

A Tri-Modal Directional Modem Transducer

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Abstract- With suitable voltage distribution, higher order extensional modes of a piezoelectric cylinder can be excited which produce directional radiation patterns. These modal radiation patterns can then be combined to synthesize desired beam patterns which may be steered by incrementing the excitation. This paper describes a directional modem communication transducer which uses the combined acoustic response of the first three extensional modes of vibration of a piezoelectric ceramic cylinder, a method of synthesizing a desired radiation pattern and an experimental implementation of a two-ring directional modem transducer that uses these techniques. This tri-modal transducer has a smooth response in the band from 15 kHz to 20 kHz, with a frequency-independent 90 degree beam width which may be steered in 45 degree increments from a coded input. The interior of the transducer contains the electronics and the unit may be deployed from a Type A launch tube. (Works supported by a Phase II, SBIR from ONR and SPAWAR)

I. INTRODUCTION

Radially poled piezoelectric cylinders with continuous electrodes on the inner and outer surfaces are widely used in underwater acoustic transducers as single mode devices. As projectors, cylinder transducers are operated typically in the vicinity of their first breathing mode resonance, where the surface velocity is uniform and the sound field generated is omni-directional in the plane perpendicular to the axis. Although usually operated below resonance, most cylindrical hydrophones also use this omni-directional mode. Ehrlich and Frelich [1] describe a cylindrical sensor that has a pattern of four electrodes, each covering a quadrant of the cylinder, that, when combined, generate orthogonal directional dipoles and omni receive beam patterns. Spherical sensors with patterned electrodes [2] as well as dual mode flextensional transducers [3] have also been used to achieve directionality through the omni and dipole modes. Although higher order modes have been used for super directive microphones and considered for PVDF hydrophones [4], the implementation of higher modes in cylindrical projectors and hydrophones appears to have been overlooked. In this paper we present a simple means for creating directional beams from a cylinder using the quadrupole mode as well as the omni and dipole modes. The resulting transducer may be steered by changing the voltage amplitude rather than the phase on the electrodes of the cylinder.

The dynamics of the multimode cylinder can be represented by the two dimensional planar motion of a ring. This is a reasonable approximation for a cylinder at resonance that is short and thin walled. The 'breathing' mode of a ring is the fundamental extensional mode which occurs when extensional waves in the structure of the ring complete one wavelength around the circumference. The extensional strain is then in phase at all circumferential locations and this uniform extensional strain is translated, by the geometry, into a purely radial displacement of the ring wall. The resonance frequency of this fundamental mode f_0 is given by $c / \pi D$ where c is the speed of sound for extensional waves in the ring wall and D is the mean diameter.

The higher order extensional modes of a ring [5] have a radial displacement that is harmonic in the azimuth angle φ with an even component proportional to $\cos n\varphi$ and an odd component proportional to $\sin n\varphi$, where n is the mode number. The fundamental 'breathing' mode has a mode number $n = 0$. The second mode, $n = 1$, has two nodes with the radial motion of one half of the ring out of phase with the other and has a resonance frequency $f_1 = \sqrt{2} f_0$. The third mode with, with $n = 2$, has four nodes and a resonance frequency $f_2 = \sqrt{5} f_0$. The n 'th extensional mode has $2n$ vibrational nodes and a resonance frequency, $f_n = f_0(1+n^2)^{1/2}$. Figure 1 illustrates the first three even extensional modes of a ring.

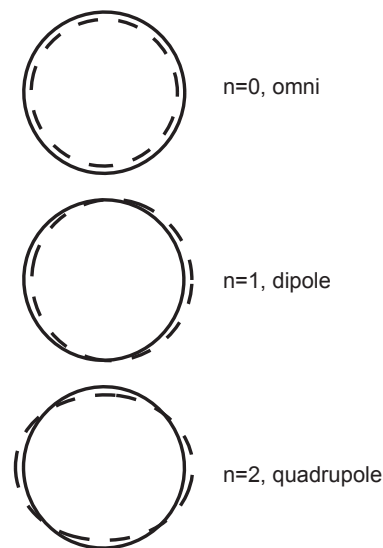


FIG. 1 Static (—) and deformed (_ _ _) displacements of omni, dipole and quad modes.

This multimode transducer has application for directional acoustic links in an underwater acoustic modem system which requires compact directional acoustic transducers both for transmit and receive [6]. Underwater communication networks support telemetry of sensor data from distributed stations within the ocean, and are the basis for future undersea communication and navigation grids for autonomous undersea vehicles and submarines. The use of directional transducers substantially benefits network performance by increasing signal-to-noise ratio, decreasing multi-access interference, reducing battery-energy consumption, and improving transmission security [7]. The need for compact directional transducers has revived interest in the use of multimode transducers. This paper describes a method for exciting a cylindrical transducer for operation as a directional multimode compact acoustic source and receiver which uses the first three extensional modes of vibration and operates in a band of frequencies between the dipole and quadrupole resonance frequencies.

II. TRANSDUCER ANALYTICAL MODEL

Symmetrical beams are of particular interest and the analysis will be restricted to the even $\cos n\varphi$ components which requires an even excitation of the cylinder. The $\cos n\varphi$ radial motion of the n 'th extensional mode will generate a radiated acoustic pressure with a $\cos n\varphi$ azimuthal directional factor in a plane perpendicular to the axis of the cylinder. The fundamental, $n = 0$, mode will generate an omnidirectional beam pattern, the $n = 1$ mode will generate the two-lobed $\cos \varphi$ dipole beam pattern and the $n = 2$ mode will generate the four-lobed $\cos 2\varphi$ quadrupole beam pattern.

A theoretical model for the radiation from the cylinder may be based on the Laird-Cohen model [8] with infinite rigid extensions. In this case the azimuthal radial velocity distribution, u_r , on the cylinder surface may be expanded as the Fourier series

$$u_r = e^{-i\omega t} \sum_{n=0}^{\infty} u_n \cos(n\varphi) \quad (1)$$

where φ is the azimuth angle. In the plane perpendicular to the axis of the cylinder at a radial distance, r , we may also write the pressure as

$$p(r, \varphi) = e^{-i\omega t} \sum_{n=0}^{\infty} p_n(r) \cos(n\varphi) \quad (2)$$

where the modal pressure

$$p_n(r) = u_n 2\rho c L (e^{ikr} / \pi r) e^{-in\pi/2} / H'_n(ka) \quad (3)$$

and ρ is the density, c is the sound speed in the medium, k is the wave number, L is the half-length of the cylinder, a is

the radius of the cylinder and $H'_n(ka)$ is the derivative of the cylindrical Hankel function of the first kind of order n .

Equation (3) allows us to write the n 'th order modal value in terms of the zero order omni mode value as

$$u_n / u_0 = (p_n / p_0) e^{-in\pi/2} H'_n(ka) / H'_0(ka) \quad (4)$$

Since Eq. (2) is a Fourier series we can obtain the modal values, p_n , from the far-field pressure, $p(r, \varphi)$, as

$$p_n(r) = (\delta_n / \pi) \int_0^{\pi} p(r, \varphi) \cos(n\varphi) d\varphi \quad (5)$$

where $\delta_0 = 1$ and $\delta_n = 2$ for $n > 0$.

Thus, in theory, any specific directivity function or beam pattern can be synthesized through Eq. (5) with the required modal velocity ratios given through Eq. (4). A large number of p_n coefficients may be necessary if the desired beam pattern function $p(\varphi)$ changes abruptly or is narrow. In our case of interest we use only a three term expansion of $\cos n\varphi$ corresponding to the first three extensional modes. Here the pressure function expansion may be written as

$$p(\varphi) = \frac{p(0)(1 + A \cos \varphi + B \cos 2\varphi)}{1 + A + B} \quad (6)$$

where the normalized beam pattern function is $p(\varphi)/p(0)$. Substitution of Eq. (6) into Eq. (5) yields the solutions $p_0/p(0) = 1/(1+A+B)$, $p_1/p(0) = A/(1+A+B)$ and $p_2/p(0) = B/(1+A+B)$, which is also evident if the pressure function expansion is simply compared with Eq. (2). Results can then be substituted into Eq. (4) to determine the required velocity ratios, u_n/u_0 , for selected values of the weighting coefficients A and B .

Although a finite element model was used in modeling the transducer, an approximate analytical model was also developed for comparison with the finite element model and for more rapid computation of transducer design variations. This model is based on the Fourier representation by Gordon, Parad and Butler [9] for dipole mode operation of a piezoelectric ring, extrapolated to cover the case of higher order modes. Accordingly, for the n 'th mode and an applied even electrical field E_n of angular frequency ω , the radial velocity may be written as

$$u_n = \frac{j\omega\omega_0^2 a d_{31} E_n}{\left[(1 + n^2)\omega_0^2 - \omega^2 + \left(1 - \frac{n^2\omega_0^2}{\omega^2} \right) \frac{j\omega Z_n}{\rho_0 t} \right]} \quad (7)$$

where ω_0 is the fundamental angular resonance frequency, a is the mean radius of the cylinder, ρ_0 is the density of the of the ring, t is the wall thickness of the cylinder and d_{31} is the piezoelectric coefficient. Z_n is the specific acoustic modal impedance, p_n/u_n , where p_n is the modal pressure.

There is no analytical expression for either the modal specific acoustic impedance, Z_n , or the far field sound pressure amplitude for a finite un-baffled cylindrical transducer. However, there are useful approximations that can be made for an analytical model. The most accurate assumes that the transducer is embedded in an infinitely long rigid cylinder as modeled by Laird and Cohen [8], given earlier. In this case the Fourier series method by Butler and Butler [10] may be used to obtain the radiation impedance for each cylindrical mode. If the cylinder is long compared to the acoustic wavelength, the simpler model for an infinitely long cylinder could also be used to obtain an approximation for Z_n . On the other hand, if the cylinder is acoustically small, a spherical model of the same surface area could be used. Although the Laird-Cohen model is probably the most accurate, we chose the spherical analytical model since our initial design was based on an acoustically small cylinder; and moreover, with this model algebraic expressions for the impedance as well as the far field pressure expansion could be used resulting in a rapid design program. The simple spherical model allowed many design iterations before selected specific designs were evaluated using a more accurate Finite Element Model (FEM).

The approximate spherical model for the modal specific acoustic impedance for Eq. (7) may be obtained from a spherical wave expansion. If the surface of a sphere of radius a_s is vibrating with an axially symmetric radial velocity [11], the specific acoustic modal impedance is then

$$Z_n = p_n / u_n = -j\rho c h_n(ka_s) / h'_n(ka_s). \quad (8)$$

where h_n is the spherical Hankel function of the second kind of order n . The approximate impedances for the multimode cylinder are obtained by equating the curved radiating areas of the cylinder, $4L\pi a$, and the sphere, $4\pi a_s^2$, yielding the equivalent sphere radius $a_s = (aL)^{1/2}$ where, as before, a is the radius and L is the half length of the cylinder. Equation (8) when substituted into Eq. (7) yields the modal velocities, u_n in terms of the electric field intensities, E_n .

The simplest directional beam pattern synthesis is the classic cardioid beam pattern. The cardioid directional factor is the superposition of an omni and a dipole directional factor, $\cos \varphi$, of equal pressure amplitude and phase so that $A=1$ and $B=0$ in Eq. (6). It has a strong null at its rear with corresponding normalized beam pattern function

$$p(\varphi) / p(0) = (1 + \cos \varphi) / 2 \quad (9)$$

The cardioid beam pattern is widely used in microphones and has been used in underwater acoustics where unidirectional hydrophone characteristics are desirable. Adding higher order modes to the synthesis can widen the range of possible beam patterns. Adding omni, dipole and quadrupole modes in the ratio 1:2:1 ($A=2$, $B=1$) gives a

super cardioid beam pattern. The corresponding beam pattern function from Eq. (6) is

$$\begin{aligned} p(\varphi) / p(0) &= (1 + 2 \cos \varphi + \cos 2\varphi) / 4 \\ &= \cos \varphi (1 + \cos \varphi) / 2 \end{aligned} \quad (10)$$

with the second equality revealing an alternative interpretation of the pattern as a product of dipole and cardioid beam patterns with nulls at ± 90 and 180 degrees.

A superposition with comparatively smaller dipole and quadrupole components, in the ratio 1:1:0.414, ($A=1$, $B=0.414$) is illustrated in Fig. 2 with the beam pattern function

$$p(\varphi) / p(0) = \frac{(1 + \cos \varphi + 0.414 \cos 2\varphi)}{2.414} \quad (11)$$

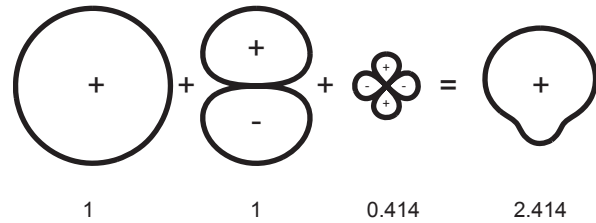


FIG. 2 The synthesis of a "quadrant" beam pattern.

This combination of modes was chosen to give a pattern with a 90 degree beam width and a 15 dB front-to-back ratio as illustrated in Fig. 3. This pattern will be called the quadrant beam pattern; it is suitable for 45 degree incremental steering and is used in our implementation of the tri-modal directional transducer concept.

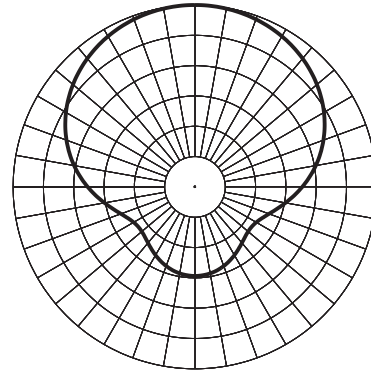


FIG. 3 Quadrant beam pattern distribution. Radial divisions are 5 dB and rotational divisions are 10 degrees.

III. BEAM SYNTHESIS MODEL

In an earlier implementation of a tri-modal transducer, three different size cylinders were used in the form of a coaxial array [12]. The diameter of each cylinder

was chosen to make the resonance frequency of the quadrupole mode, of the largest cylinder, and the dipole mode, of the next largest cylinder, coincide with the omni mode resonance frequency of the smallest cylinder. With this arrangement, each cylinder could be operated at resonance with approximately the same transmit voltage response. The voltage distributions required for a given beam pattern were then relatively easy to implement, with the required omni voltage distribution applied to the smallest cylinder, the dipole voltage distribution applied to the next largest and the quadrupole voltage distribution applied to the largest. However, as the acoustic center of each cylinder was offset by its location in the array, these offsets and phase reversals on the back side of the cylinders created undesirable rear grating lobes in the superimposed beam patterns.

In the present method all cylinders are of the same diameter and the cylinders are excited to simultaneously operate in three modes of vibration. An appropriate distribution of voltages around the piezoelectric cylinder is required to excite the desired modes of vibration. A circumferential electric field voltage distribution $V(\phi) = V_0 \cos n\phi$ will excite the $\cos n\phi$ extensional mode. A continuous voltage distribution is difficult to implement but can be approximated by a set of equally spaced segmented inner electrodes with one electrode on the outside. A particularly useful arrangement for the first three modes has 8 electrodes. With even symmetry the 8 electrodes can be grouped into 4 pairs as in Fig. 4.

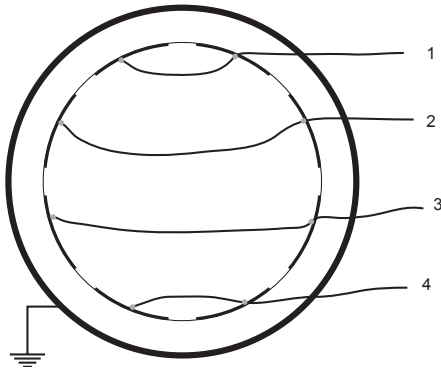


FIG. 4 A 31 mode piezoelectric ceramic cylinder with eight inner electrodes connected in pairs.

If the voltage distribution for the four pairs of electrodes is a four element vector, then the distribution $[1, 1, 1, 1]$ will excite only the $n = 0$ omni mode. The distribution $[1, 1, -1, -1]$ will excite primarily the $n = 1$ dipole mode while the distribution $[1, -1, -1, 1]$ will excite primarily the $n = 2$ quadrupole mode. The three basic voltage distributions, shown in Table I, were used to excite the transducer in each of the three fundamental extensional modes.

Table I: Basic voltage distributions that excite each fundamental mode.

Mode	V_1	V_2	V_3	V_4	SUM
omni	1	1	1	1	4
dipole	1	1	-1	-1	0
quadrupole	1	-1	-1	1	0

In order to implement a given directional response, a voltage distribution is required that results in a superposition of the far field pressure response of each mode in the required relative amplitudes with the same phase. If V_o is the complex amplitude of the omni modal voltage and T_o is the complex amplitude of the transmit response, then the complex far field pressure, p_o , due to V_o is $p_o = T_o V_o$. Similarly, $p_d = T_d V_d$ is the pressure due to the dipole modal voltage, V_d , and $p_q = T_q V_q$ is the pressure due to the quadrupole voltage distribution. With the desired beam pattern limited to the first three modes; the omni, dipole and quadrupole modal voltages for the normalized directivity pressure function of Eq. (6) may be written as

$$V_o = 1/T_o, V_d = A/T_d, V_q = B/T_q \quad (12)$$

The coefficients $A = 1$ and $B = 0.414$ are for the specific case of the “quadrant” beam shown in Fig. 3.

The voltage distribution around the ring, for the electrodes of Fig. 4, may be determined with the help of Fig. 5 which shows the electrode phase reversals necessary for excitation of the dipole and quadrupole modes.

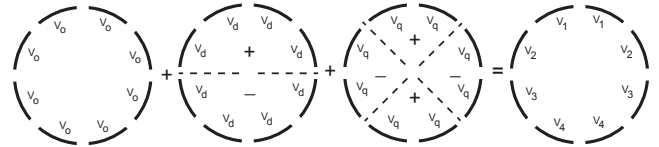


FIG. 5 An illustration of the addition of three basic monopole, dipole and quadrupole voltage distributions to give the broadband voltage distribution.

This mode summation leads to the complex voltage distribution

$$V_1 = V_o + V_d + V_q \quad (13a)$$

$$V_2 = V_o + V_d - V_q \quad (13b)$$

$$V_3 = V_o - V_d - V_q \quad (13c)$$

$$V_4 = V_o - V_d + V_q \quad (13d)$$

for the four paired electrodes voltages, V_1, V_2, V_3 and V_4 for Fig. 4.

The voltage distribution over the cylinder for a required beam pattern can be found for any given

frequency through Eqs. (12) and (13a,b,c,d). Since T_o , T_d and T_q vary with frequency, V_o , V_d and V_q and consequently V_1 , V_2 , V_3 and V_4 must change with frequency to maintain the required beam pattern over a band of frequencies. Table II shows the values calculated from the transducer model for the omni, dipole and quadrupole modes yielding quadrant beam patterns at 15 kHz, 17.5 kHz and 20 kHz, based on Eq. (6).

Table II. Calculated omni, dipole and quadrupole modal voltages.

Mode Voltage	15 kHz	17.5 kHz	20 kHz
Omni, V_o	1.0 + j0.0	1.0 + j0.00	1.0 + j0.00
dipole, V_d	0.1 - j0.2	0.4 - j0.10	0.55 - j0.05
Quadrupole, V_q	-1.3 + j0.3	-0.6 + j0.10	-0.25 + j0.05

The voltage distribution results, shown in Table III, were calculated from the Eqs. (13a,b,c,d) using the results listed in Table II.

Table III: Calculated voltage distributions for the quadrant beam pattern at three frequencies together with approximate constant broadband distribution.

Voltage	15 kHz	17.5 kHz	20 kHz	Broadband
V_1	-0.2 + j0.1	0.8 + j0.0	1.3 + j0.0	1.5 + j0.0
V_2	2.4 - j0.5	2.0 - j0.2	1.8 - j0.1	1.9 + j0.0
V_3	2.2 - j0.1	1.2 + j0.0	0.7 + j0.0	0.5 + j0.0
V_4	-0.4 + j0.5	0.0 + j0.2	0.2 + j0.1	0.1 + j0.0
SUM	4.0 + j0.0	4.0 + j0.0	4.0 + j0.0	4.0 + j0.0

The approximate broadband distribution of Table III was obtained from inspection of the distribution at the three separate frequencies mindful that the sum of each is and must be identically $4 + j0$ since the dipole and quadrupole mode voltage distribution sum to zero, as illustrated in Table I. It was found that the distribution at 20 kHz yielded nearly satisfactory patterns at 15 kHz and 17.5 kHz and that the imaginary parts were negligible. The distribution at 20 kHz was then modified to obtain nearly equal 90 degree beam patterns at 15 kHz and 20 kHz with approximately 10 dB down level at 90 and 180 degrees. Note that the selected broadband voltage distribution is purely real and that the distribution sums to 4. The single broad band distribution is a significant improvement in the implementation of the tri-modal transducer allowing one simple real voltage distribution (1.5, 1.9, 0.5, 0.1) for the desired quadrant beam pattern over a band of frequencies from 15 kHz to 20 kHz.

IV. TRANSDUCER IMPLEMENTATION

An earlier narrow beam implementation with a 27° vertical beam used three rings [13]. In the new implementation, shown in Fig. 6, the transducer array is a stack of only two cylinders of equal size. The cylinders are poled through the thickness of the wall. Each cylinder is 108 mm (4.25") in diameter, 50.8 mm (2.0") high with a wall thickness of 7.48 mm (0.19"). The outside of each cylinder has one continuous electrode, the inner surface has eight equally spaced electrodes around the circumference. The two cylinders are axially decoupled and wired in parallel to make an effective cylinder 102 mm (4.0") high,

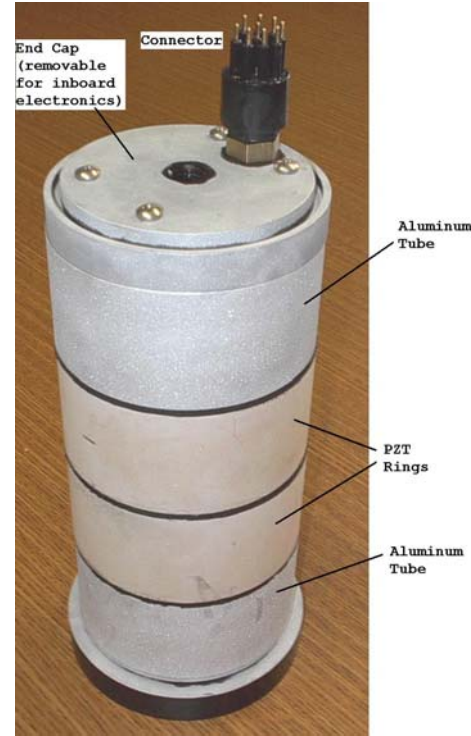


FIG. 6 Assembled modem before potting

all encapsulated in polyurethane. The array height was chosen to yield a vertical beam width of approximately 40 degrees to minimize the effects of tilting when communicating with other units. The diameter was chosen so the unit could be deployed from standard launch tubes. During evaluation, the eight inner electrodes were connected in pairs, as in Fig. 4.

Measurements were made at the Naval Post Graduate School in Monterey, CA and at FSI/Acoustikos in Cataumet, MA. The transducer was initially wired, according to Table I, for the separate excitation of the omni dipole and quad modes. Corresponding measured beam patterns of the dipole and quad modes are shown at 20 kHz in Fig. 7.

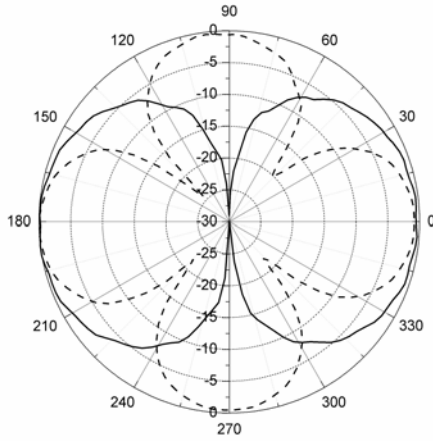


FIG. 7 Horizontal beam patterns for dipole (—) and quadrupole (---) modes at 20 kHz.

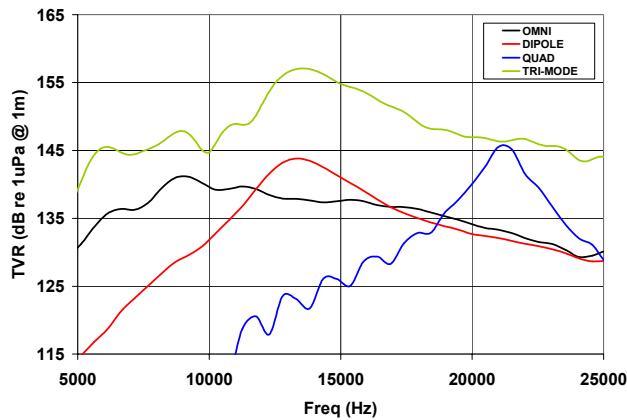


FIG. 8 Modem TVR operating in Omni, Dipole, Quadrupole and synthesized Tri-modal mode.

The wideband measured TVR for each mode is shown in Fig. 8. Measurements were also made with a transformer with a step up ratio of 2.5 and secondary distribution of 1.5, 1.9, 0.5, 0.1, from the broadband distribution in Table III. This distribution produced omni, dipole and quadrupole far field pressures in ratios that approximate 1, 1, 0.414 for synthesized excitation of the quadrant beam pattern of Fig. 3. The corresponding transformer coupled TVR is also shown in Fig. 8 and is the result of all three modes.

Although the measured response curves of Fig. 8 are from 5 kHz to 25 kHz, the current band of interest is only from 15 kHz to 20 kHz. The purely real broadband voltage distribution given in Table III is consistent with the calculated phase distribution of the pressure in the far field shown in Fig. 9 for the omni dipole and quadrupole modes and the measured responses in Fig. 8. In the band of frequencies from 15 kHz and 20 kHz, between the dipole and quadrupole resonances, the transmit response of the first three modes are of the same order of magnitude.

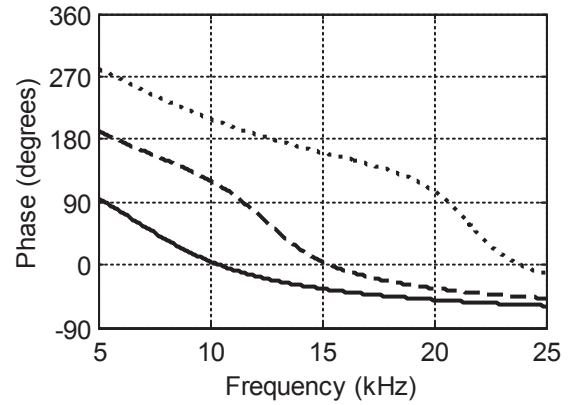


FIG. 9 Phase of the transmit response for the transducer driven by the basic voltage distributions of Table I for the omni (—), dipole (---) and quad (. . .) modes.

At low frequencies, the radial motion of the cylinder wall is in phase for all vibration modes, but as shown in Fig. 9, the low frequency far field transmit pressure response of the quadrupole mode leads the dipole by 90 degrees and the dipole leads the omni by 90 degrees. However, between 15 kHz and 20 kHz, the omni and dipole modes have gone through their resonances and are approximately in phase while both are out of phase with the quadrupole. This observation allows the direct addition of the modes with only a 180 degree phase reversal needed for the quadrupole mode in the band between the dipole and quadrupole resonant frequencies. The horizontal measured beam patterns with the transformer in place are shown in Fig. 10 at 15, 17.5 and 20 kHz and, as seen, approximate the idealized calculated beam patterns of Fig. 3.

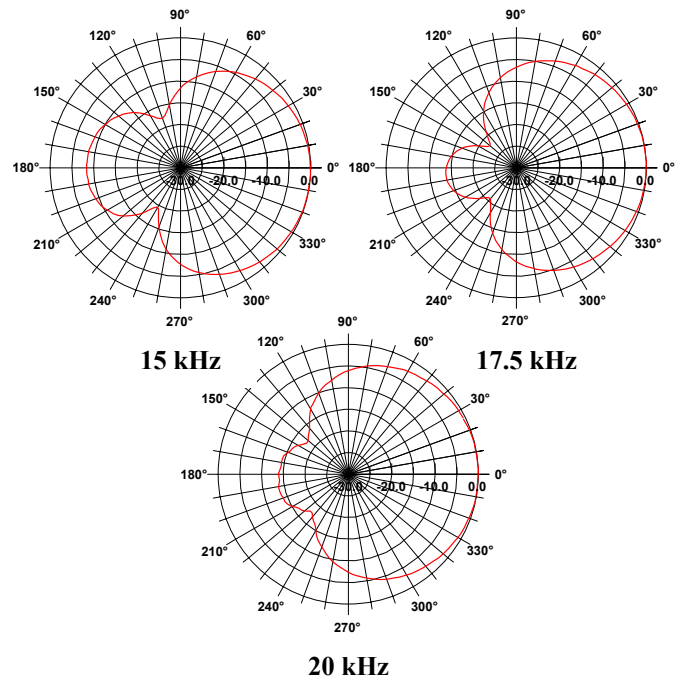


FIG. 10 Horizontal beam patterns for Tri-mode at 15 kHz, 17.5 kHz and 20 kHz.

The measured vertical beam pattern at 17.5 kHz is shown in Fig. 11 with a resulting $DI = 6.5$ dB and a vertical beam width of approximately 40° .

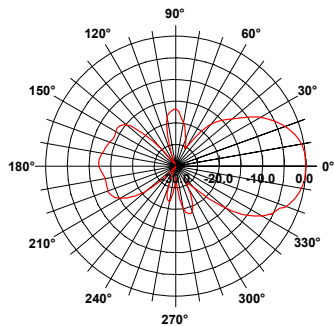


FIG. 11 Vertical beam pattern for Tri-mode at 17.5 kHz.

The two ring transducer housing was designed to accept acoustic modem boards through the addition of the two aluminum tubes shown in Fig. 6. The circuit board performs both transmit and receive functions as well as 45 degree incremental steering through a simple coded input. The board is fitted with four dual channel 10w/ch Tripath Class T power amplifiers in addition to the preamplifiers, tuning and transformers, diodes and coded activated switching. The switching allows the incremental steering by applying the four broadband voltage levels to the respective eight electrodes of the transducer through a coded input.

V. CONCLUSIONS

A simple, compact, easily steered, reasonably broadband directional sonar transducer has been implemented by exploiting higher order extensional modes of a piezoelectric cylinder. The beam pattern responses of each higher order mode can be combined to produce a directional beam pattern. One particularly useful directional beam pattern is the quadrant directional beam formed by a superposition of the first three extensional modes of the cylinder (omni, dipole and quadrupole modes with weightings 1:1:0.414). With segmented electrodes on a ceramic piezoelectric cylinder, a simple, easily implemented voltage distribution can be used to obtain nearly constant directional beam patterns over a significant band of frequencies. A transducer was fabricated, using this technique, which yielded a smooth transmit response from 15 kHz to 20 kHz, with a nearly constant 90 degree beam width and 10 dB front to back ratio across the band.

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