Broadband Beamspace DOA Estimation Algorithms

Yixin Yang[1], Chunru Wan[1] and Chao Sun[2]

[1]School of Electrical and Electronic Engineering
Nanyang Technological University, Singapore, 639798
[2]Institute of Acoustic Engineering,
Northwestern Polytechnical University, Xi'an, China, 710072
E-mail: eyxyang@ntu.edu.sg, ecrwan@ntu.edu.sg, csun@nwpu.edu.cn

***Abstract*-Two kinds of broadband beamspace high-resolution DOA estimation algorithms are studied in this paper. The first one is broadband beamspace coherent high-resolution processing method based on constant beamwidth beamformers. In this method, conventional narrowband beamspace high-resolution DOA estimation methods such as beamspace MUSIC can be applied to the output of constant beamwidth beamformers. Because of the fixed beamformers used in this method, its performance will be reduced when there exist strong interference signals outside the beam coverage region. To solve this problem, broadband incoherent high-resolution processing method based on subband beamformers is proposed in this paper. The broadband signals are divided into subbands via frequency decomposition. In each subband, we use MVDR beamformers to suppress the strong interference signals outside the beam coverage region. To overcome the beampattern distortion problem due to the inaccuracy estimation of the covariance matrix, diagonal loading is applied to the MVDR beamformers. Beamspace MUSIC is used to get the spatial spectrum of each subband. The final DOA estimates result from the incoherent combination of spatial spectrum of each subband. Simulation results are presented to show the performance of these two algorithms.**

I. INTRODUCTION

Most of the underwater acoustic systems use beam outputs to estimate the target directions [1]. When there are several targets and the angular separation between two arbitrary targets is less than the beamwidth, traditional direction estimation methods fail to get the target directions correctly. High-resolution DOA estimators such as MUSIC [2-4] can be used to overcome this problem in narrowband systems. To reduce the computation complexity of the MUSIC algorithm, beamspace MUSIC was discussed in several references, such as in [5-7]. Unfortunately, these algorithms are only suitable for narrowband signals. Several methods of broadband DOA estimation have been proposed based on the coherent signal subspace approach introduced by Wang and Kaveh [8] (for example, [9-13]). The CSS method is based on decomposing the wideband data into several nonoverlapping narrowband frequency bins and finding focusing matrices that transform the data in each bin to a reference frequency bin. The focused data is then combined to form a composite covariance matrix. Conventional narrowband DOA estimators may then be directly applied by eigendecomposition of the composite covariance matrix.

Lee [14] proposed a CSS method based on beamspace processing. For each frequency bin, Lee constructs a beamforming matrix chosen such that the resulting beampatterns are essentially identical for all frequencies. The dimension of the beamspace is usually much smaller than the element number so the computational complexity is reduced and the statistical stability of the estimate of the covariance matrix is improved. Furthermore, for non-white spatial noise that we assume unknown, if we divide the spatial space into some sectors by using parallel beamspace processors, the noise can be modeled as white with an unknown variance in each of these sectors, so that the assumption of the additional white Gaussian noise will be valid for most of the high-resolution DOA estimation algorithms. Ward [15] proposed another broadband beamspace DOA estimation algorithm that performs broadband beamforming using time-domain processing. The broadband beamformers have constant beamwidth over a wide frequency band by using the method in [16]. The performance of this algorithm is very similar to that proposed by Lee [14].

In this paper, we studied two kinds of broadband beamspace high-resolution DOA estimation algorithms. The first one is broadband beamspace coherent high-resolution processing method based on constant beamwidth beamformers. The idea of this algorithm is from Lee [14] directly, but we use beamspace MUSIC instead of beamspace root-MUSIC in Lee's paper. The constant beamwidth beamformers not only transform the element space signals into beamspace, but also focus the different frequency components on a reference frequency. Unfortunately, when the beamformers are fixed, they can not remove the strong interference signals from outside the beam coverage region. To solve this problem, we proposed the second method, which is broadband incoherent high-resolution processing method based on subband beamformers. In this method, the broadband signals are divided into subbands via frequency decomposition. In each subband, MVDR beamformers are used to suppress the strong interference signals outside the beam coverage region. To overcome the beampattern distortion problem due to the inaccuracy estimation of the covariance matrix, diagonal loading is applied to the MVDR beamformers. Beamspace MUSIC is used to get the spatial spectrum of each subband. The final DOA estimates result from the incoherent combination of spatial spectrum of each subband.

0-933957-31-9

The paper is organized as follows. In Section II, broadband array signal modeling is briefly reviewed. In Section III, the broadband beamspace coherent high-resolution processing method based on constant beamwidth beamformers is presented. In section IV, the broadband incoherent high-resolution processing method based on subband MVDR beamformers is proposed. Section V presents simulation results demonstrating the performance of the proposed methods, and Section VI concludes the paper.

II. BROADBAND ARRAY SIGNAL MODELING

Consider an array composed of M elements with a known arbitrary geometry receiving plane wavefronts generated by P wideband sources at locations $\theta_1, \theta_2, \cdots, \theta_P$ with overlapping spectra of bandwidth BW, in the presence of additive white noise. The signal received at the mth element can be denoted as

$$x_m(t) = \sum_{p=1}^{P} s[t - \tau_m(\theta_p)] + n_m(t), \; m = 1, \cdots, M \tag{2.1}$$

where $s_p(t)$ is the unknown signal radiated by the pth source, $n_m(t)$ is the additive noise received at the mth element, and $\tau_m(\theta_p)$ is the propagation delay associated with the pth source and the mth element. If a discrete Fourier transform is applied to time duration T_d of $x_m(t)$, then we may write

$$\boldsymbol{X}(f_j) = \boldsymbol{A}(f_j, \Theta)\boldsymbol{S}(f_j) + \boldsymbol{N}(f_j) \tag{2.2}$$

where

$$\boldsymbol{X}(f_j) = [X_1(f_j) \quad \cdots \quad X_M(f_j)]^{\mathrm{T}} \tag{2.3a}$$

$$\boldsymbol{S}(f_j) = [S_1(f_j) \quad S_2(f_j) \quad \cdots \quad S_P(f_j)]^{\mathrm{T}} \tag{2.3b}$$

$$\boldsymbol{N}(f_j) = [N_1(f_j) \quad \cdots \quad N_M(f_j)]^{\mathrm{T}} \tag{2.3c}$$

$$\boldsymbol{A}(f_j, \Theta) = [\boldsymbol{a}(f_j, \theta_1) \quad \boldsymbol{a}(f_j, \theta_2) \quad \cdots \quad \boldsymbol{a}(f_j, \theta_P)] \tag{2.3d}$$

$$\boldsymbol{a}(f_j, \theta_p) = [e^{-j2\pi f_j \tau_1(\theta_p)} \quad e^{-j2\pi f_j \tau_2(\theta_p)} \quad \cdots \quad e^{-j2\pi f \tau_M(\theta_p)}]^{\mathrm{T}} \tag{2.3e}$$

Here $X_m(f_j)$, $S_p(f_j)$ and $N_m(f_j)$ are the discrete Fourier coefficients of $x_m(t)$, $s_p(t)$ and $n_m(t)$, respectively, at frequency f_j. $\boldsymbol{A}(f_j, \Theta)$ is the $M \times P$ direction matrix, and the vectors $\boldsymbol{a}(f_j, \theta_p)$ are referred to as the steering vectors of the array. We assume that the length T_d is long enough compared to the correlation times of both the signals and the noise, so that the discrete Fourier transform coefficients are uncorrelated. The noise samples are assumed to be statistically independent in time and space, so the spatial covariance matrix of the noise for each frequency is $\sigma^2 \boldsymbol{I}$, where σ^2 is an unknown scalar and $\boldsymbol{I}$ is the identity matrix.

If we have K length-T_d observations, then the sample covariance matrix at the frequency f_j can be formed as

$$\hat{\boldsymbol{R}}(f_j) = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{X}_k(f_j)\boldsymbol{X}_k^{\mathrm{H}}(f_j) \tag{2.4}$$

III. DOA ESTIMATION BASED ON CONSTANT BEAMWIDTH BEAMFORMERS

Assume that we apply constant beamwidth beamforming to the received array data in frequency domain. The outputs of B conjunctive beamformers at the frequency f_j are given by

$$\begin{aligned} \boldsymbol{Y}(f_j) &= \boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{X}(f_j) \\ &= \boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{A}(f_j, \Theta)\boldsymbol{S}(f_j) + \boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{N}(f_j) \end{aligned} \tag{3.1}$$

where

$$\boldsymbol{W}(f_j) = [\boldsymbol{w}_1(f_j) \,|\, \boldsymbol{w}_2(f_j) \,|\, \cdots \,|\, \boldsymbol{w}_B(f_j)] \tag{3.2}$$

is the beamforming matrix of the B conjunctive beamformers. Because the beamformers are designed to have constant beamwidth over the interested frequency band, $\boldsymbol{W}(f_j)$ satisfies the following relation,

$$\boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{A}(f_j) \cong \boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{A}(f_0) \tag{3.3}$$

where f_0 is the reference frequency. Hence, $\boldsymbol{Y}(f_j)$ can be approximately expressed as

$$\boldsymbol{Y}(f_j) \cong \boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{A}(f_0, \Theta)\boldsymbol{S}(f_j) + \boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{N}(f_j) \tag{3.4}$$

The sample covariance matrix of constant beamwidth beamformers at the frequency f_j is given by

$$\hat{\boldsymbol{R}}_y(f_j) = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{Y}_k(f_j)\boldsymbol{Y}_k^{\mathrm{H}}(f_j) \tag{3.5}$$

The broadband sample covariance matrix is then constructed by coherently combining the sample covariance matrices

$$\begin{aligned} \hat{\boldsymbol{R}}_y &= \frac{1}{J} \sum_{j=1}^{J} \alpha_j \hat{\boldsymbol{R}}_y(f_j) = \frac{1}{JK} \sum_{j=1}^{J} \alpha_j \sum_{k=1}^{K} Y_k(f_j)Y_k^{\mathrm{H}}(f_j) \\ &\cong \boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{A}(f_0)\left(\frac{1}{J}\sum_{j=1}^{J} \alpha_j \boldsymbol{S}(f_j)\boldsymbol{S}^{\mathrm{H}}(f_j)\right)\boldsymbol{A}^{\mathrm{H}}(f_0)\boldsymbol{W}(f_0) + \hat{\boldsymbol{R}}_n \end{aligned} \tag{3.6}$$

where α_j is the weighting factor for the sample covariance matrix at the frequency f_j, $\hat{\boldsymbol{R}}_n$ is the noise covariance matrix

$$\hat{\boldsymbol{R}}_n = \frac{\sigma^2}{J} \sum_{j=1}^{J} \alpha_j \boldsymbol{W}^{\mathrm{H}}(f_j)\boldsymbol{W}(f_j) \tag{3.7}$$

If the signal-to-noise ratio (SNR) is the same at each frequency, the weighting factor can be uniformly applied to covariance matrices. For different SNRs at each frequency, optimal Eckart coefficients [1, pp. 154] can be used as the weighting factors. In this case, the prior information about the spectra of signals and noise is needed to form the Eckart coefficients. For simplicity, we use uniform weighting in the simulation in Section V.

Now the broadband sample covariance matrix of (3.6) is in a form in which conventional eigen-based DOA estimators may be applied. Denote the eigen-decomposition of $\{\hat{\boldsymbol{R}}_y, \hat{\boldsymbol{R}}_n\}$ as

$$\hat{\boldsymbol{R}}_y \boldsymbol{E}_B = \boldsymbol{\Lambda} \hat{\boldsymbol{R}}_n \boldsymbol{E}_B \tag{3.8}$$

where $\boldsymbol{\Lambda}$ is the diagonal matrix of sorted eigenvalues, $\boldsymbol{E}_B = [\boldsymbol{E}_{BS} \mid \boldsymbol{E}_{BN}]$ is the corresponding eigenvectors, $\boldsymbol{E}_{BS}$ is eigenvectors corresponding to the largest P eigenvalues, $\boldsymbol{E}_{BN}$ is eigenvectors corresponding to the smallest $J-P$ eigenvalues. The ranges of $\boldsymbol{E}_{BS}$ and $\boldsymbol{E}_{BN}$ are referred to as the signal and noise subspaces, respectively. Specifically, for the beamspace MUSIC algorithm, the source directions are given by the P peak positions of the following B-MUSIC spatial spectrum:

$$P_{\text{B-MUSIC}}(\theta) = \frac{\boldsymbol{a}^{\mathrm{H}}(f_0,\theta)\boldsymbol{W}(f_0)\boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{a}(f_0,\theta)}{\boldsymbol{a}^{\mathrm{H}}(f_0,\theta)\boldsymbol{W}(f_0)\boldsymbol{E}_{BN}\boldsymbol{E}_{BN}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{a}(f_0,\theta)} \tag{3.9}$$

where $\boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{a}(f_0,\theta)$ is the transformed steering vector in constant beamwidth beamspace.

IV. DOA ESTIMATION BASED ON SUBBAND MVDR BEAMFORMERS

The broadband beamspace coherent subspace method based on constant beamwidth beamformers has lower computation complexity than the CSSM proposed by Wang and Kaveh [8]. Another advantage of this method is that it does not need to pre-estimate the approximate DOAs. Unfortunately, because the beamformers are fixed, they cannot remove the strong interference signals from outside the beam coverage region. Adaptive beamforming techniques such as Minimum Variance Distortionless Response (MVDR) beamformers should be used to automatically remove the influence of the interference signals. We will study the broadband incoherent high-resolution processing method based on subband MVDR beamformers in this section.

A. Incoherent Broadband Beamspace DOA Estimation

Assume we apply conventional beamforming to the received array data in frequency domain. The outputs of B conjunctive beamformers at the frequency f_j are given by

$$\boldsymbol{Y}_C(f_j) = \boldsymbol{W}_C^{\mathrm{H}}(f_j)\boldsymbol{X}(f_j) \tag{4.1}$$

The sample covariance matrix of conventional beamformers at the frequency f_j is given by

$$\hat{\boldsymbol{R}}_{Cy}(f_j) = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{Y}_{Ck}(f_j)\boldsymbol{Y}_{Ck}^{\mathrm{H}}(f_j) \tag{4.2}$$

If the eigenvectors corresponding to the smallest $J-P$ eigenvalues are denoted by $\boldsymbol{E}_{CBN}(f_j)$, then we can form the beamspace null spectrum of MUSIC for frequency f_j,

$$P_{\text{MUSIC}}(f_j,\theta) = \frac{\boldsymbol{a}^{\mathrm{H}}(f_j,\theta)\boldsymbol{W}_C(f_j)\boldsymbol{E}_{CBN}(f_j)\boldsymbol{E}_{CBN}^{\mathrm{H}}(f_j)\boldsymbol{W}_C^{\mathrm{H}}(f_j)\boldsymbol{a}(f_j,\theta)}{\boldsymbol{a}^{\mathrm{H}}(f_j,\theta)\boldsymbol{W}_C(f_j)\boldsymbol{W}_C^{\mathrm{H}}(f_j)\boldsymbol{a}(f_j,\theta)} \tag{4.3}$$

The final broadband spatial spectrum is obtained by incoherently combining the beamspace null spectrum,

$$P(\theta) = \frac{1}{\frac{1}{J}\sum_{j=1}^{J}P_{\text{MUSIC}}(f_j,\theta)} \tag{4.4}$$

The source directions are given by the P peak positions of $P(\theta)$.

B. Adaptive Beamforming

It is well known that MVDR beamformer can produce a null at the direction of a strong interference signal. The interference signal component in the output of MVDR beamformer will be removed as much as possible. If there are strong interference signals outside the beam coverage region, then MVDR beamformers can be used to null the interference signal. In this case, the beamforming vector for frequency f_j is

$$\boldsymbol{w}(f_j,\theta_s) = \frac{\hat{\boldsymbol{R}}^{-1}(f_j)\boldsymbol{a}(f_j,\theta_s)}{\boldsymbol{a}^{\mathrm{H}}(f_j,\theta_s)\hat{\boldsymbol{R}}^{-1}(f_j)\boldsymbol{a}(f_j,\theta_s)} = \mu\hat{\boldsymbol{R}}^{-1}(f_j)\boldsymbol{a}(f_j,\theta_s) \tag{4.5}$$

where θ_s is the beam steering direction, $\hat{\boldsymbol{R}}(f_j)$ is defined in (2.4), as the estimate of covariance matrix. Inevitably, there will be inaccuracy in the estimation of $\hat{\boldsymbol{R}}(f_j)$, as well as the array perturbations in practical systems. All these factors will result in the distortion of MVDR beams. The most common way to overcome this problem is the so-called diagonal loading. The diagonal loaded sample covariance matrix at frequency f_j is

$$\boldsymbol{R}_{DL}(f_j) = \hat{\boldsymbol{R}}(f_j) + \sigma_L^2\boldsymbol{I} \tag{4.6}$$

where σ_L^2 is the loading factor, selected as to satisfy the following equation

$$10\log_{10}(\max_m(R_{mm}(f_j))/\sigma_L^2) = 15 \text{ dB} \tag{4.7}$$

Compared to many alternative selections of σ_L^2 [4], the value of σ_L^2 defined in (4.7) is easy to get. We will use this selection in our computer simulation.

The new MVDR beamforming vector after diagonal loading is

$$\mathbf{w}_{DL}(f_j,\theta_s)=\frac{\mathbf{R}_{DL}^{-1}(f_j)\mathbf{a}(f_j,\theta_s)}{\mathbf{a}^{\mathrm{H}}(f_j,\theta_s)\mathbf{R}_{DL}^{-1}(f_j)\mathbf{a}(f_j,\theta_s)}=\mu\mathbf{R}_{DL}^{-1}(f_j)\mathbf{a}(f_j,\theta_s) \tag{4.8}$$

Interference signals can be removed by using this robust MVDR beamformer as preprocessor in each subband.

V. COMPUTER SIMULATION

Computer simulations are presented to ascertain the performance of the two broadband beamspace DOA estimation algorithms in this section.

A. Simulation for DOA Estimation based on Constant Beamwidth Beamforming

Consider a uniform linear array consisting of $M=15$ elements with a half-wavelength spacing at the center frequency of $f_c = 10$ kHz. The bandwidth of the receiver was 4 kHz, with f_c as the center frequency. Two source signals came from $[9°,12°]$. We consider the situation of coherent sources, so that we have $s_2(t)=s_1(t-t_0)$, where $s_1(t)$ is assumed to be bandpass white Gaussian process with flat spectral density of height S_0. Spatially white Gaussian bandpass noise also with a flat spectral density height of N_0, independent of the received signal, was present at each array element. The *SNR* in decibels per array element was defined as $10\log_{10}(S_0/N_0)$. The received signals are divided into $J=33$ subbands using DFT, so the output data from each bin can be modeled as narrowband. For each execution of DOA estimation, N = 32 snapshots were collected at each of the 33 bins.

The reference frequency for constant beamwidth beamforming was selected as $f_0=f_L$ =8 kHz. The three beams designed for the reference frequency steered at $0°$, $7.7°$ and $15.5°$ are shown in Fig. 1. The beams at frequency f_j ($j=1,\cdots,33$) were designed using the constant beamwidth beamforming method proposed in [14]. Results of the three beams at each of the 33 frequency bins are shown in Fig. 2.

The broadband beamspace DOA estimation algorithm based on constant beamwidth beamforming (denoted as B-MUSIC) was used in the simulation. For the purpose of performance comparison, CSSM using rotational signal-subspace focusing matrix [10,11] was also exploited to resolve the source directions. Fig. 3 shows the spatial spectra of both algorithms when the *SNR* of the two source signals was 6 dB. The advantage of B-MUSIC is that it has lower background and lower valley than that of CSSM.

In order to compare the probability of resolution and *RMSE* (Root Mean Square Error) of both algorithms at different *SNR*s, 200 independent trials were done at each *SNR* from –10 dB to 10 dB with 2 dB step. The resulting probability of resolution is shown in Fig. 4(a), and the sample *RMSE* of the DOA estimates are shown in Fig. 4(b) for various *SNR* levels (CSSM: dashed line; B-MUSIC: dotted line). These results show that the performance of B-MUSIC is a little bit better than that of CSSM. It should be noted that by using only threes beams in B-MUSIC, the computation complexity of B-MUSIC is much less than that of CSSM.

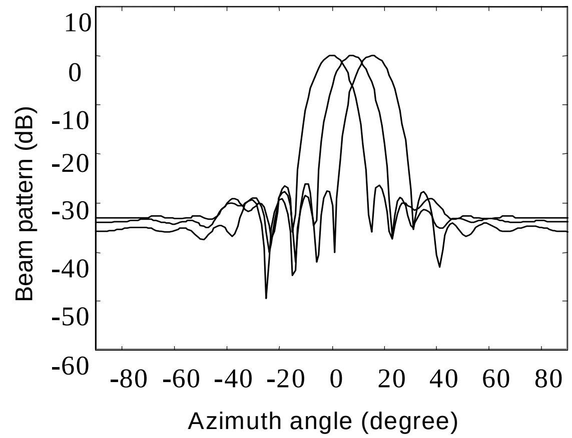


Fig. 1. Superposition of three beams steered at $0°$, $7.7°$ and $15.5°$ at reference frequency

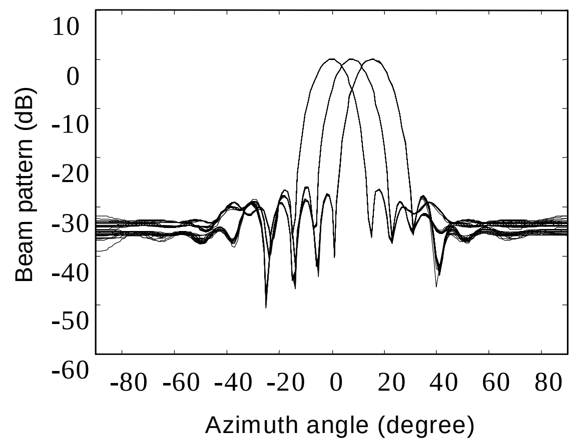


Fig. 2. Superposition of constant beamwidth beams steered at $0°$, $7.7°$ and $15.5°$ at J = 33 frequency bins

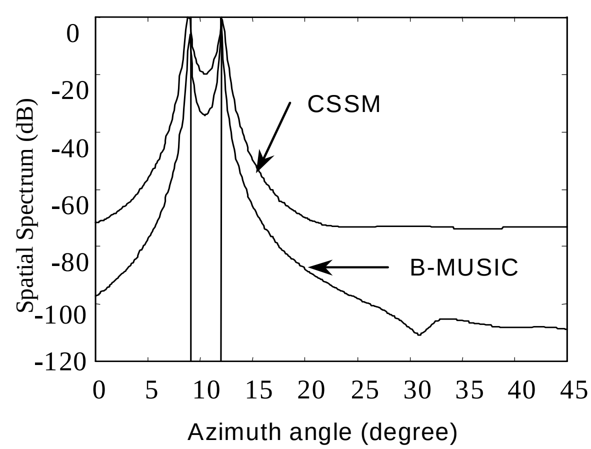
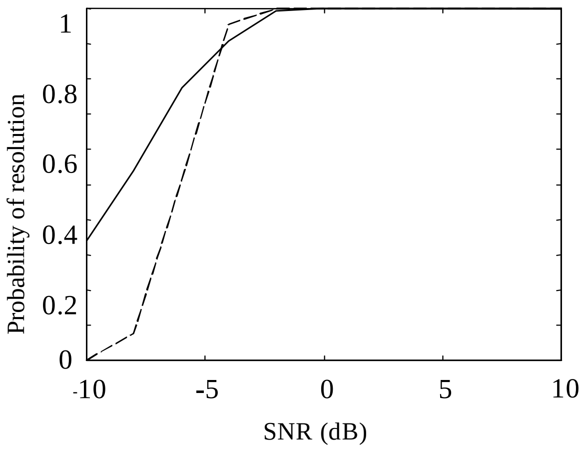
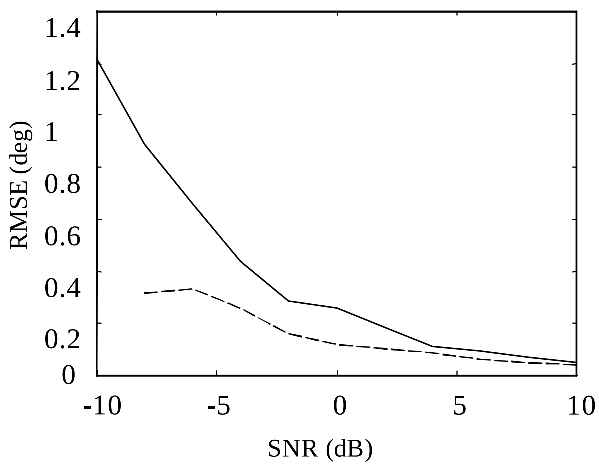


Fig. 3. Typical spatial spectrum of CSSM and B-MUSIC



(a) Probability of resolution



(b) *RMSE*

Fig. 4 Performance comparison for CSSM and B-MUSIC

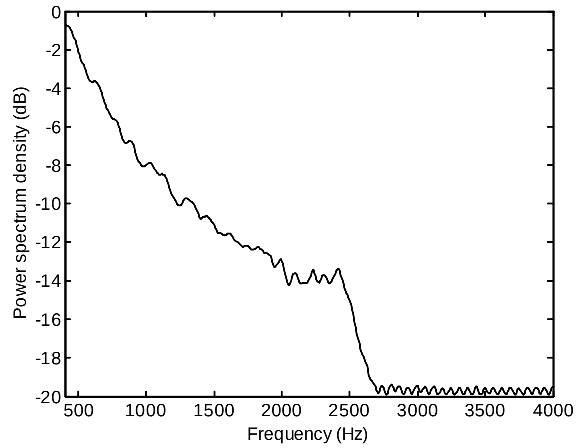
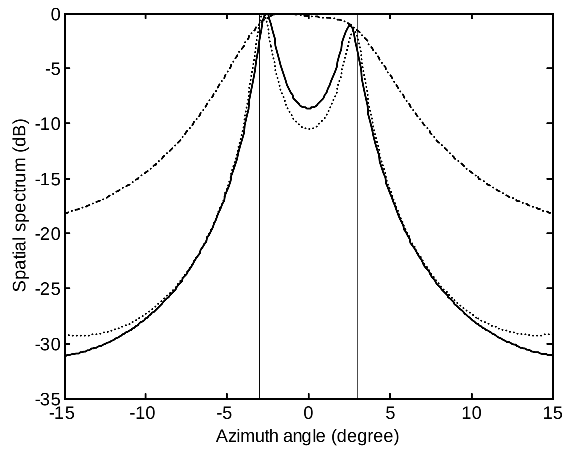
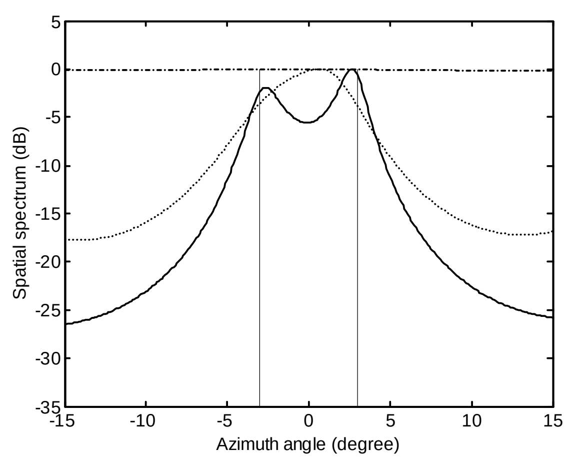


Fig. 5. The power spectral density of the source signals (the power spectrum density of sea noise is also modeled as this shape in simulation [1])



(a) Without interference signal



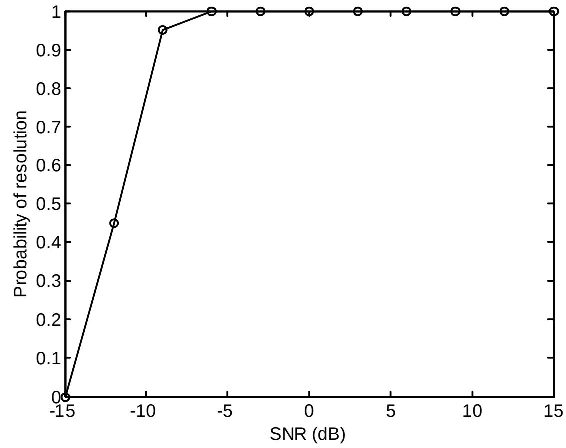
(b) With interference signal

Fig. 6. Typical spatial spectrum of E-MUSIC, CB-MUSIC and MVDR-MUSIC

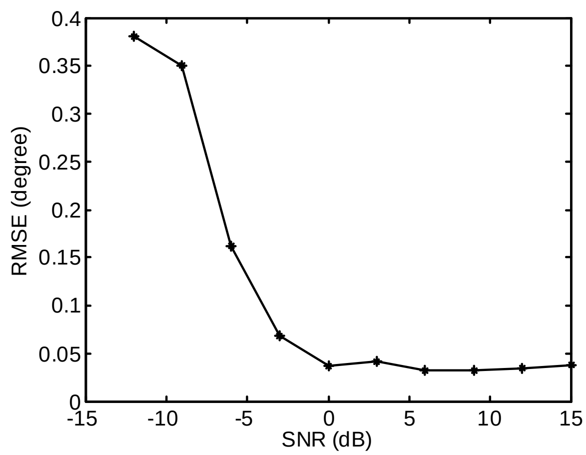
B. Simulation for DOA Estimation based on Subband MVDR Beamformers

Assume a 24-element uniform circular array receiving two source signals arrive from –3° and 3°. The radius of the array is 1.5 m. Two source signals assumed are uncorrelated with each other, and the spectrum density of each signal is decreasing with 6 dB per octave, as shown in Fig. 5. The sea noise is assumed to have the same shape of power spectrum density as that of the source signal radiated from underwater vehicle. Three methods are used in the computer simulation. The first method is the conventional incoherent element space MUSIC (denoted as E-MUSIC, dash-dotted in figures), the second one is the incoherent beamspace MUSIC based on conventional beamforming (denoted as CB-MUSIC, dotted line in figures), and the third one is the incoherent beamspace MUSIC based on diagonal loaded MVDR beamforming (denoted as MVDR-MUSIC, solid line in figures). Three beams steered at –7.5° , 0° and 7.5° were used in beamspace processing. Fig. 6(a) shows the spatial spectra of these three methods when the *SNR* of the two source signals was 0 dB. We can see that CB-MUSIC and MVDR-MUSIC can correctly resolve the directions of the two source signals, but E-MUSIC failed to do so. We further assumed that there was a strong interference signal located at 90°. The interference signal was uncorrelated with the source signals of interest, and the Interference-to-Signal Ratio (*ISR*) was 25 dB. Fig. 6(b) shows the spatial spectra of the three methods. In this case, only MVDR-MUSIC can resolve the two interested source signals.

The probability of resolving the two source signals when there is strong interference signal at 90° with *ISR* = 25 dB is shown in Fig. 7(a) for various *SNR* levels. Results are based on 200 independent trials at each *SNR*. The resulting *RMSE* of the DOA estimate is shown in Fig. 7(b) for various *SNR* levels. As a demonstration, the results of single trial at *SNR*s from –15 dB to 15 dB with 1 dB step is shown in Fig. 8. These results show that the MVDR-MUSIC has good performance when strong interference signal exists.



(a) Probability of resolution



(b). *RMSE*

Fig. 7. Performance of MVDR-MUSIC algorithm

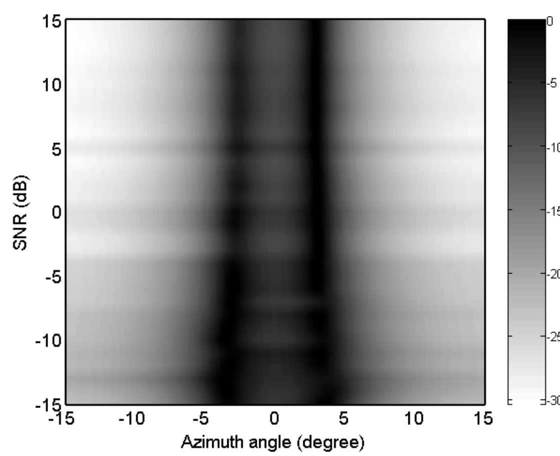


Fig. 8. Estimation results of single trial for various *SNR* levels

VI. CONCLUSION

The broadband beamspace DOA estimation algorithm based on constant beamwidth beamforming can reduce the computation complexity of CSSM, but it will fail to work properly when there exist strong interference signals outside the beam coverage region. In this paper, we proposed a broadband incoherent beamspace algorithm based on subband beamformers to automatically remove the strong interference signals. Performance of these two algorithms is demonstrated by computer simulation.

The main disadvantage of the incoherent algorithm proposed in this paper lies in its huge computation. Our future work will concentrate on the design of frequency invariant beams with adaptive interference suppression ability for arbitrary arrays.

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