate based on the system configuration, component reliability estimates, and the variance of those estimates. The variance of the system-reliability estimate was determined as a function of the component reliability estimates and the associated variance measures for systems with s-independent component reliability estimates [12], and for systems with arbitrarily repeated components [13].

The objective of variance decomposition is to separate the system-reliability variance estimate into decomposed subcomponents that can be assigned or attributed to the individual system-components. Sobol' [17] presented a technique to decompose the variance of a model with random inputs, by decomposing the variance into additive subcomponents for the individual random variables and every possible interaction term. Homma and Saltelli [18] extend this approach to investigate the influence of individual parameters. Wheeler and Spulak [19] describe variance decomposition for failure-frequency estimates to be used in probabilistic risk assessments. A similar approach is used here to decompose system-reliability estimate variance.

1) Reliability Performance Measures: When there is a meaningful mission time or time period of interest (e.g., warranty period, useful life), then system-reliability measures are a convenient performance measure. A system-reliability estimate is determined by using $\hat{\mathbf{r}}$. If $\hat{R}(t)$ is an unbiased estimate, then

$$\begin{aligned}\mathrm{Var}[R(t, \hat{\mathbf{r}})] &\equiv \mathrm{E}\left[(R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r}))^2\right]\\ \mathrm{SD}[R(t, \hat{\mathbf{r}})] &\equiv \sqrt{\mathrm{Var}[R(t, \hat{\mathbf{r}})]}.\end{aligned} \tag{1}$$

Decomposition of the variance is based on a Taylor series expansion of $(R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r}))^2$. Equation (2) is a restated but equivalent form of the system-reliability expansion in [20]. It assumes that the system-reliability estimate is s-unbiased. In practice, the following method can still be applied to obtain useful priority classifications as long as $R(t, \hat{\mathbf{r}})$ is approximately unbiased

$$\begin{aligned}&[R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r})]^2\\ &\quad\approx [R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r})]^2\vert_{\mathbf{r}=\hat{\mathbf{r}}}\\ &\qquad + \sum_i \frac{\partial}{\partial r_i}[R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r})]^2\vert_{\mathbf{r}=\hat{\mathbf{r}}} \cdot (r_i - \hat{r}_i)\\ &\qquad + \sum_i \frac{\partial^2}{\partial r_i^2}[R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r})]^2\vert_{\mathbf{r}=\hat{\mathbf{r}}} \cdot \frac{(r_i - \hat{r}_i)^2}{2}\\ &\qquad + \sum_i \sum_{j:\, j\neq i} \frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j}[R(t, \hat{\mathbf{r}}) - R(t, \mathbf{r})]^2\vert_{\mathbf{r}=\hat{\mathbf{r}}}\\ &\qquad \cdot \frac{(r_i - \hat{r}_i)\cdot(r_j - \hat{r}_j)}{2}.\end{aligned} \tag{2}$$

The s-expected value of the Taylor series expansion is used to estimate the variance. If the component-reliability estimates are s-independent and s-unbiased, then the s-expected value of the expansion can be simplified. Under these conditions,

$$\begin{aligned}\mathrm{Var}[\hat{r}_i] &= \mathrm{E}\left[(r_i - \hat{r}_i)^2\right],\\ \mathrm{E}[(r_i - \hat{r}_i)] &= 0,\\ \mathrm{E}[(r_i - \hat{r}_i)\cdot(r_j - \hat{r}_j)] &= \mathrm{Cov}[\hat{r}_i, \hat{r}_j] = 0.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathrm{Var}[R(t;\,\hat{\mathbf{r}})] &= \mathrm{E}\left[(R(t,\,\hat{\mathbf{r}}) - R(t,\,\mathbf{r})^2\right] \\ &\approx \sum_i \frac{\partial^2}{\partial r_i^2}\,[R(t,\,\hat{\mathbf{r}}) - R(t,\,\mathbf{r})]^2|_{\mathbf{r}=\hat{\mathbf{r}}} \cdot \frac{\mathrm{Var}[\hat{r}_i]}{2} \\ &\approx \sum_i \left[\frac{\partial}{\partial r_i}\,R(t,\,\mathbf{r})|_{\mathbf{r}=\hat{\mathbf{r}}}\right]^2 \cdot \mathrm{Var}[\hat{r}_i]. \end{aligned} \tag{3}$$

The system-reliability estimate variance can now be approximately represented as a linear combination of the variance estimates of the individual components; see also [19].

The $\delta_i \cdot \sigma_i^2$ represents the approximate decrease in the system-reliability estimate variance if r_i is known with certainty, i.e., $\sigma_i^2 = 0$. The decomposed variance is,

$$\mathrm{Var}[R(t,\,\hat{\mathbf{r}})] \approx \sum_i \delta_i \cdot \sigma_i^2. \tag{4}$$

The δ_i equals the Birnbaum sensitivity measure squared. Therefore, although fundamentally different, variance decomposition and reliability sensitivity measures are related.

The variance decomposition is based on the first several terms of a Taylor series expansion. If the component reliability distributions are from very differently-shaped distributions with large variance, the approximate variance estimation error can become large, and this can affect the rankings. In practice, it is often difficult to determine the shape of the distribution, based on limited data. However, if it is known that the distributions are heavily skewed, an approach can be developed similar to log-regression approach [16]. Alternatively, the Taylor series expansion can include additional terms to improve the approximation [19].

2) MTTF Performance Measures: Some systems are intended to be used for unspecified and prolonged periods of time. For these systems, there is no meaningful mission time to use as a basis for reliability estimation, *viz,* probability of success. For these applications, analogous variance decomposition methods have been developed based on the variance of the system MTTF estimate. This decomposed MTTF estimate variance is more difficult to use and more restrictive, compared to the corresponding reliability estimate decomposition.

The equations in this Section II-A-2 apply when all component-failure times have an exponential distribution. System MTTF can be estimated based on a system model and estimated exponential distribution parameters for the components within the system

$$\mathrm{MTTF} = \int_0^\infty R(t;\,\boldsymbol{\lambda})\,dt, \quad \widehat{\mathrm{MTTF}} = \int_0^\infty R(t;\,\hat{\boldsymbol{\lambda}})\,dt.$$

The variance of the system-MTTF estimate can be decomposed using the same approach as for the system-reliability estimate decomposition. The decomposition is based on μ_i. For exponentially distributed failure times, the variance decomposition expression yields closed-form expressions. System variance decomposition requires the variance of the estimate of $\mu_i = 1/\lambda_i$. This can be estimated based on experience, or given time-to-failure data; the asymptotic variance can be obtained from the inverse of the Fisher information matrix

$$\begin{aligned}\mathrm{Var}\left[\widehat{\mathrm{MTTF}}\right] &\approx \sum_i \left[\frac{\partial}{\partial \mu_i}\int_0^\infty R(t;\,\boldsymbol{\lambda})\,dt\right]^2\Bigg|_{\boldsymbol{\lambda}=\hat{\boldsymbol{\lambda}}} \cdot \mathrm{Var}[\hat{\mu}_i] \\ &\approx \sum_i \left[\frac{\partial}{\partial \lambda_i}\int_0^\infty R(t;\,\boldsymbol{\lambda})\,dt\right]^2\Bigg|_{\boldsymbol{\lambda}=\hat{\boldsymbol{\lambda}}} \cdot \frac{\mathrm{Var}[\hat{\mu}_i]}{\hat{\mu}_i^4}. \end{aligned} \tag{5}$$

The system-MTTF estimate variance can now be approximated as a linear combination of the variance estimates of the individual components

$$\begin{aligned}\delta_i &= \left[\frac{\partial}{\partial \mu_i}\int_0^\infty R(t;\,\boldsymbol{\lambda})\,dt\right]^2\Bigg|_{\boldsymbol{\lambda}=\hat{\boldsymbol{\lambda}}} \\ &= \frac{\left[\frac{\partial}{\partial \lambda_i}\int_0^\infty R(t;\,\boldsymbol{\lambda})\,dt\right]^2\Big|_{\boldsymbol{\lambda}=\hat{\boldsymbol{\lambda}}}}{\hat{\mu}_i^4} \qquad \sigma_i^2 = \mathrm{Var}[\hat{\mu}_i]. \end{aligned} \tag{6}$$

The $\delta_i \cdot \sigma_i^2$ represents the approximate decrease in the system-MTTF estimate variance if μ_i is known with certainty: $\sigma_i^2 = 0$. The decomposed variance equation is

$$\mathrm{Var}\left[\widehat{\mathrm{MTTF}}\right] \approx \sum_i \delta_i \cdot \sigma_i^2.$$

B. RPPI

RPPI is a dimensionless parameter that represents the approximate proportion of the system-reliability estimate variance that can be attributed to a particular component. Since the goal of reliability prediction is to produce a system-reliability estimate with minimum variance, the components with the highest RPPI are the best candidates to improve the reliability prediction by testing, or by additional and/or more thorough analyzes. RPPI is computed by normalizing the variance expansion so that the terms add to 1:

$$\mathrm{RPPI}_i = \alpha_i, \qquad \sum_i \alpha_i = 1. \tag{7}$$

RPPI_i is proportional to δ_i which is related to Birnbaum sensitivity measures. If σ_i^2 is the same for all components, then RPPI and Birnbaum rankings are the same. However, often σ_i^2 varies substantially among the components within a system.

RPPI is used to assign components into priority group 1 or priority group 2. Components in priority group 1 are those that require further analysis or dedicated life-testing because the system-reliability estimation variability can be meaningfully decreased if the component reliability estimates have lower variance. For those components in priority group 2, the system-reliability estimate variance is not decreased in any meaningful way by a better reliability estimate. Therefore, for those components in priority group 2, the preliminary estimates do not necessarily require updating, even if they are highly variable. Limited analysis and testing budgets can be allocated

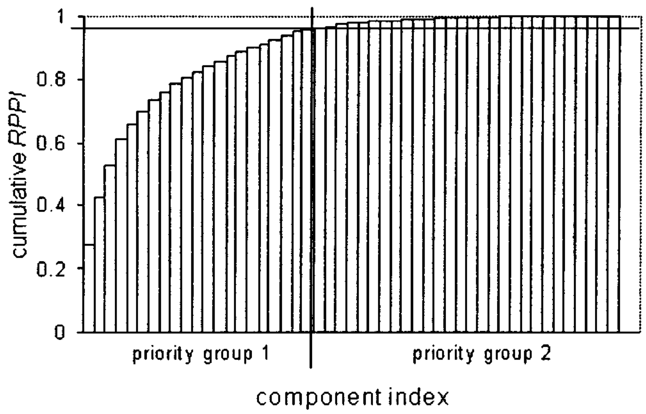

Fig. 1. Cumulative RPPI plot.

to those components in priority group 1, where the benefits are more readily evident.

Once RPPI_i has been computed for each component, they are rearranged from highest to lowest, in accordance with a Pareto analysis. Then, the priority groups are determined as shown in Fig. 1.

The components are assigned to a priority group based on one of two selection strategies:

- threshold value for cumulative RPPI,
- a pre-selected fraction of components.

If a threshold value (θ) is selected (e.g., 0.99, 0.999), then all component indexes with a cumulative RPPI index less than θ are defined as priority group 1. Components with a higher cumulative RPPI index are defined as priority group 2.

The $0 \leq \theta \leq 1$ is a relative measure;

- $\theta = 0$ means that all preliminary reliability predictions are satisfactory (group 2),
- $\theta = 1$ means that none of the preliminary reliability predictions are satisfactory (group 1).

Higher θ values should be chosen for design problems where it is more critical to have an accurate system-reliability prediction. Often some components dominate the system-variance decomposition and it is advisable to select a $\theta \geq 0.99$.

For some system-reliability design problems, a relatively few components dominate the RPPI. In this case, an alternative priority group selection strategy is to pre-select some fraction of components (e.g., 20%, 50%) to analyze in more detail. The components are ranked and the highest fraction (pre-selected) are subjected to further analysis.

When applicable, the best policy is to allocate RPPI_i measures and determine priority groups iteratively as the design evolves. The RPPI indicates the potential for decreasing system-level variability, but it does not measure the feasibility or ease of decreasing component-level estimation variance.

The RPPI procedure does not consider the cost of the additional analyzes or testing required to decrease component-estimate variance. In practice, this is important. The method could be revised by dividing α_i by a cost metric to yield cost-adjusted RPPI metrics.

C. Override Principle

The RPP strategy is useful to rank the potential for improvement. Priority group 1 components are recommended for further and more detailed analysis or testing. Some components should be grouped into priority group 1 without regard to RPPI calculation; this override principle is invoked when at least 1 of these 4 conditions apply:

- Failure, mode and effects analysis (FMEA) classifies the component in the highest (most severe) severity class.
- Component technology is unproven and has yet to be demonstrated in fielded systems.
- The component is in a redundant configuration and the effect of common-cause failures is believed to be important.
- Other system or operational specific condition dictates more detailed analyzes, independent of the RPPI.

III. Estimation of Component-Reliability Estimate Variance

Component-reliability estimates have various degrees of uncertainty depending on the nature and source of the data. Several sources of component-reliability predictions are company-specific field data, accelerated life-test results, physics-of-failure models, publicly available data [e.g., Reliability Analysis Center (RAC)], empirical models (e.g., Bellcore, MIL-HDBK-217F), and extrapolation of observed degradation patterns.

The magnitude and source of the reliability-estimate uncertainty can be very different for each source of component-reliability information.

- For company-specific field data, there could very likely be statistical uncertainty because of a low number of failures.
- For accelerated testing results, there will also be statistical uncertainty but there is additional uncertainty caused by the required extrapolation and the difference between the actual and test environment.
- For publicly-available generic data, there can be low statistical uncertainty (because of many failures) but an overall high degree of uncertainty because the associated operational and environmental stresses are unknown or merged.
- For empirical and physics-of-failure models, there are other sources of uncertainty.

Computation of RPPI requires an estimate of the variance for each component-reliability estimate used in the system-reliability model. These variance estimates can be calculated for empirical estimates of component reliability using well-established techniques. If there are time-to-failure and associated censored data for the components of interest, then the reliability estimate and an estimate of the variance can be made using Kaplan–Meier [21] estimates, or by considering the binomial distribution. When accelerated testing or reliability models are used, other means are necessary to estimate the variances. Finally, if there is no information available to compute these variance values, then worst-case estimates are available that assume that the component reliability estimates are unbiased.

Test data can result from testing conducted at accelerated environmental stress levels. The Kaplan–Meier or binomial data estimates are appropriate for accelerated life test data if the extrapolation factors do not introduce any additional estimation error. The mission time and failure times can be measured in compressed or accelerated time units and then extrapolated to reflect actual usage conditions. However, in most cases, the extrapolation factors introduce estimation error and the variance is underestimated if it is assumed otherwise. Estimation error is introduced by both the scarcity of empirical data and the extrapolation factor.

System-reliability predictions sometimes use failure rate data from internal field data reporting systems or published data compilations [22], [23]. The constant failure-rate has generally been assumed for these data sources. These failure rates can be used in a system-reliability prediction if the data originated from the same type of components operating under the same environmental conditions. Unfortunately, it is often unclear whether the environmental and usage conditions are the same or even analogous. Additionally, the data sources might pool individual data records together using an implicit homogeneity assumption. It is often unclear whether that assumption is valid.

The variance estimate should not be based solely on the pooled estimate unless the data are assuredly from homogeneous sources with homogeneous usage profiles. If there is doubt, it is important to consider the variability of the individual failure rate estimates (approximated by $\hat{\lambda}_i^2/n$) and compute the variance of the weighted average failure rate. If a homogeneous population of components is exposed to nonhomogeneous usage profiles, a conservative estimate of the failure rate variance can be estimated. Equation (8) pertains to the case where data $(n_i, \hat{\lambda}_i)$ are collected from different sources which may have differences in usage and environmental stress

$$\begin{aligned} \sigma_\lambda^2 &= \mathrm{Var}[\overline{\lambda}] \\ &= \mathrm{Var}\left[\sum_i \eta_i \cdot \hat{\lambda}_i\right] \\ &\approx \frac{1}{N} \cdot \sum_i \eta_i \cdot \hat{\lambda}_i^2; \\ N &\equiv \sum_i n_i; \qquad \eta_i \equiv \frac{n_i}{N}. \end{aligned} \tag{8}$$

A variance estimate for the component-reliability estimate can then be based on a s-normal distribution assumption for $\hat{\lambda}$ (and equivalently, a lognormal distribution for $\hat{r}$):

$$\mathrm{Var}[\hat{r}] = \exp\left(-2\overline{\lambda} \cdot t + \sigma_\lambda^2 \cdot t^2\right) \cdot \left[\exp\left(\sigma_\lambda^2 \cdot t^2\right) - 1\right]. \tag{9}$$

If no information is available to estimate the component-reliability estimate variance, then an upper-bound estimate can be computed using (10). There are different upper-bound variance values depending on whether an LB $(r_{L,i})$ for component reliability can be defined. The upper-bound values can be used as a conservative surrogate for the unknown variance. This makes it more likely that the component is categorized into priority group 1

$$\sigma_i^2 \le \begin{cases} \hat{r}_i \cdot (1 - \hat{r}_i), & \text{no LB for } \hat{r}_i \\ (\hat{r}_i - r_{L,i}) \cdot (1 - \hat{r}_i), & \text{LB can be estimated} \\ & \quad (r_{L,i} \le r_i \le 1). \end{cases} \tag{10}$$

These upper-bound values for estimate variance assume that $\hat{r}_i$ is an unbiased estimate. It is derived by considering that the component-reliability estimate is a Bernoulli r.v. (*viz,* 2 possible outcomes, $r_{L,i}$ and 1.0 with the mean $= \hat{r}_i$). In practice, the distribution of the component-reliability estimate is more likely to be some continuous distribution centered somewhere about the actual reliability value. Thus, the Bernoulli assumption is not realistic, but it is useful because it produces the maximum variance, which then can be used to provide an upper-bound for component-reliability estimate variance when no other information is available.

IV. Illustrative Examples

RPP is demonstrated using 2 examples. Example 1 is an automatic train control system. Example 2 is a bridge system from [5].

A. Automatic Train Control (ATC)

The ATC system is stationed in a fixed location and monitors the speed of bullet trains as they pass. It detects a preceding train, generates an upper-limit speed, and then, controls the train operation by the upper-limit speed signal. The system has redundancy for signal transmission and for logic to generate upper-limit speeds. There is a fail-safe system for automatic braking. The example is a hypothetical system based on an actual ATC [24].

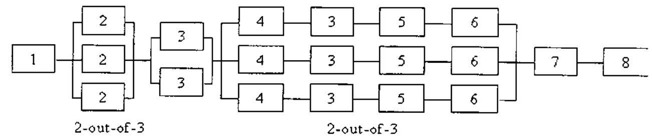

Fig. 2. ATC system-reliability block diagram.

The ATC system-reliability block diagram is in Fig. 2 and component-level information is in Table I. The reliability estimates are based on constant failure-rate assumptions. The variance estimate for the brake is based on field data, while the other components and assemblies use worst-case variance estimates, with LB on reliability where indicated in Table I. The ATC uses 2-out-of-3: G redundancy in two locations in the design. The amplifier (component 3) is used in several places within the design, although there is a single estimate available. The ATC system reliability is,

$$\begin{aligned} R(t;\mathbf{r}) = r_1 \cdot \left(3r_2^2 - 2r_2^3\right) \cdot \left[1 - (1 - r_3)^2\right] \\ \cdot \left[3(r_3 \cdot r_4 \cdot r_5 \cdot r_6)^2 - 2(r_3 \cdot r_4 \cdot r_5 \cdot r_6)^3\right] \\ \cdot r_7 \cdot r_8. \end{aligned} \tag{11}$$

The predicted reliability for the ATC is 0.9815 and the approximate variance of that estimate is $7.56 \cdot 10^{-5}$ for a mission time

TABLE I
ATC SYSTEM-COMPONENT DATA

i	assembly	failure/hr	$r_i(t)$	r_L	σ_i^2	$\partial R/\partial r$	δ_i	$\delta_i \cdot \sigma_i^2$	RPPI
1	Train Detection	$3.0 \cdot 10^{-8}$	.9995	.96	.0000207	.9820	.9643	$2.00 \cdot 10^{-5}$	.2643
2	Logic Unit	$1.0 \cdot 10^{-7}$	.9982	–	.0017474	.0103	.0001	$1.85 \cdot 10^{-7}$	.0024
3	Amplifier	$3.0 \cdot 10^{-7}$	.9948	.90	.0004967	.0470	.0022	$1.10 \cdot 10^{-6}$	.0145
4	Synchronism Detection	$1.0 \cdot 10^{-8}$	.9998	.90	.0000175	.0366	.0013	$2.34 \cdot 10^{-8}$	.0003
5	Bandpass Filter	$4.0 \cdot 10^{-8}$	.9993	.90	.0000670	.0366	.0013	$9.31 \cdot 10^{-8}$	.0012
6	Control Logic	$1.0 \cdot 10^{-8}$	.9998	–	.0001752	.0366	.0013	$2.34 \cdot 10^{-7}$	.0031
7	Axle Rotation Detection	$3.0 \cdot 10^{-8}$	.9995	.96	.0000207	.9820	.9643	$2.00 \cdot 10^{-5}$	.2643
8	Brake	$1.0 \cdot 10^{-6}$	.9826	–	.0000341	.9988	.9976	$3.40 \cdot 10^{-5}$	.4499

TABLE II
AUTOMATIC TRAIN CONTROL SYSTEM RPPI RESULTS

i	assembly	RPPI	cum RPPI	Priority Group
8	Brake	.4499	.4499	1
1	Train Detection	.2643	.7141	1
7	Axle Rotation Detection	.2643	.9784	1
3	Amplifier	.0145	.9929	1
6	Control Logic	.0031	.9960	2
2	Logic Unit	.0024	.9985	2
5	Bandpass Filter	.0012	.9997	2
4	Synchronism Detection	.0003	1.0000	2

of 2 years of constant operation (17 520 hours). The initial reliability prediction was based on a combination of assembly-level and component-level reliability predictions.

Table II shows the RPPI analysis results. Partial derivatives of $R(t; \mathbf{r})$ were computed for each component to determine δ_i and $RPPI_i$. As shown in Table II, the brake contributes almost half of the system-reliability estimate variance even though the component-reliability estimate variance is relatively low. However, the brake reliability is low compared to the other components and the brake is not part of a redundant configuration. Even small variability of the brake-reliability estimate affects the system-level estimate appreciably.

A threshold value, θ, of 0.99 was pre-determined for the system. This means that the components that contribute 0.99 of the decomposed approximate variance measure will be subjected to more detailed analyzes or additional testing. In accordance with the Table II results, components 8, 1, 7, 3 should be in priority group 1. The reliability of these components should be further analyzed, and dedicated life testing should be contemplated.

B. Bridge System

Fig. 3 and Table III show the bridge system and corresponding data. This system was analyzed [5] to demonstrate its sensitivity measures. Several shortcomings with Birnbaum sensitivity measures [1] were identified, and revised sensitivity measures were proposed. Only component reliability point-estimates were originally provided. Typical variance estimates were assumed, and presented in Table III, to demonstrate the RPP process. Table III also presents the data and the analysis results. If a $\theta = 0.9$ was selected, components 1, 2, 3, 4 would be assigned to priority group 1, and component 5 would be assigned to priority group 2.

To compare RPPI indexes with other sensitivity measures, Table III includes Birnbaum sensitivity measures and 2 revised sensitivity measures from [5]:

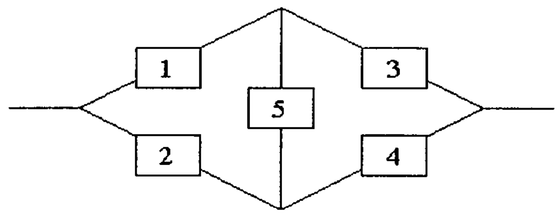

Fig. 3. Bridge system.

- (labeled X and S redundant) is the increase in system reliability if a redundant component is individually added for each component in the system;
- (labeled X and S perfect) is the increase in system reliability if each component is replaced by an equivalent component with a reliability of 1.0.

Birnbaum sensitivity measures are the partial derivative of system reliability with respect to component reliability (which is equivalent to the difference in system reliability if the component has a reliability of 1.0 compared to a reliability of 0.0).

Table IV presents the rankings of the 5 components within the bridge system with respect to each of the rankings. As the rankings in the table demonstrate, different components are ranked differently depending on the particular index. This is not unanticipated because the particular indexes measure different characteristics.

Birnbaum [1] and Xie and Shen [5] indexes are intended to characterize how sensitive system reliability is to changes in component reliability. Therefore, they are useful to assess the impact of improving component reliability. Alternatively, RPPI indicates how sensitive the variance of the system-reliability estimate is to changes in the variance of the component-reliability estimate. Thus, Birnbaum [1] and Xie and Shen [5] indexes are useful to decide which components to improve, whereas RPPI is useful to determine which components should be subjected to more testing or detailed analyzes to decrease the variance of the component-reliability estimate.

V. DISCUSSION

RPPI and priority groups have been defined to compare components and to provide a relative ranking of components based on their potential for meaningful improvement at the system-level. RPPI is based on a decomposition of the variance of the system reliability or MTTF estimate. RPPI is approximately the proportion of the system-reliability estimate variance that can be attributed to an individual component. The goal of reliability

TABLE III
BRIDGE SYSTEM RPPI AND SENSITIVITY MEASURE CALCULATIONS

(i)	RPPI Data				Sensitivity Measures		
	$r_i(t)$	σ_i^2	$\delta_i \cdot \sigma_i^2$	RPPI	Birnbaum [1]	X&S redundant [5]	X&S perfect [5]
1	0.8	0.0011	0.00032	0.124	0.545	0.087	0.109
2	0.3	0.0053	0.00038	0.150	0.270	0.057	0.189
3	0.6	0.0060	0.00119	0.465	0.445	0.107	0.178
4	0.7	0.0084	0.00053	0.206	0.250	0.053	0.075
5	0.5	0.0050	0.00014	0.055	0.168	0.042	0.084
system	0.673	–	0.002553	1.000	–	–	–

TABLE IV
SENSITIVITY MEASURE RANKINGS

rank	Birnbaum [1]	X&S [5] redundant	X&S [5] perfect	RPPI
1	1	3	2	3
2	3	1	3	4
3	2	2	1	2
4	4	4	5	1
5	5	5	4	5

prediction is to estimate system reliability with minimal variance. Therefore, components with higher RPPI should be exposed to additional testing or analyzes to improve the reliability prediction because the improvement will be more readily apparent at the system-level.

Reliability sensitivity measures and RPPI are fundamentally different because they are intended to measure different characteristics. However, the Birnbaum sensitivity measures and RPPI are mathematically related. The δ_i from (4) is the [Birnbaum sensitivity measure]^2. RPPI is proportional to δ_i; thus if σ_i^2 is the same for all components, then the RPPI and Birnbaum rankings are the same. For many systems, these indexes often do correspond. However, for other systems they differ, as in Table IV. The RPPI indexes are most useful when component reliability estimates are from a variety of sources with varying variance levels.

Birnbaum sensitivity measures and RPPI can also be conceptually related. If the Birnbaum sensitivity measure is small, the system reliability is insensitive to relatively large shifts in component reliability. If this is true, then it is logical to conclude that prediction errors (analogous to real increases or decreases in component reliability) would also have smaller effect. Since the RPPI rankings are equivalent to Birnbaum sensitivity rankings when σ_i^2 is the same for all components, the Birnbaum measures can be considered as a special case of the RPPI. An expanded and generalized RPPI sensitivity measure could be developed to characterize formally both the effect of improvements in component-reliability and in component-reliability estimation variance.

System-reliability prediction and the associated sensitivity measures are generally based on the assumption that components, and thus the system, are in either a failed or a full operational state. In practice, there can be multiple degraded states. In the degraded state,

- the system can continue to provide full operational capability, but observed deterioration can allow for a prediction of reliability before experiencing any failures, or
- the system might not provide full operational capability, but it can still provide some partial operational capability.

In practice, both of these scenarios can be useful for modeling complex systems. Extension of the RPPI concept to address this reliability modeling will be particularly useful because extrapolation of observed degradation trends can introduce prediction uncertainty.

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