Theoretical and Experimental Studies on Broadband Constant Beamwidth Beamforming for Circular Arrays

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Abstract – **In this paper, we deal with the broadband constant beamwidth beamforming problem for circular arrays. The response vector of a circular array is expanded in the form of sum of an infinite series, whose core function is the first kind Bessel function. High order terms of this series are truncated so that the array response vectors at different frequency components can be transformed to be approximately equal to that at the reference frequency. Constant beamwidth beamforming vectors can be constructed by using the transforming procedure, so that beams at different frequencies are same as the reference beam. The effectiveness of this method is firstly verified by computer simulation. To further investigate the performance of the proposed method, lake-experiment was carried out for a 24-element uniform circular array with radius = 1.5 m. Based on the data collected from the experiment, constant beamwidth beams were successfully formed at eight different frequencies covering one-octave frequency band. Although big amplitude difference among hydrophones was observed during the experiment, the constant beamwidth beams were still correctly formed except for some distortions at sidelobe region of the beampattern. From the experiment result, we can conclude that constant beamwidth beamforming approach proposed for circular array is applicable to practical systems and has good error tolerance.**

I. INTRODUCTION

Broadband signals are often encountered in both active sonar systems and passive sonar systems [1]. An active sonar system often uses broadband signals to improve the range resolution without reducing the velocity resolution, and a passive sonar system will take advantage of the broadband spectrum of noise radiated from targets of interest to detect their existence. In these systems, beamforming is used to discriminate between signals based on the physical locations of the signal sources [2]. The conventional broadband sonar system uses delay-weight-and-sum method to realize the beamforming procedure [1]. In such a system, the patterns of spatial filters at different frequencies are different, which results in the different frequency responses in spatial domain except for the looking direction. When the target signals incident from directions other than the looking direction, linear distortion is unavoidable. Distortion in the received signal will degrade the performance of detection and estimation as well as that of classification of targets. The basic approach to solve this problem is using constant beamwidth beamformers, where the array system has the same spatial frequency responses for different frequency components of broadband signal [3-8]. Most of the existing approaches of designing constant beamwidth beamformers are for linear arrays. The main ideas behind these approaches are either to change the sensor number of the array, or to change the effective array aperture. Unfortunately, very few of the existing approaches are suitable for circular array.

In this paper, we will deal with the broadband constant beamwidth beamforming problem for circular arrays. The response vector of a circular array is expanded in the form of sum of an infinite series, whose core function is the first kind Bessel function. The high order terms of this series are truncated so that the array response vectors can be partitioned into two parts. In the two parts, one is always the same regardless of frequencies and the other is changing with frequency. We can use the changing part to transform the array response vectors at different frequencies to be equal to that at reference frequency. From the transforming procedure, the constant beamwidth beamforming vectors can be formed, so that the beams at different frequencies are the same as the reference beam.

The effectiveness of this method is verified by computer simulation. To further investigate the performance of the proposed method, lake-experiment was carried out for a 24-element uniform circular array with radius = 1.5 m. Based on the data collected from the experiment, constant beamwidth beams were successfully formed at eight different frequencies covered one-octave frequency band. During the experiment, big amplitude difference among hydrophones was observed, but the constant beamwidth beams were still correctly formed except for some distortions at sidelobe regions of the beampatterns. From the experiment result, we can conclude that constant beamwidth beamforming approach proposed in this paper for circular array is applicable to practical systems and has good error tolerance.

II. PROBLEM FORMULATION

Consider an uniform circular plane array with M omnidirectional elements receiving plane wavefronts radiated from point sources in the far-field of the array, as shown in Fig. 1. Assume that the elements of the array have the same amplitude and phase responses. The position vector of the mth element is

$$\boldsymbol{r}_m = (r_c \cos\theta_m, r_c \sin\theta_m, 0)\,,\ m = 1,\cdots,M \tag{2.1}$$

where $\theta_m = 2\pi m/M$, r_c is the radius of the circular array. If the plane wavefront incidents from $-\boldsymbol{e}$ direction to the array, where

$$\boldsymbol{e} = (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi) \tag{2.2}$$

then the time delay between the signals received at the mth element and the reference point (origin of circular array) is

$$\tau_m = -\boldsymbol{e}\cdot\boldsymbol{r}_m / c = -r_c \sin\phi\cos(\theta_m - \theta)/c \tag{2.3}$$

where c is the sound propagation velocity. If the signal received at the reference point is $s(t)$ then the output of the mth element of the array is

$$x_m(t) = s(t-\tau_m) + n_m(t) \tag{2.4}$$

where $s(t)$ is a broadband signal with frequency band $f \in [f_L, f_U]$, $n_m(t)$ is the additive noise on the mth element. Taking Fourier transform of (2.4), we have

$$X_m(f) = e^{-j2\pi f\tau_m} S(f) + N_m(f) \tag{2.5}$$

The frequency domain output of array denoted in matrix form is given by

$$\boldsymbol{X}(f) = \boldsymbol{A}(f,\theta,\phi)\boldsymbol{S}(f) + \boldsymbol{N}(f) \tag{2.6}$$

where $\boldsymbol{X}(f) = [X_1(f) \ \cdots \ X_M(f)]$ is the received array signal vector, $\boldsymbol{N}(f) = [N_1(f) \ \cdots \ N_M(f)]$ the additive noise vector, $\boldsymbol{A}(f,\theta,\phi)$ the response vector of circular array,

$$\boldsymbol{A}(f,\theta,\phi) = [e^{j2\pi f r_c \sin\phi\cos(\theta_1-\theta)/c}, \cdots, e^{j2\pi f r_c \sin\phi\cos(\theta_M-\theta)/c}]^{\mathrm{T}} \tag{2.7}$$

Without loss of generality, now we only consider the case where the incident signal is in the same plane as the circular array, i.e., $\phi = \pi/2$. In this case, the array response vector is

$$\boldsymbol{A}(f,\theta) = [e^{j2\pi f r_c \cos(\theta_1-\theta)/c}, \cdots, e^{j2\pi f r_c \cos(\theta_M-\theta)/c}]^{\mathrm{T}} \tag{2.8}$$

The beamformer output is formed by applying a weight vector to the received array data, giving

$$y_i = w_i^{\mathrm{H}} X(f_i) \tag{2.9}$$

where "H" denotes Hermitian transpose, and $\boldsymbol{w}_i$ is the M-vector of array weights to apply to the ith frequency bin. The spatial response of the beamformer is given by

$$B(f_i,\theta) = \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{A}(f_i,\theta) \tag{2.10}$$

which defines the transfer relation between a source at location $\theta \in [-\pi, \pi)$ and the beamformer output. The problem of designing constant beamwidth beamformer can now be formulated as finding the array weights in each frequency bin such that the resulting spatial response remains constant[1] over all frequency bins of interest.

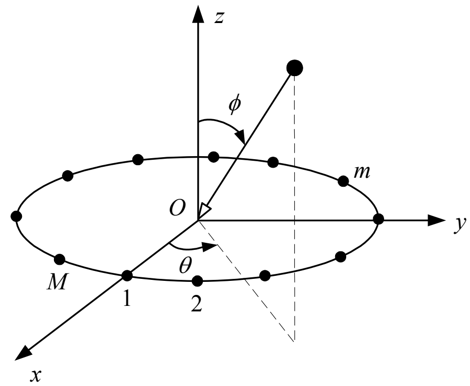


Fig. 1. Geometry of circular array

III. THEORETICAL SOLUTION

The conventional approach of beamforming is to compensate the time delay between elements so that the received signals can be added with the same phase. Generally, amplitude weighting is applied to control the beam shapes. For a circular array, the weighting coefficients for the frequency bin f_0 making the beam steered to direction θ_S is

$$\boldsymbol{w}_{0_0} = \mathrm{diag}(\boldsymbol{w})\boldsymbol{A}(f_0,\theta_S) \tag{3.1}$$

where $\boldsymbol{w} = [w^1 \ \cdots \ w^M]$ is the M-vector of amplitude weighting, $\mathrm{diag}(\boldsymbol{w})$ means a diagonal matrix whose diagonal elements are the elements of $\boldsymbol{w}$. The beampattern of the array at frequency bin f_0 is

$$B(f_0,\theta) = |\, \boldsymbol{w}_0^{\mathrm{H}} \boldsymbol{A}(f_0,\theta)\,|^2 \tag{3.2}$$

If the incident signal covers the frequency band $f \in [f_L, f_U]$, the beampatterns at different frequency bins are obviously different. In this case, the beam output signal will be linearly distorted except for the signal comes from the looking direction. The essential role of constant beamwidth beamforming is to select the weighting vectors at each frequency bin in the frequency band of interest so that

$$B(f_i,\theta_S) = |\, \boldsymbol{w}_i^{\mathrm{H}} \boldsymbol{A}(f_i,\theta)\,|^2 \cong B(f_0,\theta_S) \tag{3.3}$$

For circular arrays, we can use the decomposition formula of plane waves,

$$e^{jz\cos\Psi} = \sum_{n=-\infty}^{\infty} J_n(z)(j)^n e^{-jn\Psi} \tag{3.4}$$

so that each element of the response vector of a circular array can be represented as the summation form of an infinite series, whose core function is the first kind Bessel function,

$$\begin{aligned} a(f,\theta;\theta_m) &= e^{j2\pi f r_c \cos(\theta_m-\theta)/c} \\ &= \sum_{n=-\infty}^{\infty} (j)^n J_n(2\pi f r_c / c) e^{jn\theta_m} e^{-jn\theta} \end{aligned} \tag{3.5}$$

[1] At least constant in the mainlobe region of the spatial response, i.e., constant beamwidth

where $m = 1, \cdots, M$. Hence, the array response vector can be partitioned into two parts

$$A(f, \theta) = \widetilde{T}(f)\widetilde{w}(\theta) \tag{3.6}$$

where

$$[\widetilde{T}(f)]_{m,n} = (j)^n J_n(2\pi f r_c / c) e^{jn\theta_m}, \ m = 1, \cdots, M, \tag{3.7}$$

$$[\widetilde{w}(\theta)]_n = e^{-jn\theta} \tag{3.8}$$

According to (3.5), $n = 0, \pm 1, \pm 2, \cdots \pm \infty$ in (3.7) and (3.8).

For the Bessel function $J_n(z)$, $z = 2\pi f r_c / c$, we now consider its plots in Fig. 2. We see that for high order Bessel functions, we will have $\max |J_{n_\varepsilon}(z)| < \varepsilon$ in a certain interval, where ε is a small value selected according to the accuracy requirement. Actually, the interval of $z = 2\pi f r_c / c$ in (3.7) is determined by the frequency band. For circular arrays,

$$z \in [z_{\min}, z_{\max}] = [2\pi f_{\min} r_c / c, 2\pi f_{\max} r_c / c] \tag{3.9}$$

For certain $z_{\max} = 2\pi f_{\max} r_c / c$, we can find an n_ε to satisfy $|J_{n_\varepsilon}(z)| < \varepsilon$ for all z in (3.9). With this observation, we can truncate $\widetilde{T}(f)$ and $\widetilde{w}(\theta)$ at $|n| = n_\varepsilon$, so that

$$A(\theta, f) \cong T(f) w(\theta) \tag{3.10}$$

where $T(f)$ is an $M \times (2n_\varepsilon + 1)$ matrix, $w(\theta)$ is a $(2n_\varepsilon + 1) \times 1$ vector, which is frequency invariant. The beamforming vector of frequency bin f_i can be designed using the invariance of $w(\theta)$,

$$w_i = T^{\mathrm{H}} w_0 \tag{3.11}$$

where $T = T(f_0) T^{+}(f_i) = T(f_0)[T^{\mathrm{H}}(f_i) T(f_i)]^{-1} T^{\mathrm{H}}(f_i)$. Now the beampattern of frequency bin f_i is

$$\begin{aligned} B(f_i, \theta_S) &= |w_0^{\mathrm{H}} A(f_i, \theta)|^2 \\ &\cong |w_0^{\mathrm{H}} T(f_0)[T^{\mathrm{H}}(f_i) T(f_i)]^{-1} T^{\mathrm{H}}(f_i) T(f_i) w(\theta)|^2 \\ &= |w_0^{\mathrm{H}} T(f_0) w(\theta)|^2 \cong |w_0^{\mathrm{H}} A(f_0, \theta)|^2 \end{aligned} \tag{3.12}$$

which satisfies the constant beamwidth beamforming condition in (3.3).

IV. COMPUTER SIMULATION

A 24-element uniform circular array with radius $r_c = 1.5$m was considered in computer simulation. The design was for a frequency band of 960~1920 Hz. In this case, $z_{\max}$ in (3.9) was equal to $2\pi f_{\max} r_c / c = 12.06$, and we chose $n_\varepsilon = 15$. The reference frequency is chosen as $f_0 = 1600$ Hz. We designed the beampattern at this reference frequency, as is shown in Fig. 3. The frequency band was divided into 16 frequency bins, each with center frequency $f_n = 960 + 64n$ Hz ($n = 0, 1, \cdots, 15$). The beamforming vector for each frequency bin was designed using (3.11). With these beamforming vectors, constant beamwidth beampatterns were formed at each frequency, which is shown in Fig. 4. The –3 dB beamwidth of the beams in Fig. 3 and Fig. 4 is $17.6°$.

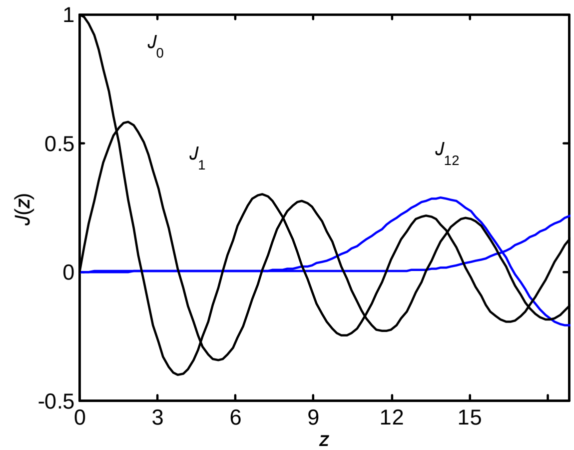


Fig. 2. Graphs of first kind Bessel function

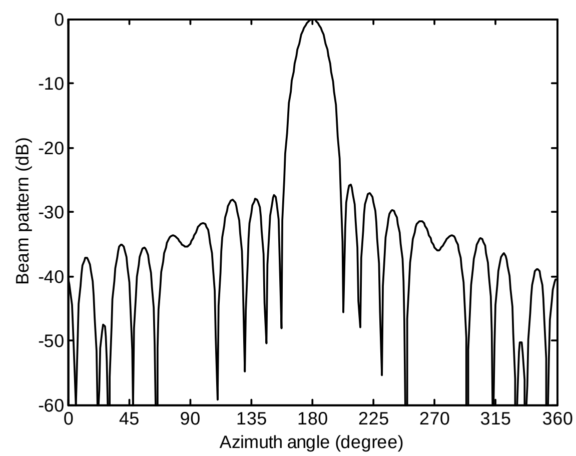


Fig. 3. The beampattern designed for the reference frequency

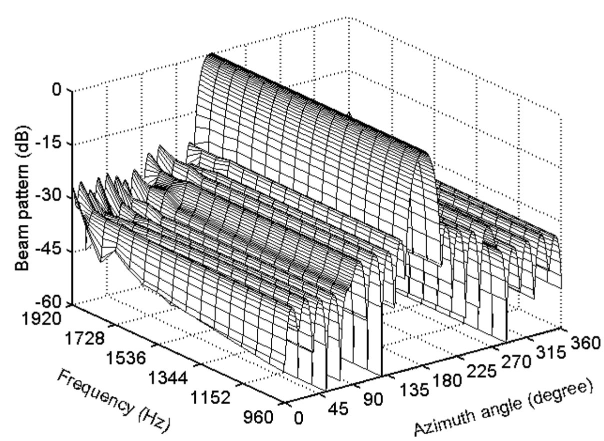


Fig. 4. Broadband beampattern

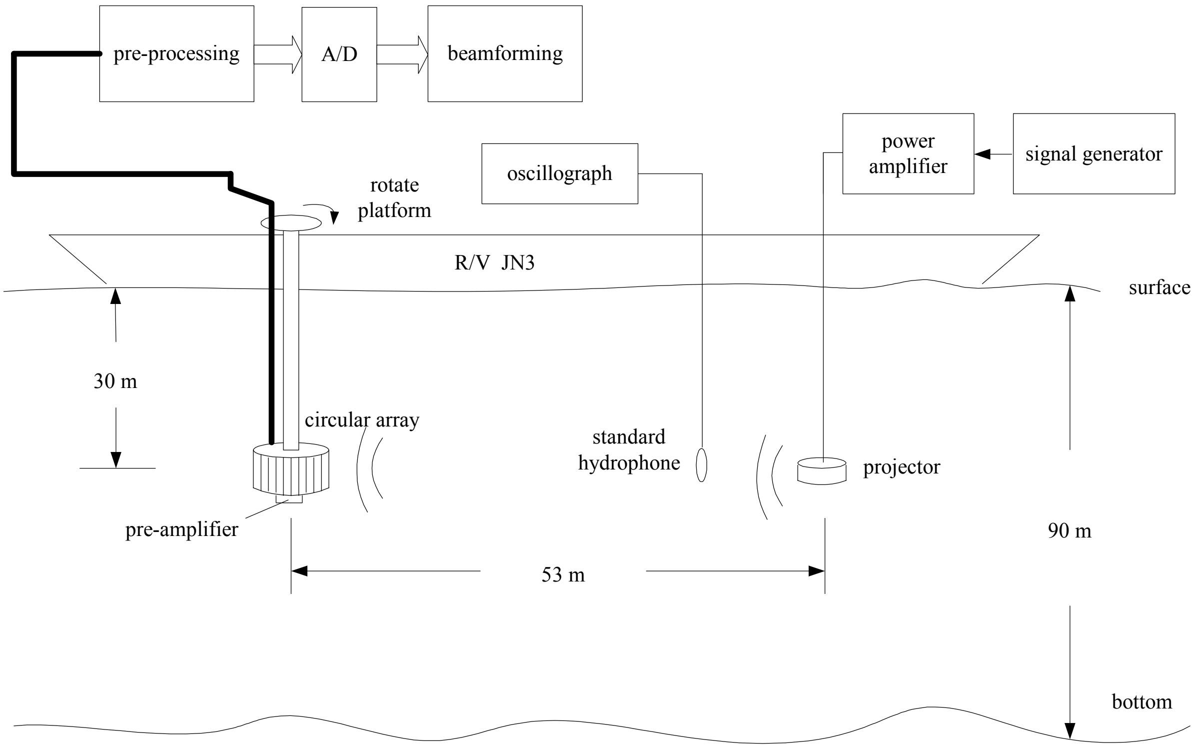


Fig. 5. The experiment configuration

V. LAKE EXPERIMENT

To further investigate the performance of the constant beamwidth beamforming approach, lake-experiment was carried out for a 24-element uniform circular array with radius = 1.5 m in a lake located at east China. Fig. 5 shows the experiment configuration. The circular array was put at 30 m below the lake surface, and the source signal was located at the same depth, 53 m in distance from the array. The signal transmitted by the projector was rectangular window modulated single frequency continuous wave (CW). The impulse repetition period is 200 ms. 8 frequencies covering one octave were selected and shown in Table 1 with the corresponding temporal rectangular window length L used.

During the experiment, the array was rotated in the horizontal plane to receive signals from different directions. The rotation was from 0° to 360° with 2.5° step. The received signals were pro-processed by anti-alias filter, and then sampled by A/D converters. The data from the A/D converter were saved to PC as data files. With these data in hand, we obtained the array manifold (a set of array response vectors for different directions) of this experimental circular array by the method described in Appendix. Based on the real array manifold, beams can be formed. Fig. 6 showed the beams at the 8 selected frequencies. The conventional beams of the same circular array at each frequency were also shown in Fig. 6. The –3 dB beamwidth of the beams was calculated, and shown in Table 2, where CBBW denotes the beamwidth of constant beamwidth beams, and CBW denotes the beamwidth of conventional beams. From Table 2, we can see that the difference of –3 dB beamwidth of the conventional beams between the highest frequency and the lowest frequency is 10.3° , whereas it is only 2.6° for constant beamwidth beams. Theoretically, the constant beamwidth beams will have the same –3dB beamwidth at each frequency bin. The experimental results deviated from the theoretical results because system errors unavoidably existed in the circular array used in experiment.

TABLE 1

PARAMETERS OF THE TRANSMITTED SIGNAL

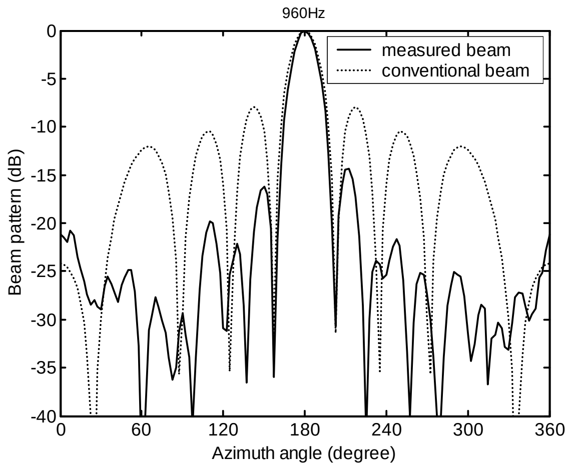
f (Hz)	960	1152	1280	1408	1472	1600	1728	1920
L (ms)	12.5	13	15.6	14.2	13.6	15.0	15.6	15.6

TABLE 2

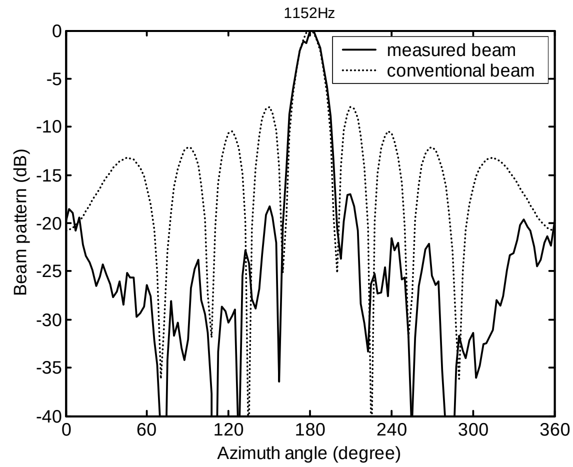
BEAMWIDTH OF BEAMS IN FIG. 6

f (Hz)	960	1152	1280	1408	1472	1600	1728	1920
CBBW (Deg)	18.3	18.3	18.0	19.3	18.0	19.4	18.5	20.6
CBW (Deg)	21.3	17.7	15.9	14.5	13.9	12.7	11.7	10.7

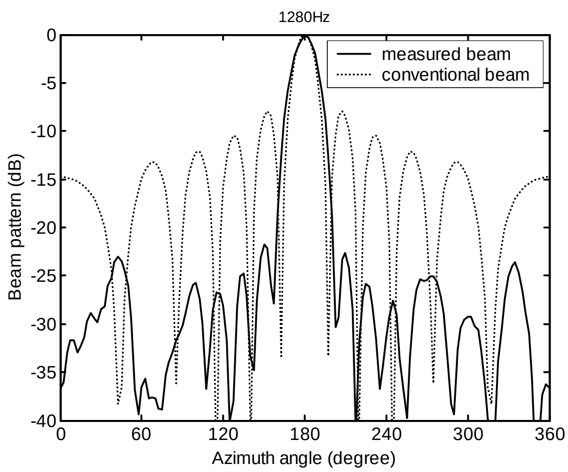
We have made some efforts to guarantee the quality of the hydrophones used in the experiment array. The sensitivity of the hydrophones was measured using a standard hydrophone with known sensitivity as reference. Good results were obtained. For example, the sensitivity of 24 hydrophones at 960 Hz was between –200 dB and –199 dB, and each can be considered to be omnidirectional due to low spatial variation



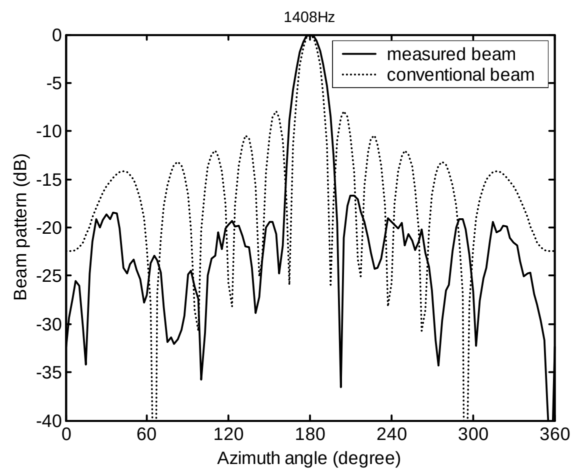
(a) $f = 960$ Hz



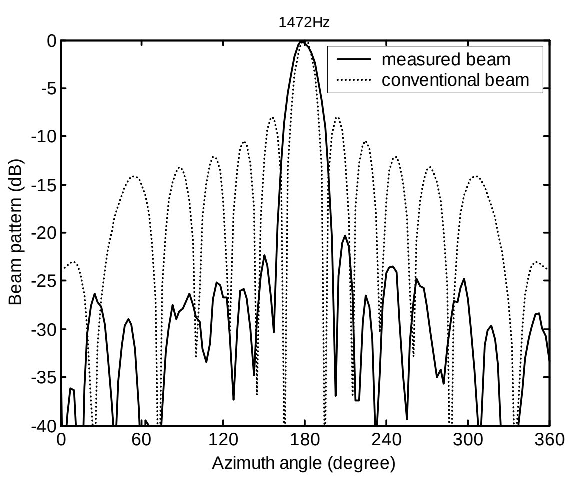
(b) $f = 1152$ Hz



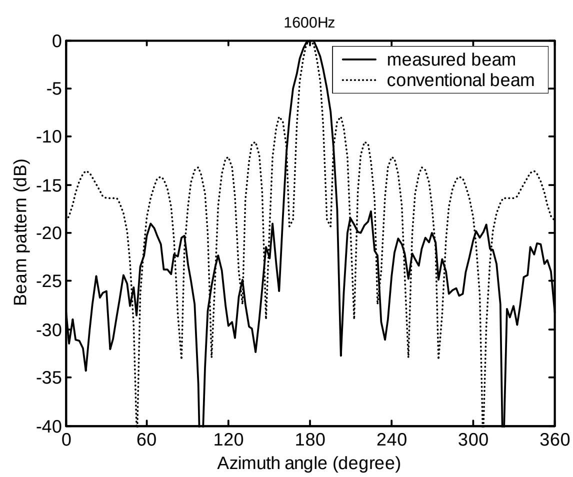
(c) $f = 1280$ Hz



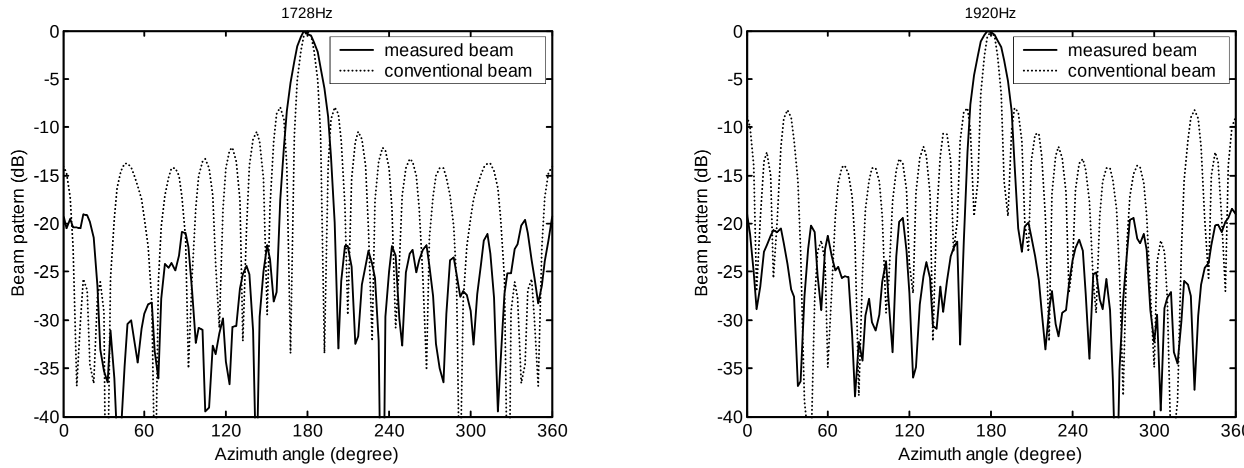
(d) $f = 1408$ Hz



(e) $f = 1472$ Hz



(*f*) f = 1600 Hz



(g) f = 1728 Hz

(h) f = 1920 Hz

Fig. 6. Beams formed using experiment data

in the spatial responses of individual hydrophone. When all the 24 hydrophones were installed on the circular array, the situation changed because of influence of the structure of circular array. Now difference of amplitude and phase responses among hydrophones will be greater than that measured individually. Fig 7. shows the amplitude responses of the 5th and the 22nd elements at 1024 Hz. We can see that the largest difference at some directions could be $20\log(1/0.7)=3$ dB, much bigger than 1 dB. Fig. 7 also shows that the amplitude response with respect to the azimuth angle changed slowly, and the fluctuation was also bigger than 1 dB.

Though there were big system errors in the circular array, the constant beamwidth beams were still correctly formed except for some distortions at sidelobe regions of the beampatterns (Fig.6). The conclusion can be made that the constant beamwidth beamforming approach used in this paper for circular array has good error tolerance.

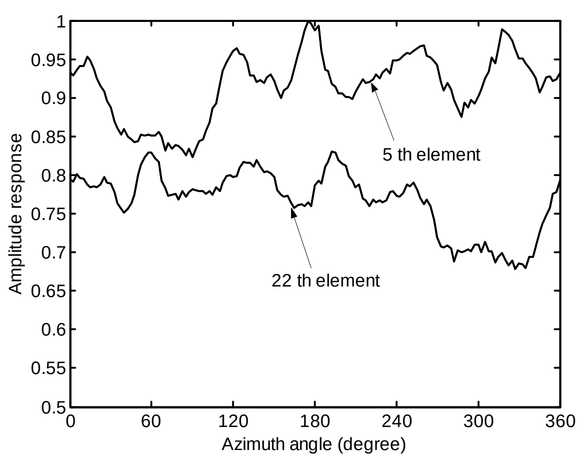


Fig. 7. Amplitude response of 5th and 22nd element

VI. CONCLUSIONS

Circular array is widely used in broadband sonar systems. We have presented the theoretical and experimental results for a broadband circular array in which the beamwidth of the far-field beam pattern is constant over a desired frequency range. It is expected that the detection and estimation ability of such a broadband sonar system will be improved by exploiting constant beamwidth beamforming.

The beamforming vectors obtained in this paper are suitable for frequency domain beamforming. There are several references [11-13] deal with the broadband beamforming in time domain. Those methods can be used to implement constant beamwidth beamforming in time domain.

ACKNOWLEDGMENTS

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APPENDIX: MEASURING ARRAY MANIFOLD

In order to measure the array response vector of the circular array at frequency *f*, the received testing signal of each element can be transformed using the procedure in Fig. 8. Assume that the received signal of the element after A/D converter is

$$x(n) = A\cos(2\pi fn + \varphi) \qquad \text{(A.1)}$$

where $f = f_c / f_s$ is the digital frequency of the signal, f_s the sampling frequency, n the index of sample points. $x(n)$ is multiplied by a complex exponential signal with the same frequency to get $y(n)$ as,

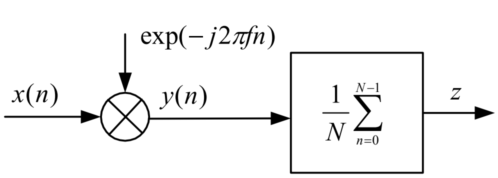

Fig. 8. Measure the array response vector

$$\begin{aligned} y(n) &= x(n)\exp(-j2\pi fn) \\ &= 0.5A\{\exp[j(2\pi fn+\varphi)]+\exp[-j(2\pi fn+\varphi)]\}\exp(-j2\pi fn) \\ &= 0.5A\exp(j\varphi)+0.5A\exp[-j(4\pi fn+\varphi)] \end{aligned} \tag{A.2}$$

Then $y(n)$ is summed from n=0 to N to get the final output z,

$$z = 0.5A\exp(j\varphi)+\frac{A}{2N}\exp(-j\varphi)\sum_{n=0}^{N-1}\exp(-j4\pi fn) \tag{A.3}$$

The second term in the right hand of (A.3) is summation of a geometric progression. After simple calculation, we get

$$\begin{aligned} z &= 0.5A\exp(j\varphi)+ \\ &\frac{A\sin(2\pi Nf)}{2N\sin(2\pi f)}\exp\{-j[2\pi(N-1)f+\varphi]\} \end{aligned} \tag{A.4}$$

The second term in the right hand of (A.4) will be zero if $N = m/2f$ ($m = 1,2,\cdots$). In this case, (A.4) is simplified to

$$z = 0.5A\exp(j\varphi) \tag{A.5}$$

Hence the amplitude and initial phase of the received signal is obtained, which represents the amplitude and phase response of this sensor to the signal from a certain direction. If $m/2f$ is not an integer, then N can be selected as an integer nearest $m/2f$, which guarantees (A.5) be satisfied approximately.

When the received signal at each element is processed using the above approach, the array response vector for a certain direction is obtained. Varying the direction of the incident testing signal, we can form the array manifold of the circular array.

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