COMPARISON OF QUADRATIC PHASE COUPLING DETECTORS ON SONAR DATA

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ABSTRACT

Detection and estimation of processes with mixed spectra and coupled harmonics is a well-known problem in the analysis of time-series and is of particular interest in classification of sonar signals. In this paper a comparison is made between two quadratic phase coupling (QPC) detection techniques analytically. One technique adopts a segment and average approach to obtain estimates of the signal bicoherence and then uses a step-wise outlier rejection method to select only relevant data points. The other technique is a single record phase coupling detector which removes the need for multiple independent records. It is shown that analytically the techniques are identical.

1. INTRODUCTION

In sonar signal processing identification and classification of different underwater sources are crucial. In passive sonar analysis the estimation problems are often focussed on spectral separation and characteristaion. In particular on the narrow-band signals from the engine, the drive and the hull. There are times when the power spectrum is unable to separate these sources. Coupling of frequencies due to a variety of physical mechanisms are often useful in providing additional features for classification in such circumstances. In particular, using the normalised bispectrum to detect the presence of quadratic phase coupling arising as a result of nonlinear mechanisms. This paper discusses the relative performance of two detectors, one which requires multiple independent records and one which does not.

2. THE ZHOU AND GIANNAKIS PHASE COUPLING DETECTOR (ZGPCD)

The Zhou and Giannakis Phase Coupling Detector (ZGPCD) will be briefly described, but the reader is referred to the original texts [1–3] for comprehensive details. In keeping with the approach used by Zhou and Giannakis, the special case of quadratic phase coupling (QPC), detected using the 3rd-order FS polyspectrum, will be considered here.

The ZGPCD aims to detect QPC between coupled harmonics in noise. The discrete time signal is assumed to have the form [2, (2.1), p15]

$$x(t) = \sum_{l=1}^{3} A_l e^{j(\omega_l t + \phi_l)} + v(t), t = 0, 1, ..., T-1, \quad (1)$$

where $v(n)$ is the noise (which may be non-Gaussian). If QPC occurs then the phases of the harmonics are related such that $\phi_3 = \phi_1 + \phi_2$.

In [2] the test for QPC is developed in a hypothesis-testing framework in terms of ϕ, the phase of the 3rd FS polyspectrum [2, (2.18), p20], with the following hypotheses [2, (2.44), p30]

H_0: $\phi = 0$ (QPC present),

H_1: $\phi \neq 0$ (QPC not present).

The ZGPCD is applied to the *estimated* phase, denoted $\hat{\phi}$. The hypothesis H_0 is rejected if [2, (2.50),p31]

$$\hat{\phi}^2 > \phi_c^2,$$

where

$$\phi_c^2 = \Gamma(\alpha)\sigma_{\hat{\phi}}^2 / T. \quad (2)$$

In (2), $\Gamma(\alpha)$ is determined from a $\chi^2(1)$ table for a chosen *significance level* α, T is the data length, and $\sigma^2_{\hat{\phi}}$ is estimated from [2, (2.46), p30]

$$\hat{\sigma}^2_{\hat{\phi}} = \frac{1}{2}\left[\frac{S_{2v}(\omega_3)}{A_3^2} + \frac{S_{2v}(\omega_2)}{A_2^2} + \frac{S_{2v}(\omega_1)}{A_1^2}\right], \qquad (3)$$

in which $S_{2v}(\omega)$ is the 2nd-order spectrum of the noise, and the A's are the harmonic amplitudes as given by (1).

3. FALSE-ALARM PERFORMANCE OF ZGPCD

In any hypothesis testing situation, there are two types of error which can occur [4];

- **Type I error:** The hypothesis is true but is rejected (i.e. there is QPC, but we reject H_0). The probability of a Type I error is measured by the significance level of the test α (i.e. $P[\hat{\phi}^2 > \phi_c^2|\text{QPC}] = \alpha$). In other words the probability that H_0 will be rejected when H_0 is in fact true is α. α is (we believe) erroneously called the "Probability of False Alarm" by Zhou and Giannakis.
- **Type II error:** The hypothesis is false but is accepted (i.e. there is no QPC, but we accept H_0). We believe the probability of *this* error is the true Probability of False Alarm. The probability of this type of error is measured by the *power* of the test β [4, p1261] (i.e. $P[\hat{\phi}^2 < \phi_c^2|\text{no QPC}] = \beta$).

Now, although Zhou and Giannakis consider the probability of a Type I Error, there is no discussion of the probability of Type II Errors. In practical applications the probabilities of both these types of errors must be evaluated, since they present conflicting requirements. Indeed, discussion of the detector without reference to the probability of Type II errors is meaningless, because a detector which detects QPC **all the time** (regardless of whether or not the signal exhibits QPC) will have the apparently "ideal" performance of $P[\text{Type I}] = 0$. The failings of such a detector would only show up in $P[\text{Type II}]$. Now β, the probability of a Type II error, can be determined easily from the value of ϕ_c^2. This will now be explained. The ZGPCD is described in terms of the squared phase angle ϕ^2 of the FS polyspectrum. Conceptually it is easier to consider the phase angle ϕ rather than its square. The critical value ϕ_c from (2) thus defines a *QPC-acceptance* region in the complex plane, as shown in Figure 1. The width of the acceptance region is $2\phi_c$ in which the factor of 2 accounts for negative *and* positive angles.

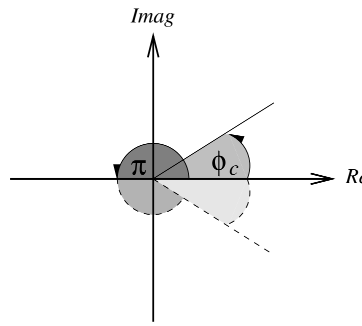

Figure 1: Schematic diagram showing complex plane. ϕ_c defines the critical acceptance region for the phase of the FS polyspectrum.

The probability of a Type II Error is the probability that, where there is no QPC, ϕ (the phase of the 3rd-order FS polyspectrum) falls within the acceptance region defined by ϕ_c. If there is no QPC, ϕ will be uniformly distributed as $U(-\pi, +\pi]$, so β is given by

$$\beta = \frac{\text{angular width of } H_0 \text{ acceptance region}}{2\pi},$$

which from (2) is

$$\begin{aligned} \beta &= \frac{2\sqrt{\Gamma\sigma^2_{\hat{\phi}}/T}}{2\pi} \\ &= \frac{1}{\pi}\sqrt{\Gamma\sigma^2_{\hat{\phi}}/T}. \end{aligned} \qquad (4)$$

Thus it can be seen, that if α is chosen (say at 0.05), then β will vary according to $\sigma^2_{\hat{\phi}}$. Now $\sigma^2_{\hat{\phi}}$ is closely related to the signal to noise (SNR) ratio, so we see that as the SNR varies, so the performance of the ZGPCD will vary.

4. NOISE PERFORMANCE

One aspect not addressed in Zhou and Giannakis's work is that of performance in noise of the ZGPCD. It has already been shown above that the power of the ZGPCD will change as the SNR changes. A further complication arises since the quantities which determine the width of the *QPC-acceptance* region in (2) have themselves to be estimated. In this section these matters are considered.

The discussion of Type I and Type II errors above *assumed* that the quantities $S_{2v}(\omega)$ and A_l in (3) were known *a priori*. This is not in fact the case in practice, and these quantities have to be estimated from the data. As a result, it may be expected that the incidences of Type I and Type II errors observed in practice differ from the theoretical α and β given in Section 3. A heuristic argument will be presented concerning how these errors will vary with respect to the SNR.

Zhou and Giannakis suggest that the harmonic amplitudes (the A_l's in (1)) can be estimated from the 2nd-order FS polyspectrum, and that the noise variance σ_v^2 can be estimated by averaging the spectrum over frequencies where harmonics are absent.

We consider the following quantities

$$\begin{aligned}\phi_c &= \sqrt{\frac{\Gamma(\alpha)\sigma_{\hat{\phi}}^2}{T}} \\ &= \sqrt{\frac{\Gamma(\alpha)}{2T}\left[\frac{S_{2v}(\omega_3)}{A_3^2}+\frac{S_{2v}(\omega_2)}{A_2^2}+\frac{S_{2v}(\omega_1)}{A_1^2}\right]} \\ &= \sqrt{\frac{\Gamma(\alpha)\sigma_v^2}{2T}\sum_{l=1,2,3}\frac{1}{A_l^2}} \qquad (5)\end{aligned}$$

where it has been assumed for simplicity that the noise is white and so $S_{2v}(\omega_l)=\sigma_v^2$ $(l=1,2,3)$. The quantity $2\phi_c$ is the *theoretical* width of the ZGPCD acceptance region as shown in Figure 1. The *estimated* width of the acceptance region will be denoted $2\hat{\phi}_c$ and will be given by

$$\hat{\phi}_c = \sqrt{\frac{\Gamma(\alpha)\hat{\sigma}_v^2}{2T}\sum_{l=1,2,3}\frac{1}{\hat{A}_l^2}}. \qquad (6)$$

Note that (5) and (6) differ only in that the second equation uses estimates of the noise variance and harmonic amplitudes. A new quantity ζ can be formed which measures how well the acceptance region is estimated;

$$\begin{aligned}\zeta &= \frac{\hat{\phi}_c}{\phi_c} \\ &= \sqrt{\frac{\hat{\sigma}_v^2\sum_{l=1,2,3}\frac{1}{\hat{A}_l^2}}{\sigma_v^2\sum_{l=1,2,3}\frac{1}{A_l^2}}}. \qquad (7)\end{aligned}$$

A qualitative discussion of the practical implications of the ZGPCD will now be given.

If the SNR is high, then it is reasonable to assume that, notwithstanding the effects of leakage terms, the harmonic amplitudes will be well estimated (since the effect of noise will be low). However, the noise variance estimate will be biased, since the noise energy estimate will be positively biased by the energy which leaks out of the bins from the harmonics. With a high probability the noise energy will be *over-estimated.* Consequently $\zeta \geq 1$ (from (7)), and the estimated acceptance region will be *wider* than the true acceptance region. Thus the probability that QPC will be detected when it is present should in fact be *higher* than $1-\alpha$, and in a similar way the probability that QPC will be detected when it is absent should also be *higher* than β. That is,

$$\begin{aligned}\zeta &\geq 1 \\ \Rightarrow \hat{P}[\text{detect}|\text{ QPC}] &\geq 1-\alpha \qquad (8)\\ \Rightarrow \hat{P}[\text{detect}|\text{no QPC}] &\geq \beta \qquad (9)\end{aligned}$$

("detect" means here detect QPC, i.e. select hypothesis H_0), where $\hat{P}$ denotes the experimental detection probability, and the equalities in each of the equations above will hold if the harmonic amplitudes and noise variance can be accurately estimated.

If the SNR is low, then following the converse of the above argument, the noise variance will be well estimated, but the harmonic amplitudes will be *over-estimated.* Thus

$$\begin{aligned}\zeta &\leq 1 \\ \Rightarrow \hat{P}[\text{detect}|\text{QPC}] &\leq 1-\alpha \qquad (10)\\ \Rightarrow \hat{P}[\text{detect}|\text{no QPC}] &\leq \beta. \qquad (11)\end{aligned}$$

Thus as a result of this heuristic argument, the following results might be expected

- **For signals exhibiting QPC:** At high SNR's we expect to see a detection rate greater than $1-\alpha$, and at low SNR's we expect to see a detection rate lower than $1-\alpha$.

- **For signals not exhibiting QPC:** At high SNR's we expect to see a detection rate greater than β, and at low SNR's we expect to see a detection rate lower than β.

5. SIMULATIONS

To investigate whether the claims made in this correspondence are true, some simulations were carried out. The simulations described here are similar in to those presented by Zhou [2, page37]. The signal consists of three real sinusoids [1] with related frequencies with $T=1024$, $\alpha=0.05$, $\Gamma=3.841$, $\omega_1=0.4$, $\omega_2=0.9$, $\omega_3=1.3$, $f_s=\pi$, $A_1=A_2=A_3=1$. We consider the following two cases

Case 1: There is QPC; $\phi_3=\phi_1+\phi_2$,

Case 2: There is **no** QPC; $\phi_3=U(-\pi,\pi]$,

[1] Zhou uses complex exponentials rather than real sinusoids, but we believe that the real signal case is of more practical use.

and count the number of detections in 200 Monte-Carlo runs at the bifrequency ω_1, ω_2 for different SNR's [2], where the noise is white and Gaussian. The theoretical P_{FA} β is given by (5) as

$$\begin{aligned}\beta &= \frac{1}{\pi}\sqrt{\frac{\Gamma(\alpha)\sigma_v^2}{2T}\sum_{l=1,2,3}\frac{1}{A_l^2}} \\ &= \frac{1}{\pi}\sqrt{\frac{\Gamma(\alpha)\sigma_y^2}{2T10^{SNR/10}}\sum_{l=1,2,3}\frac{1}{A_l^2}} \\ &= \frac{3}{2\pi}\sqrt{\frac{\Gamma(\alpha)}{T10^{SNR/10}}},\end{aligned}$$

since the variance of a sine wave of amplitude A is $\frac{A^2}{2}$.

Figure 2 shows the performance of the ZGPCD in both the cases described above (i.e. with QPC and without QPC).

The results show that at high SNR's, the QPC hypothesis is accepted when QPC is present, and rejected when QPC is absent, with high probability. However, as the SNR falls the probability that a signal with QPC is accepted falls rapidly, and equally importantly, the probability that a signal without QPC is accepted rises – leading to a serious performance degradation.

In this sense the ZGPCD is not "fail-safe," i.e. at low SNR it tends to detect QPC erroneously in many signals which do not in fact exhibit QPC.

For the signals exhibiting QPC, it is evident that at high SNR

$$\hat{P}[\text{detect}|\text{QPC}]_{highSNR} > 1-\alpha$$

as predicted in (8), and that at low SNR

$$\hat{P}[\text{detect}|\text{QPC}]_{lowSNR} < 1-\alpha$$

as predicted in (10).

For signals not exhibiting QPC, it is evident from the Figure that at high SNR $\beta \to 0$, and that

$$\hat{P}[\text{detect}|\text{no QPC}]_{highSNR} \approx \beta$$

and that at low SNR

$$\hat{P}[\text{detect}|\text{no QPC}]_{lowSNR} > \beta$$

which disagrees with the prediction in (11).

It is not clear at this stage why the experimental detection rates do not behave quite in the way expected according to the qualitative argument presented earlier. It seems plausible that the distribution of the

[2]The SNR is defined by $SNR = 10\log_{10}\frac{\sigma_y^2}{\sigma_v^2}$ where σ_y^2 is the signal variance and σ_v^2 is the noise variance.

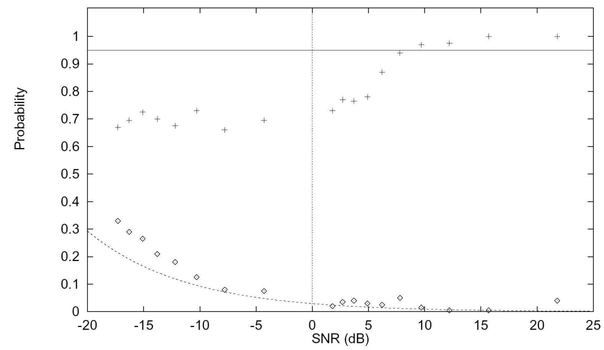

Figure 2: Comparison between theory and experiment for the ZGPCD. Significance level $\alpha = 0.05$. $1 - P[\text{Type I error}]$: theory (solid line) experiment (crosses). $P[\text{Type II error}]$: theory (dashed line) experiment (diamonds).

phase in practice does not agree with the theoretical approximation derived by Zhou, especially as the SNR decreases. This may in turn explain why the probability of erroneous detection at low SNR's is higher than the prediction made in (11).

It is worth adding that different methods (such as inverse filtering) for estimating the noise variance may give improved results, and this remains a topic for future investigation.

6. EQUIVALENCE WITH THE FACKRELL-MCLAUGHLIN-WHITE DETECTOR (FMWD)

In this section it is demonstrated that, for the case of signals like with real sinusoids in AWGN (additive second-order white Gaussian noise), the ZGPCD detector [1–3] is in fact identical to the Fackrell-McLaughlin-White detector (FMWD) [6]. This is shown by reformulating the expression for $\sigma^2_{\hat{\Phi}}$, the biphase variance at (f_1, f_2) from [5,6], in terms of the model parameters A_i and the noise variance σ_v^2 used in [1–3].

The two expressions to be reconciled are;

1. The empirical variance of the biphase estimate at (f_1, f_2) computed for a real signal over K independent frames [5];
$$\sigma^2_{\hat{\Phi}}[\text{FMWD}] = \frac{1}{2K}\left(\frac{1}{b^2_{f_1,f_2}} - 1\right), \qquad (12)$$
2. The corresponding theoretical variance of the biphase estimate at (f_1, f_2) for a single record (of length

M) of a signal consisting of complex harmonics in noise. This is the asymptotic variance of $\sqrt{M}(\hat{\Phi} - \Phi)$ [1,3] divided by M,

$$\sigma^2_{\hat{\Phi}}[\text{ZGPCD}] = \frac{1}{2M}\sum_{i=1}^{3}\frac{\sigma_v^2}{A_i^2}. \qquad (13)$$

For the purpose of this illustration it is assumed that the additive noise $v(n)$ is AWGN with variance σ_v^2, but what follows should be equally applicable to any second-order white noise with a symmetric pdf.

6.1. Noise free case

If there is no noise $\sigma_v^2 = 0$ then it is easy to show that $X_k = Y_k$, and that at bifrequency $(k = f_1, l = f_2)$ the expected values of the components of the bicoherence will be [7];

$$\begin{aligned} |E[X_{f_1}X_{f_2}X^*_{f_3}]|^2 = |E[Y_{f_1}Y_{f_2}Y^*_{f_3}]|^2 &= \prod_{i=1}^{3}\frac{A_i^2}{4}, \\ E[|X_{f_1}X_{f_2}|^2] = E[|Y_{f_1}Y_{f_2}|^2] &= \prod_{i=1}^{2}\frac{A_i^2}{4}, \\ E[|X_{f_3}|^2] = E[|Y_{f_3}|^2] &= \frac{A_3^2}{4}, \end{aligned}$$

from which it easily follows that $b^2_{f_1,f_2} = 1$.

6.2. Noisy case

If there is some additive background noise $\sigma_v^2 \neq 0$ then the DFT becomes $X_k = Y_k + V_k$. The numerator of the bicoherence is unchanged;

$$|E[X_{f_1}X_{f_2}X^*_{f_3}]|^2 = |E[Y_{f_1}Y_{f_2}Y^*_{f_3}]|^2 = \prod_{i=1}^{3}\frac{A_i^2}{4}, \qquad (14)$$

since the cross terms between signal and noise which arise all have zero expectations, and the noise has zero expected bispectrum since it is assumed Gaussian. However, the two terms on the denominator do get affected by noise; the first term on the denominator becomes

$$\begin{aligned} E[|X_{f_1}X_{f_2}|^2] &= E[|(Y_{f_1}+V_{f_1})(Y_{f_2}+V_{f_2})|^2] \\ &= E[|Y_{f_1}Y_{f_2} + Y_{f_1}V_{f_2} + V_{f_1}Y_{f_2} + V_{f_1}V_{f_2}|^2] \\ &= E[Y_{f_1}^2Y_{f_2}^2 + Y_{f_1}^2V_{f_2}^2 + Y_{f_2}^2V_{f_1}^2] \\ &= E[Y_{f_1}^2Y_{f_2}^2] + \frac{\sigma_v^2}{M}\left(E[Y_{f_1}^2] + E[Y_{f_2}^2]\right) \\ &= \prod_{i=1}^{2}\frac{A_i^2}{4} + \frac{\sigma_v^2}{M}\sum_{i=1}^{2}\frac{A_i^2}{4}, \end{aligned} \qquad (15)$$

where the independence of the noise $v(n)$ from the sinusoids has been used, and the fact that, for second-order white noise $E[V^2(k)] = \sigma_v^2/M$. The second term on the denominator becomes

$$E[|X_{f_3}|^2] = E[|Y_{f_3} + V_{f_3}|^2] = E[Y_{f_3}^2] + \frac{\sigma_v^2}{M} = \frac{A_3^2}{4} + \frac{\sigma_v^2}{M}. \qquad (16)$$

The theoretical bicoherence $b^2_{f_1,f_2}$ is given by combining Eqns. 14-16. This can then be substituted in Eqn. 12 to give a new expression for $\sigma^2_{\hat{\Phi}}[\text{FMWD}]$;

$$\sigma^2_{\hat{\Phi}}[\text{FMWD}] = \frac{1}{2KM}\left(\sum_{i=1}^{3}\frac{4\sigma_v^2}{A_i^2} + \frac{16\sigma_v^4}{M}\left[\frac{1}{A_2^2A_3^2} + \frac{1}{A_1^2A_3^2}\right]\right),$$

and provided M is reasonably large, the square bracketed term can be neglected to give

$$\sigma^2_{\hat{\Phi}}[\text{FMWD}] \approx \frac{1}{2KM}\sum_{i=1}^{3}\frac{4\sigma_v^2}{A_i^2} = \frac{4\sigma_v^2}{2KM}\sum_{i=1}^{3}\frac{1}{A_i^2}. \qquad (17)$$

Now from the asymptotic distribution of the biphase of a signal composed of *complex* harmonics $y(n) = \sum_i A_i \exp(-j2\pi f_i n + \phi_i)$ in *complex* noise [1,3], an expression for *real* signals can be formed. The results derived in [1,3] are (we believe) still valid, as long as each instance of the squared sinusoid amplitude A_i^2 is scaled down by a factor of 4. This is because $E[|X_{f_i}|^2] = A_i^2$ for a complex harmonic signal [3, Eqn.24], but $E[|X_{f_i}|^2] = A_i^2/4$ for a real signal (the total energy of the real signal is half that of the complex signal, and the real signal has half of its energy mirrored above the folding frequency). The biphase variance is then given by

$$\sigma^2_{\hat{\Phi}}[\text{ZGPCD}] \Rightarrow \frac{1}{2M}\sum_{i=1}^{3}\frac{\sigma_v^2}{A_i^2/4} = \frac{4\sigma_v^2}{2M}\sum_{i=1}^{3}\frac{1}{A_i^2}.$$

Furthermore, if K independent realisations are available, then the biphase variance is scaled down by a factor of K. The asymptotic biphase expression then leads to the following expression for the variance of the biphase averaged over K independent segments of $x(n)$;

$$\sigma^2_{\hat{\Phi}}[\text{ZGPCD}] \Rightarrow \frac{4\sigma_v^2}{2KM}\sum_{i=1}^{3}\frac{1}{A_i^2}. \qquad (18)$$

The equivalence of Eqns. 17 and 18 shows that for the AWGN case, the two QPC detectors are the same. In other words the FMWD detector is a special case of the more general ZGPCD detector.

7. CONCLUSIONS

In this correspondence the presence of Type II errors in the Zhou and Giannakis Phase Coupling Detector have been discussed, and an expression for the probability of these errors has been derived. Simulations have shown a general agreement between theory and experiment, but the matter is complicated because the critical region for detecting phase coupling is itself estimated, and the estimate is sensitive to noise. Further work is clearly warranted on the theoretical clarification of the ideas expressed here, and ways of alleviating the problems which arise in the estimation of the QPC acceptance limits. Moreover, we have shown that for the case of real sinusoids in white Gaussian noise, the Zhou and Giannakis Phase Coupling Detector is identical to the Fackrell-McLaughlin-White Detector. This was done by comparing analytically the asymptotic expressions for the variance of the two underlying biphase estimators.

8. ACKNOWLEDGEMENTS

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