Tracking Highly Correlated Sources via Multi-stage Constant Modulus Array

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***Abstract*- An effective method is proposed for "half blindly" recovering coherent signals impinging on a uniform linear array (ULA) and estimating their directions of arrival (DOAs). The generalized steering vector is blindly estimated from the CM array, where the steering vectors to all coherent signals are reconstructed directly by using the property of sub-array shift-invariance. When used as the array weighting, the steering vectors will provide recovered signals in corresponding beams. DOAs can be estimated from the angular spectrum subsequently. Results from the computer simulations and the real data analysis are in good agreement with theoretical analysis. Our method not only can resolve closely spaced coherent signals, but also is shown to be more robust to system errors compared to some other DOA estimation algorithms.**

Ⅰ. INTRODUCTION

Beamforming is fundamental to most of the sensor array signal processing problems [1]. It requires knowledge of the array geometry and calibration and information of incident signals to form one or more beams pointing to the wanted directions. Used as spatial filters, beams formed in adaptive ways have deep nulls in directions of strong interferes while retaining proper responses to signal directions so as to give out better filtered results. The performance of beamformers depends on the prior knowledge of directions of incident signals which, by themselves, constitutes one of the main themes of the estimation problem. As one aspect of the rapidly developed blind signal processing techniques, blind beamforming algorithms [2], [3] have the ability to recover the desired signals from sensor array outputs without any prior knowledge of the Directions-Of-Arrival (DOAs). Furthermore, the angular spectrum can be obtained and therefore the DOAs be estimated if the array manifold is known in some way [4].

The Constant Modulus Algorithm (CMA) [2] is one of the blind algorithms that have been intensively studied. With CMA implemented, the multi-stage Constant Modulus (CM) array [5] is in fact an adaptive beamformer providing recovered signals at the output of each stage without using any training signal. An adaptive signal canceller is included in each stage to remove a captured signal from the input to next stage. In this way, several independent narrowband signals can be sequentially extracted at successive stages of the system. Unfortunately, coherent signals, which are frequently encountered in the problem of detection and estimation of underwater sources especially in shallow water environment, are unrecoverable if the CM array is directly applied.

Although blind beamforming algorithm cannot directly handle the coherency problem, they can estimate the generalized steering vector which is a linear combination of the steering vectors to the coherent signals and contains sufficient information on the DOAs of the signals [3]. Recently, many solutions to have been proposed for DOAs estimation of coherent signals by using the generalized steering vector, such as the four order statistics-based forward/backward linear prediction method [6] and extend virtual ESPRIT algorithm [7]. However, these methods are limited to solve the coherent signals location problem, and cannot be directly applied in beamforming to recover the incident signals.

In this paper, we propose a new method to "half blindly" recover coherent signals impinging on a uniform linear array (ULA) and estimate their DOAs. Similar to the idea of the spatial smoothing technique [8], the steering vectors of coherent signals are reconstructed directly from the estimated generalized steering vector by using the property of sub-array shift-invariance. When used as the array weighting, the steering vectors will provide reconstructed signals in corresponding beams. Then DOAs can be estimated from the angular spectrum. Spatial smoothing techniques can be applied not only on the linear array, but also on a certain class of nonlinear array geometry [9]. As long as the array geometry of the sonar system satisfies the conditions of spatial smoothing techniques, this method can be used to form tracking beams to coherent signals and provide recovery of incident signal with less gain loss and distortion.

This paper is organized as follows. Section Ⅱ defines the data model and the generalized steering vector. In section Ⅲ, a new method is proposed to reconstruct the steering vectors from the generalized steering vector obtained from the multi-stage CM array. Computer simulations and water tank experiments are presented in section Ⅳ, where the DOA estimation performance are illustrated and compared. Finally, concluding remarks are given in section Ⅴ.

Ⅱ. DATA MODEL

Consider a ULA of M sensors with inter-element spacing Δ. Assume that there are d non-Gaussian signals $\{s_i(k)\}_{i=1}^{d}$ with negative kurtosis impinging on the array from directions $\{\theta_i\}_{i=1}^{d}$. The array input data can be put in a vector $\boldsymbol{x}(k)=[x_1(k),\cdots,x_M(k)]^T$ written as

$$\boldsymbol{x}(k)=\boldsymbol{a}(\theta_1)s_1(k)+\cdots+\boldsymbol{a}(\theta_d)s_d(k)+\boldsymbol{n}(k) \tag{2.1}$$

or

$$\boldsymbol{x}(k)=\boldsymbol{A}(\Theta)\boldsymbol{s}(k)+\boldsymbol{n}(k) \tag{2.2}$$

where the superscript T denotes the transpose operation. $\boldsymbol{s}(k)=[s_1(k),\cdots,s_d(k)]^T$ is the source signal vector, and $\boldsymbol{n}(k)=[n_1(k),\cdots,n_M(k)]^T$ is an additive Gaussian noise vector with unknown variance. $\boldsymbol{n}(k)$ are assumed to be uncorrelated with the incident signals $\boldsymbol{s}(k)$ and among sensors. $\boldsymbol{A}(\Theta)$ is referred to as the array manifold given by $\boldsymbol{A}(\Theta)=[\boldsymbol{a}(\theta_1),\cdots,\boldsymbol{a}(\theta_d)]$, which contains the information about the array geometry and responses and the directions of the impinging signals. Its column vector $\boldsymbol{a}(\theta_i)$ is the steering vector in the direction θ_i:

$$\boldsymbol{a}(\theta_i)=[1,\quad \phi_i,\quad \cdots,\quad \phi_i^{M-1}]^T, \tag{2.3}$$

where $\phi_i=\exp\{j2\pi\Delta\sin\theta_i/\lambda_i\}$, λ_i is the wavelength of the incident signals.

Assume that the array manifold matrix $\boldsymbol{A}(\Theta)$ is unambiguous, i.e. its response to a signal from a given direction is different from that to another signal from a different direction. Consider a simple scenario where there is only one source $s_1(k)$. $s_1(k)$ undergoes frequency-flat multipath propagation producing several multipath signals $s_2(k),\cdots s_d(k)$. The multipath signals are scaled replicas of the source signal $s_1(k)$, and coherent with it. Then the multipath signals can be expressed by [3]

$$s_i(k)=\beta_i s_1(k),\qquad i=1,2,\cdots,d \tag{2.4}$$

where β_i is the complex scaling factor of ith signal $s_i(k)$ relative to the source $s_1(k)$ with $\beta_i\neq 0$ and $\beta_1=1$. From (2.2) and (2.4), the signal $\boldsymbol{x}(k)$ can be expressed as

$$\boldsymbol{x}(k)=\boldsymbol{A}(\Theta)\boldsymbol{\beta}s_1(k)+\boldsymbol{n}(k)=\boldsymbol{b}s_1(k)+\boldsymbol{n}(k) \tag{2.5}$$

where $\boldsymbol{\beta}=[\beta_1,\beta_2,\cdots,\beta_d]^T$. Now we have the generalized steering vector as

$$\boldsymbol{b}=\sum\nolimits_{i=1}^{d}\beta_i\boldsymbol{a}(\theta_i)=\boldsymbol{A}(\Theta)\boldsymbol{\beta}. \tag{2.6}$$

Obviously $\boldsymbol{b}$ is a linear combination of the steering vectors $\{\boldsymbol{a}(\theta_i)\}_{i=1}^{d}$ to all coherent signals and contains sufficient information on $\{\theta_i\}_{i=1}^{d}$. This result shows that the case of coherent signals is equivalent to that of statistically independent signals with a modified steering vector in the blind beamforming algorithms.

The objective is to blindly estimate the steering vector and, subsequently, the DOAs of all incident signals. First, the generalized steering vector is obtained from the multi-stage CM array. Then a method is proposed to reconstruct the steering vectors to all of the coherent signals from the estimated generalized steering vector. And finally, the DOAs are extracted.

Ⅲ. STEERING VECTOR RECONSTRUCTION AND DOA ESTIMATION

A. Blind Estimation of Generalized Steering Vector using Multi-stage CM Array

A model of the cascade multi-stage CM array is shown in Fig. 1. According to the CMA [2], the cost function of CM array can be written as:

$$J(k)=E[||y(k)|-1|^2]\rightarrow \min \tag{3.1}$$

Using the adaptive steepest-descent method and replacing the gradient vectors with their instantaneous estimates, the iterative procedure of the CM array can be described by the following equations:

$$\begin{cases} y(k)=\boldsymbol{w}^H(k)\boldsymbol{x}(k) \\ e_c(k)=y(k)/|y(k)|-y(k) \\ \boldsymbol{w}(k+1)=\boldsymbol{w}(k)+\mu_{\text{CMA}}\boldsymbol{x}(k)e_c^*(k) \end{cases} \tag{3.2}$$

which is a realization of the LMS algorithm with a desired signal $y(k)/|y(k)|$. The step size $\mu_{\text{CMA}}>0$ controls the convergence rate of CM array.

The adaptive signal canceller can directly use the complex LMS algorithm, and be updated by

$$\begin{cases} y(k)=\boldsymbol{w}^H(k)\boldsymbol{x}(k) \\ \boldsymbol{e}(k)=\boldsymbol{x}(k)-\boldsymbol{u}(k)y(k) \\ \boldsymbol{u}(k+1)=\boldsymbol{u}(k)+2\mu_{\text{LMS}}y^*(k)\boldsymbol{e}(k) \end{cases} \tag{3.3}$$

where the superscript $*$ denotes the conjugation operation. The step size is bounded by $0<\mu_{\text{LMS}}<1/\sigma_y^2$, where $\sigma_y^2=E[|y(k)|^2]$ is the variance of the CM array output at the current stage. Thus, the convergence properties of the canceller weights depend on weights of the CM array,

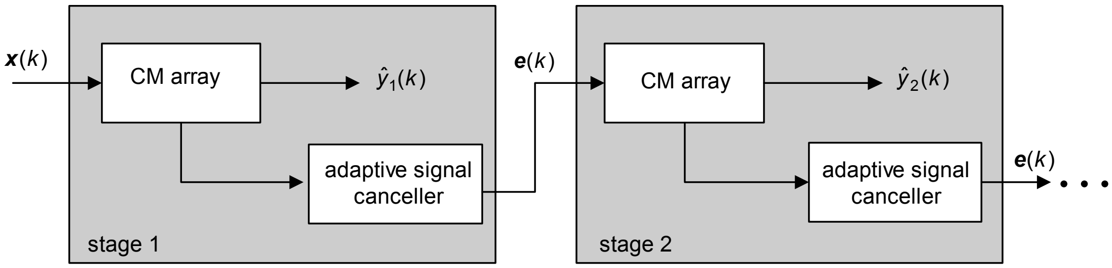


Fig. 1. Cascade multi-stage CM array

whereas the CM array weights are independent from those of the adaptive canceller.

When incident signals are all coherent, the CM array would interpret them as one, as described in (2.5), and the adaptive signal canceller removes them completely in the first stage. Therefore, only the generalized steering vector estimated in the first stage is useful. When only a part of signals are coherent, the multi-stage CM array will automatically separate the independent signals at the output of each stage. For simplicity, the order of the multi-stage CM array is assumed to be equal to the number of signals.

The generalized steering vector $\boldsymbol{b}$ can be obtained by computing the pseudo-inverse matrix of the first stage CM array weight $\boldsymbol{w}_1$. In this paper, based on blind signal reconstruction, a least squares solution of $\boldsymbol{b}$ is suggested. Having the estimated signal $\hat{\boldsymbol{s}}_1$ at hand, $\boldsymbol{b}$ can be estimated by

$$\hat{\boldsymbol{b}} = \arg\min_{\boldsymbol{A}}\{\| \boldsymbol{X} - \hat{\boldsymbol{b}}\hat{\boldsymbol{s}}_1 \|_F^2\} \tag{3.4}$$

Then we can obtain the least squares solution as

$$\hat{\boldsymbol{b}} = \boldsymbol{X}\hat{\boldsymbol{s}}_1^H\left(\hat{\boldsymbol{s}}_1\hat{\boldsymbol{s}}_1^H\right)^{-1} \tag{3.5}$$

where the superscript H denotes the complex conjugate transpose operation.

B. Steering Vectors Reconstruction

Once we obtain the generalized steering vector $\boldsymbol{b}$, the steering vectors cab be estimated and, subsequently, the DOA of each signal. From (2.6), we have $\boldsymbol{b} = \boldsymbol{A}(\Theta)\boldsymbol{\beta}$, and $\text{rank}(\boldsymbol{\beta}\boldsymbol{\beta}^H) = 1$, so the $\boldsymbol{A}(\Theta)$ cannot be obtained directly by subspace estimation of $\boldsymbol{b}\boldsymbol{b}^H$. Based on the forward/ backward Spatial Smoothing [8], a matrix can be constructed whose subspace is spanned by the columns of $\boldsymbol{A}(\Theta)$ and its rank is d.

First, vector $\boldsymbol{b}$ is partitioned into L overlapping sub-vectors with m elements in the forward or backward direction, where $L = M - m + 1$ and $m \geq d + 1$.

$$\begin{aligned}\boldsymbol{b}_{fl} &= [b_l, \quad b_{l+1}, \quad \cdots, \quad b_{l+m-1}]^T \\ &= \boldsymbol{A}_m(\Theta)\boldsymbol{\Phi}^{l-1}\boldsymbol{\beta}\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}\boldsymbol{b}_{bl} &= [b_{M-l+1}, \quad b_{M-l}, \quad \cdots, \quad b_{L-l+1}]^H \\ &= \boldsymbol{A}_m(\Theta)\boldsymbol{\Phi}^{l-1}\boldsymbol{\Phi}^{-(M-1)}\boldsymbol{\beta}^*\end{aligned} \tag{3.7}$$

Then, two matrices are formed as

$$\begin{aligned}\boldsymbol{C}_f &= [\boldsymbol{b}_{f1}, \quad \boldsymbol{b}_{f2}, \quad \cdots, \quad \boldsymbol{b}_{fL}] \\ &= \boldsymbol{A}_m(\Theta)[\boldsymbol{\beta}, \quad \boldsymbol{\Phi}\boldsymbol{\beta}, \quad \cdots, \quad \boldsymbol{\Phi}^{L-1}\boldsymbol{\beta}] \\ &= \boldsymbol{A}_m(\Theta)\boldsymbol{B}\boldsymbol{A}_L^T(\Theta)\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}\boldsymbol{C}_b &= [\boldsymbol{b}_{b1}, \quad \boldsymbol{b}_{b2}, \quad \cdots, \quad \boldsymbol{b}_{bL}] \\ &= \boldsymbol{A}_m(\Theta)\boldsymbol{\Phi}^{-(M-1)}\boldsymbol{B}^*\boldsymbol{A}_L^T(\Theta)\end{aligned} \tag{3.9}$$

where $\boldsymbol{A}_m(\Theta)$ and $\boldsymbol{A}_L(\Theta)$ are defined as the sub-matrices of $\boldsymbol{A}(\Theta)$ consisting of the first m and L rows, respectively; $\boldsymbol{\Phi} = \text{diag}\{\phi_1, \cdots, \phi_d\}$, and $\boldsymbol{B} = \text{diag}\{\beta_1, \cdots, \beta_d\}$,

Under the assumption that $\beta_k \neq 0$ and $\boldsymbol{A}(\Theta)$ is unambiguous, ranks of the diagonal matrices $\boldsymbol{B}$ and $\boldsymbol{\Phi}$ and the Vandermonde matrices $\boldsymbol{A}_m(\Theta)$ and $\boldsymbol{A}_L(\Theta)$ are given by $\text{rank}(\boldsymbol{B}) = d$, $\text{rank}(\boldsymbol{\Phi}) = d$, $\text{rank}(\boldsymbol{A}_m(\Theta)) = \min(m, d)$ and $\text{rank}(\boldsymbol{A}_L(\Theta)) = \min(L, d)$, respectively. Hence, when $d + 1 \leq m \leq M + 1 - d$, we have $\text{rank}(\boldsymbol{A}_m(\Theta)) = d$ and consequently find that ranks of $\boldsymbol{C}_f$ and $\boldsymbol{C}_b$ are both equal to d. Therefore, the subspace spanned by the columns of $\boldsymbol{A}_m(\Theta)$ is equivalent to those of $\boldsymbol{C}_f$ and $\boldsymbol{C}_b$. Utilizing the approach in [3], $\boldsymbol{A}_m(\Theta)$ can be finally extracted from the singular value decomposition of the concatenated matrix of (3.8) and (3.9). Its columns are the steering vectors $\{\hat{\boldsymbol{a}}(\theta_i)\}_{i=1}^d$ to all coherent signals with the loss of the effective array aperture.

C. Extraction of DOAs

After obtaining the steering vectors $\{\hat{\boldsymbol{a}}(\theta_i)\}_{i=1}^d$, conventional beamformers are made possible and reconstructed signals will be provided in corresponding beams. And at the same time, the DOAs are given by

$$\theta_i = \arg\max_{\theta}\{|\,\hat{\boldsymbol{a}}(\theta_i)^H \boldsymbol{a}(\theta)\,|^2\} \quad i = 1,\cdots,d \tag{3.10}$$

where $\boldsymbol{a}(\theta)$ is the scanning vector, and θ varies within the whole searching space.

Ⅳ. EXPERIMENTS RESULTS

Computer simulations and water tank experiments were conducted to investigate the DOA estimation performance of the proposed method when dealing with coherent signals. A ULA was assumed in simulation work and used in the experiments so as to be consistent with the theoretical work in earlier sections.

In this section, three algorithms are considered, including conventional beamforming with forward/backward spatial smoothing (SS-CBF), MUSIC with forward/backward spatial smoothing (SS-MUSIC), and the multi-stage CM array with the method proposed in section Ⅲ-B (SS-CMA). The former two non-blind algorithms estimate DOAs based on the spatial spectra and SS- CMA fulfills it by projecting each steering vector $\hat{\boldsymbol{a}}(\theta_i)$ onto the array manifold, as in (3.10).

In all experiments, the order of the CM array is equal to the number of incident signals, which is predetermined. The ith CM array weight was initialized to $\boldsymbol{w}_i(0) = \boldsymbol{i}$ to avoid the CM arrays converge to the same signal, where $\boldsymbol{i}$ is the unit vector with ith element being1 and all others zero. The step size was chosen as $\mu_{\text{CMA}} = 0.005$. Signal canceller weights were initialized as $\boldsymbol{u}(0) = \boldsymbol{0}$ and the initial step size $\mu_{\text{LMS}} = 0.02$. The spatial smoothing factor for all algorithms is $m = 11$. The presented results were averaged over 100 independent runs, with a sample length of 3000 in each run.

A. Computer Simulations

In the computer simulations, a ULA of 16 identical isotropic sensors spaced a half wavelength apart was used. Assume that there were 2 equal power coherent narrowband signals impinging on the array and the additive noise was spatial white Gaussian noise.

Experiment 1: Performance versus SNR. Two coherent signals arrived from 10° and 20°, and their SNR varied from -20dB to 20dB. Fig. 2 illustrates the DOA estimate biases and standard deviations of the source in 20° versus SNR. Results from three different DOA estimation algorithms are compared in these plots. SS-CBF always has a bias of DOA estimation. When the input SNR was higher than the threshold -10dB, the SS-CMA algorithm could correctly estimate DOAs. When SNR was lower than -10dB, the CM array cannot correctly estimate the generalized steering vector, so the performance of DOA estimation degrades.

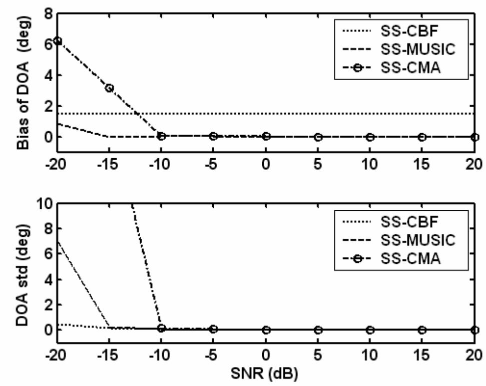


Fig. 2. DOA estimation performance for the signal in 20° at varying SNR.

Experiment 2: Performance versus Angular Separation. Two coherent signals arrived from $\theta_1 = 10^\circ$ and $\theta_2 = 10^\circ + \Delta\theta$, where the angular separation $\Delta\theta$ was varied from 1° to 15°. The SNR was fixed at 0dB. Fig. 3 shows the DOA estimate biases and standard deviations of θ_2.

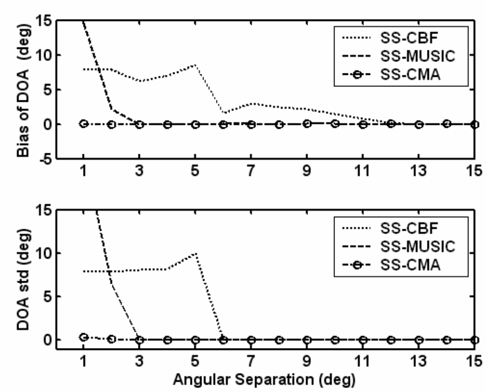


Fig. 3. DOA estimation performance for θ_2 at varying angular separations and SNR=0dB.

After spatial smoothing, the effective array aperture is decreased, and the Rayleigh beamwidth of such a 11-sensor ULA is 8.8°, i.e. SS-CBF will fail to resolve two sources when their angular separation is smaller than 8.8°. SS-MUSIC demonstrates higher resolution ability than SS-CBF in solving coherent signals at small angular separations, but its angular separation ability is not as good as that of SS-CMA. From the plots, it can be seen that the proposed method is able to reconstruct the steering vectors to two closely coherent signals and estimate their DOAs as long as the generalized steering vector can be correctly estimated. The angular separation ability of the SS-CMA is beyond the Rayleigh limitation.

B. *Water Tank Experiments*

Water tank experiments were carried out in the anechoic water tank of Northwestern Polytechnical University, whose size is $20\text{m}\times 8\text{m}\times 7\text{m}$. A 16-element ULA with 10cm inter-element spacing was used as the receiving array. A single signal generator was used to generate a sinusoid at frequency of 10kHz and drive two transmitters, one of them was connected to a power-amplifier with adjustable coefficients. Two sources were placed at distance from the array. During the experiments, the position of one source was fixed (referred to as source 1) while the other one (referred to as source 2) was placed at different positions to generate signals from varying directions. Therefore, two coherent signals from different directions and with different strengths were generated to simulate a simplified environment of multipath transmission. The sample frequency was 51.2kHz through the experiments.

Fixing two sources in -22.5° and -9.1°, Fig. 4 shows the output signal amplitudes at the first three sensors. Large variation in the gains is evident. Other errors including sensor position error and phase error were also present. To compare the robustness of different algorithms, no array calibration was conducted before data processing. The spectrum of output at the third sensor is illustrated in Fig. 5. Because two coherent signals had the same frequency, they were undistinguishable from the spectrum.

Beampatterns of the generalized steering vectors are presented in Fig. 6(a), which is directly obtained from each stage of the two-stage CM array, and the corresponding output spectra are in Fig. 6(b). From the plots, it can be seen that two peaks are formed in the true directions of two coherent signals at the output of the first stage. This is because that the two coherent signals were inseparable within the CM array and all direction information about the incident signals was included in the generalized steering vector estimated at the first stage. All energy of incident signals were completely removed by the adaptive signal canceller in the first stage and therefore the output spectrum of stage 2 is flat, as shown in Fig. 6(b). Beampatterns based on the reconstructed steering vectors by using the proposed method are plotted in Fig. 7(a). It is shown that the steering vectors of the coherent signals are correctly recovered from the generalized steering vector. Using the estimated steering vector matrix $\hat{\boldsymbol{A}}_m(\Theta)$ as the weighting matrix $\boldsymbol{W}=\hat{\boldsymbol{A}}_m(\Theta)$, the spectra of beamformer outputs at each stage are presented in Fig. 7(b). Two coherent signals are successfully separated by exploiting the spatial difference, although they occupy the same frequency.

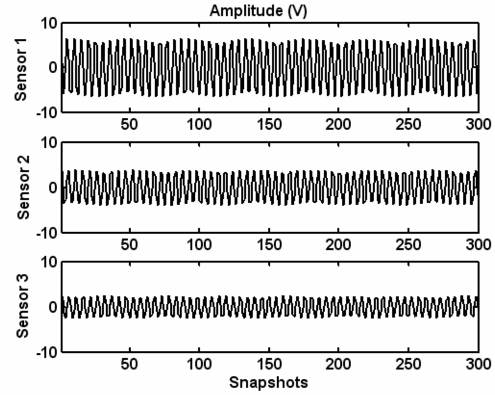


Fig. 4. Output sample signal amplitudes of the first three sensors.

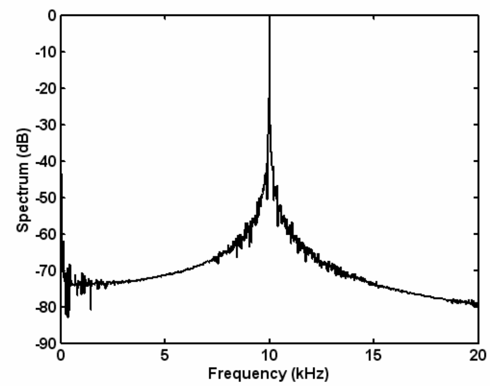
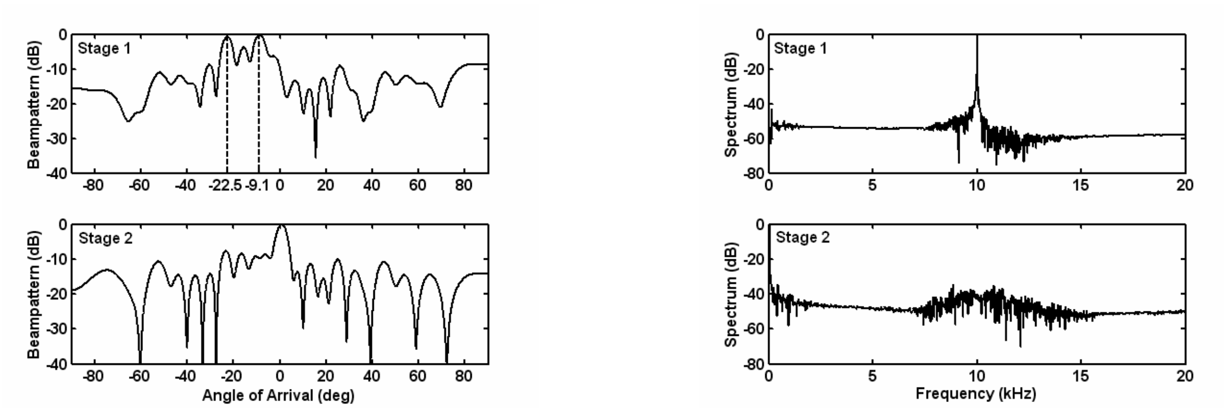


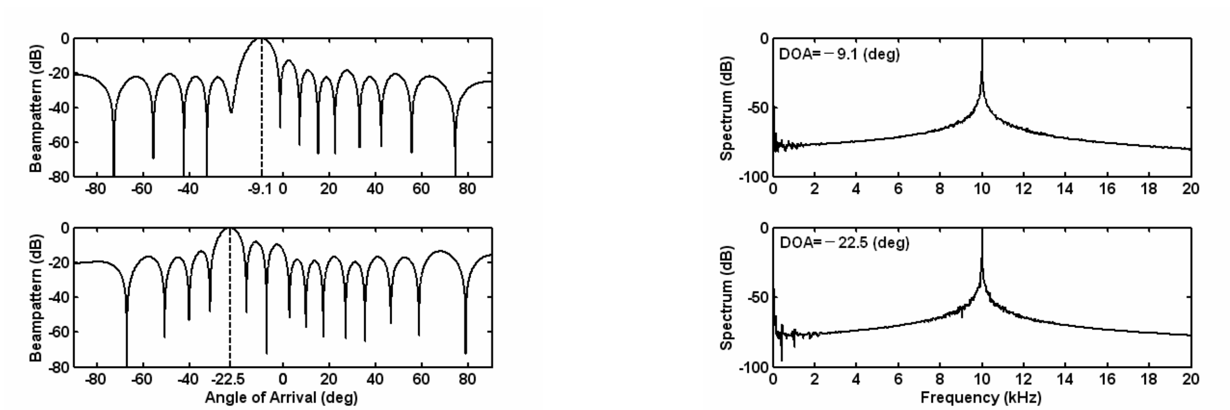
Fig. 5. Spectrum of signal at the third sensor.

To vary the angular separation of incident signals, two sources were fixed at $(-22.5^\circ,-9.1^\circ)$ and $(-22.5^\circ,-17.8^\circ)$ respectively. After spatial smoothing, the Rayleigh beamwidth of 11-sensor ULA at frequency 10kHz is 6.6°. Therefore, the ratios of angular separations to the beamwidth are 2.65 and 0.72. In order to test the DOA estimation performance of the proposed method for signals with different strengths, the transmitted power of source 2 was reduced. Results from three algorithms are compared in terms of means and standard deviations in Tables 1 and 2. In these Tables, σ_1^2 and σ_2^2 denote the transmitted powers of source 1 and source 2, respectively. '∞' implies that the relative departure between the estimated data and the true values is greater than 10°. Due to the Rayleigh limitation and system errors in the receiving array, SS-CBF and SS-MUSIC were completely ineffective at small angular separations. On the other hand, SS-CMA performed well in all situations. The water tank experiments results show that the proposed method can resolve the coherency problem, and is also more robust to the model errors in array geometry compared to SS-CBF and SS-MUSIC algorithms.



(a) (b)

Fig. 6. 16-sensor ULA receiving two coherent signals from directions $(-22.5^{\circ}, -9.1^{\circ})$. (a) Beampatterns of generalized steering vectors of each CM array stage. (b) Spectra of corresponding CM array outputs.



(a) (b)

Fig. 7. Separation of two coherent signals from directions $(-22.5^{\circ}, -9.1^{\circ})$. (a) Beampatterns based on reconstructed steering vectors. (b) Spectra of corresponding conventional beamformer outputs.

TABLE 1

PERFORMANCE COMPARISON OF DIFFERENT ALGORITHMS AT VARYING σ_2^2 ($^{\circ}$)

$\sigma_2^2 - \sigma_1^2$	$(-22.5^{\circ}, -9.1^{\circ})$					
	SS-CBF		SS-MUSIC		SS-CMA	
	Mean	Std	Mean	Std	Mean	Std
−7dB	(−22.74,−9.71)	(0.47,0.33)	(−22.90,−9.56)	(0.29,0.38)	(−22.62,−9.39)	(0.83,0.37)
−10dB	(−21.64,−8.96)	(0.37,0.25)	(−21.99,−8.78)	(0.39,0.18)	(−21.81,−8.99)	(0.73,0.16)
−13dB	(−21.83,−8.98)	(0.28,0.30)	(−22.13,−8.65)	(0.29,0.25)	(−22.04,−8.96)	(0.95,0.22)
−16dB	(−22.10,−9.30)	(0.23,0.52)	(−22.39,−8.53)	(0.17,0.66)	(−22.34,−9.98)	(0.20,1.57)
−12dB	(−22.18,−9.59)	(0.17,0.58)	(−22.56,−8.40)	(0.10,1.32)	(−22.38,−10.44)	(1.14,1.92)

TABLE 2

PERFORMANCE COMPARISON OF DIFFERENT ALGORITHMS AT VARYING σ_2^2 (°)

$\sigma_2^2 - \sigma_1^2$	(–22.5°,–17.8°)					
	SS-CBF		SS-MUSIC		SS-CMA	
	Mean	Std	Mean	Std	Mean	Std
−7dB	(–23.85,∞)	(7.41,∞)	(–22.44,–11.76)	(3.09,∞)	(–23.06,–18.38)	(2.75,2.18)
−10dB	(–21.98,∞)	(6.13,∞)	(–20.34,∞)	(4.24,∞)	(–22.82,–20.89)	(2.26,4.03)
−13dB	(–23.26,∞)	(4.88,∞)	(–22.71,∞)	(6.03,∞)	(–21.47,–19.68)	(1.48,2.27)
−16dB	(–24.58,∞)	(5.77,∞)	(–22.19,∞)	(4.18,∞)	(–21.76,–18.19)	(1.48,1.37)
−12dB	(–24.57,∞)	(5.01,∞)	(–23.88,∞)	(5.44,∞)	(–22.42,–18.11)	(0.53,0.47)

'∞' implies that the relative departure between the estimated data and the true values is greater than 10°.

Ⅴ. CONCLUSIONS

In this paper, a new method is proposed to “half blindly” recover coherent signals impinging on a ULA and estimate their DOAs by using multi-stage CM arrays. The generalized steering vector, which is a linear combination of the steering vectors to all coherent signals, is blindly estimated by the CM array. By the property of “sub-array” shift-invariance, the steering vectors are reconstructed directly from the estimated generalized steering vector. When used as the array weighting, the steering vectors reconstructed will be able to recover signals in corresponding beams. And then DOAs can be estimated from the angular spectra. The DOA estimation performance of the proposed method is analyzed by means of computer simulations and water tank experiments. Results from the computer simulations and the real data are in good agreement with theoretical analysis. Since no array calibration was carried out prior to the water tank experiments, large amplitude errors existed. The proposed method is shown to be more robust to these system errors compared to some other beamforming algorithms. Therefore, when the array geometry of the sonar system satisfies the conditions of sub-array shift-invariance, this method can be used to form tracking beams to coherent signals and provide recovery of incident signal with less gain loss and distortion.

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