CFAR Detection of Targets under Directional Noise Background

Qing Wang, Chunru Wan, Yixin Yang
Information System Research Lab, S2-B3a-06, School of EEE
Nanyang Technological University, Singapore, 639798
wangqing@pmail.ntu.edu.sg, ecrwan@ntu.edu.sg, eyxyang@ntu.edu.sg

***Abstract*--For a radar or sonar system, target detection is a basic function of the system. In sensor array processing, target detection is facilitated by examining the beamformer output of the received signal. The conventional method integrates the beam power observations by simple summation or linear integration and no statistical information is used. However, if the noise environment is directional, this will result in non-CFAR (constant false alarm rate) detection performance. In this paper, we exploit generalized likelihood ratio test (GLRT) to design the detector. We divide the data sequence from all the sensors into different segments and calculate the power observation of each segment. Then the probability distributions of the power observation under both hypotheses are derived. The unknown parameters of the signal model can be estimated by moment estimation based on statistical properties of the observations. Compared with the conventional processor, the GLRT detector can normalize the background of the output test statistic so that it shows CFAR property. Under the noise with directionality, the new detector we developed can still work well while the conventional one will show a false result due to the directionality of the noise when the *SNR* is low. Simulation result shows that the GLRT detector can also be used for multiple targets situation.**

I. INTRODUCTION

Many useful acoustic signals in the ocean are noise-like in character. The ambient ocean noise refers to the noise that remains after all easily identifiable sound sources are eliminated [1]. It can be modeled as a superposition of an anisotropic noise field from the ocean surface and an isotropic noise field. Experimental work has shown that surface noise is directional [2]. For the signal propagating in the ocean, it contains much information about the sources that produce them [3]. For a radar or sonar system, target detection is a basic function of the system. In sensor array processing, target detection is facilitated by examining the beamformer output of the received signal. In this paper, the problem is modeled as a binary hypothesis test. For H_0, only noise exists; while for H_1, a narrowband signal presents with a specific direction of arrival (DOA). The conventional method integrates the beam power observations by simple summation or linear integration [4]. It is simple and robust, and remains a powerful approach in array processing. However, if the noise environment is directional, this will result in non-CFAR (constant false alarm rate) detection performance. In this paper, we exploit generalized likelihood ratio test (GLRT) to design the detector. We derive the probability distributions of the power observation under both hypotheses and estimate the unknown parameters of the signal model by moment estimation. Compared with the conventional processor, the GLRT detector shows CFAR property.

The paper is organized as follows. Section II describes the detection problem and presents the statistical models of signal and noise. The GLRT detector is derived in Section III. In Section IV, simulations are conducted to demonstrate the detection performance of the detector. Finally, conclusion is given in Section VI.

II. DETECTION MODELS

A. Signal Model

Suppose a plane wave narrow-band signal arrives at a uniformly spaced linear array with M sensors, the received signal at baseband at the m-th sensor is

$$\begin{aligned} x_m(n) &= s_m(n) + g_m(n) \\ &= A\exp(-j2\pi f_0 m\tau) + g_m(n) \\ \text{for} \quad & m = 0,\cdots,M-1; \\ & n = 0,\cdots,Q-1 \end{aligned} \tag{2.1}$$

where A is the unknown amplitude, f_0 is the known normalized frequency and $g_m(n)$ is complex white Gaussian noise with unknown variance σ^2. τ is the delay $\tau = -d\sin\psi_0 / cT_s$, where ψ_0 is the unknown direction of arrival and T_s is the sample time interval, c is the speed of propagation and d is the separation between sensors. The geometry for the uniformly spaced linear array is shown in Fig. 1 [5][6].

Thus the received signal for the array can be expressed as

$$\begin{aligned} \mathbf{X} &= \mathbf{S} + \mathbf{G} \\ &= A\begin{pmatrix} 1 \\ \exp(-j2\pi f_0\tau) \\ \vdots \\ \exp(-j2\pi f_0(M-1)\tau) \end{pmatrix}_{M\times 1} \mathbf{I_e} + \begin{pmatrix} \mathbf{g_0} \\ \mathbf{g_1} \\ \vdots \\ \mathbf{g_{M-1}} \end{pmatrix}_{M\times Q} \end{aligned} \tag{2.2}$$

where $\mathbf{g_i}$ and $\mathbf{g_j}$ ($i \neq j$) are uncorrelated Gaussian noise and

$$\mathbf{g_i} = \left[g_i(0) \quad \cdots \quad g_i(Q-1) \right]_{1\times Q}$$
$$\mathbf{I_e} = \left[1 \quad \cdots \quad 1 \right]_{1\times Q}$$

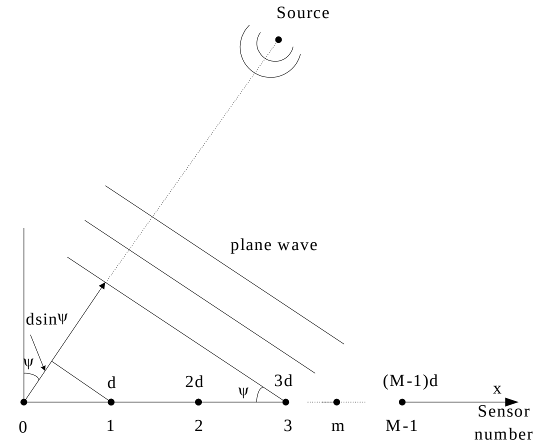

Fig. 1 Geometry for uniformly spaced linear array

Define the matrix $\mathbf{W}$ as

$$\mathbf{W}(\psi) = \begin{pmatrix} 1 \\ \exp(-j2\pi\ f_0 \tau_d) \\ \vdots \\ \exp(-j2\pi\ f_0 (M-1)\tau_d) \end{pmatrix}_{M\times 1} \qquad (2.3)$$

where $\tau_d = -\dfrac{d \sin\psi}{c T_s}$, $\psi \in [-\pi/2, \pi/2]$. The conjugate transpose of $\mathbf{W}$ is

$$W^H = \left[1 \quad \exp(j2\pi\ f_0 \tau_d) \quad \cdots \quad \exp(j2\pi\ f_0 (M-1)\tau_d) \right]_{1\times M}$$

B. Hypothesis Model

The target detection problem can be modeled as a binary hypothesis test as follows:

$$\begin{aligned} H_0 &: \quad \mathbf{X} = \mathbf{G}; \\ H_1 &: \quad \mathbf{X} = \mathbf{S} + \mathbf{G} = A\mathbf{W}\,\mathbf{I_e} + \mathbf{G} \end{aligned} \qquad (2.4)$$

For H_0, only noise exists; For H_1, a narrowband signal presents with a specific direction of arrival. Note that the noises from sensor to sensor are uncorrelated white Gaussian noise.

Divide the received data $\mathbf{X}$ into L submatrices with $M \times N$ dimension. The segment procedure is shown in Fig.2.

For the *m*-th sensor, the observation sequence $x_{m,l}$ is

$$x_{m,l}(n) = x_m(l \cdot R + n) \qquad (2.5)$$
$$\text{for} \quad n = 0, \cdots, N-1; \quad l = 0, \cdots, L-1$$

The relationship among Q, L and N is

$$L = Int[(Q-N)/R] + 1 \qquad (2.6)$$

When $R = N$, the segments are contiguous; when $R < N$, the segments overlap. The *l*-th matrix is

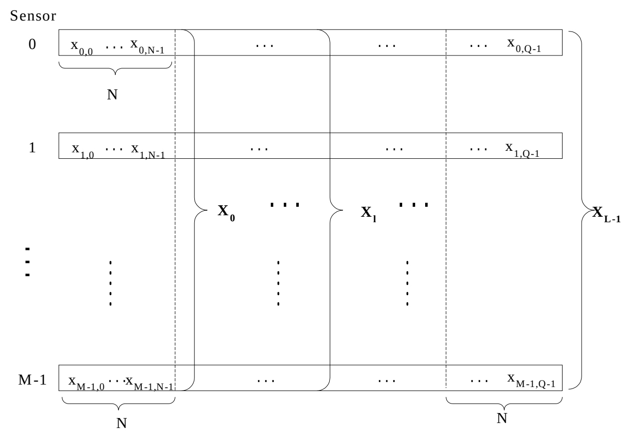

Fig. 2 Divide the received data of all sensors into matrices

$$\mathbf{X}_l = \begin{pmatrix} x_{0,lR} & x_{0,lR+1} & \cdots & x_{0,lR+N-1} \\ x_{1,lR} & x_{1,lR+1} & \cdots & x_{1,lR+N-1} \\ \vdots & \vdots & \vdots & \vdots \\ x_{M-1,lR} & x_{M-1,lR+1} & \cdots & x_{M-1,lR+N-1} \end{pmatrix}_{M\times N} \qquad (2.7)$$

The conventional beamforming algorithm estimates the signal power by

$$\begin{aligned} \mathbf{Y} &= \frac{1}{L}\sum_{l=0}^{L-1} (\mathbf{W}^H \mathbf{X}_l)(\mathbf{W}^H \mathbf{X}_l)^H \\ &= \frac{1}{L}\sum_{l=0}^{L-1} \left| \mathbf{W}^H \mathbf{X}_l \right|^2 \end{aligned} \qquad (2.8)$$

Since the direction of arrival ψ_0 is unknown, the time delay τ is unknown. We search over all the angles from $\psi \in [-\pi/2, \pi/2]$ for a linear array. The output $\mathbf{Y}$ is the function of ψ and maximal at ψ_0. The algorithm is so-called delay-and-sum beamforming [4]. It is simple and remains a powerful approach.

III. DESIGN OF DETECTOR

We resort to the statistical integration method to derive the new CFAR detector for target detection. The output test statistic is compared with a threshold γ .The peak above the threshold corresponds to the direction of arrival.

A. Statistical Properties

Define a new vector $\mathbf{Z}_l$ as

$$\begin{aligned} \mathbf{Z}_l(\psi) &= \mathbf{W}^H(\psi)\,\mathbf{X}_l \\ &= \left[z_{l0} \quad z_{l1} \quad \cdots \quad z_{l(N-1)} \right]_{1\times N} \end{aligned} \qquad (3.1)$$

It is a vector with $1 \times N$ dimension. The *j*-th element of $\mathbf{Z}_l$ is

$$z_{lj} = \sum_{i=0}^{M-1} w_i^* x_{i,lR+j} \qquad \text{for} \quad j = 0, \cdots, N-1 \qquad (3.2)$$

where w_i is the *i*-th element of $\mathbf{W}$.

For hypothesis null, only noise exists. The mean and the variance of z_{lj} are

$$E[z_{lj}] = \sum_{i=0}^{M-1} E[w_i^* x_{i,lR+j}] = 0 \tag{3.3}$$

$$Var[z_{lj}] = \sum_{i=0}^{M-1} Var[w_i^* x_{i,lR+j}] = M\sigma^2 \tag{3.4}$$

Thus z_{lj} follows complex Gaussian distribution with zero mean under H_0,

$$z_{lj} \sim CN(0, M\sigma^2) \tag{3.5}$$

For hypothesis one that the signal presents, and the looking direction ψ coinciding with the signal direction ψ_0, i. e. $\psi = \psi_0$, the mean and the variance of z_{lj} are derived as

$$E[z_{lj}] = \sum_{i=0}^{M-1} E[w_i^*(s_{i,lR+j} + g_{i,lR+j})] = MA \tag{3.6}$$

$$Var[z_{lj}] = \sum_{i=0}^{M-1} Var[w_i^* s_{i,lR+j} + w_i^* g_{i,lR+j}] = M\sigma^2 \tag{3.7}$$

Therefore, the distribution of z_{lj} under H_1 is Gaussian distribution with mean MA,

$$z_{lj} \sim CN(MA, M\sigma^2) \tag{3.8}$$

For $\psi \neq \psi_0$, signal may partially present due to the sidelobe effect. We assume that appropriate shading has been applied so that the sidelobe effect is negligible. Otherwise, the detector may reflect the sidelobe effect accordingly.

The observation of the new processor is defined as

$$y_l = \mathbf{Z}_l \mathbf{Z}_l^H = \sum_{j=0}^{N-1} z_{lj} z_{lj}^* = \sum_{j=0}^{N-1} y_{lj} \tag{3.9}$$

Since z_{lj} follows Gaussian distribution under both hypotheses, its real and imaginary part are also Gaussian. That is,

$$\begin{aligned} H_0: &\quad z_{ljR}, z_{ljI} \sim N(0, M\sigma^2/2) \\ H_1: &\quad z_{ljR} \sim N(MA\cos\theta, M\sigma^2/2) \\ &\quad z_{ljI} \sim N(MA\sin\theta, M\sigma^2/2) \end{aligned} \tag{3.10}$$

where θ is the argument of z_{lj}, $\theta = \arctan\dfrac{z_{ljI}}{z_{ljR}}$.

For the variable y_{lj},

$$y_{lj} = z_{lj} z_{lj}^* = z_{ljR}^2 + z_{ljI}^2 \tag{3.11}$$

it follows chi-square distribution under H_0 and non-central chi-square distribution under H_1. The distribution can be expressed as follows

$$\begin{aligned} H_0: &\quad y_{lj} \sim \chi_2^2(0, M\sigma^2/2) \\ H_1: &\quad y_{lj} \sim \chi_2'^2(M^2A^2, M\sigma^2/2) \end{aligned} \tag{3.12}$$

The non-centrality parameter λ_j is M^2A^2.

The observation y_l is the sum of y_{lj}, so its distribution is still chi-square. The degree of freedom (DOF) is *2N* and the non-centrality parameter is

$$\lambda = \sum_{j=0}^{N-1} \lambda_j = NM^2A^2$$

Therefore, the distribution of y_l can be expressed as

$$\begin{aligned} H_0: &\quad y_l \sim \chi_{2N}^2(0, M\sigma^2/2) \\ H_1: &\quad y_l \sim \chi_{2N}'^2(\lambda, M\sigma^2/2) \end{aligned}$$

Define $v = M\sigma^2/2$, the probability density functions (PDF) of y_l are obtained by

$$p_{y_l|H_0}(y_l) = \frac{1}{(2v)^N \Gamma(N)} y_l^{N-1} \exp\left(-\frac{y_l}{2v}\right) \tag{3.13}$$

$$p_{y_l|H_1}(y_l) = \frac{1}{2v}\left(\frac{y_l}{\lambda}\right)^{\frac{N-1}{2}} \exp\left[-\frac{y_l+\lambda}{2v}\right] I_{N-1}\left(\frac{\sqrt{\lambda\, y_l}}{v}\right) \tag{3.14}$$

where $\Gamma(\cdot)$ is gamma function and $I_{N-1}(\cdot)$ is the $N-1$ order modified Bessel function of the first kind.

B. Generalized Likelihood Ratio

The y_l from all segments form a new vector $\mathbf{Y}$,

$$\mathbf{Y} = \begin{bmatrix} y_0 & y_1 & \cdots & y_{L-1} \end{bmatrix}^T \tag{3.15}$$

The observation y_{l1} and y_{l2} ($l1 \neq l2$) are independent for

$$\begin{aligned} E[y_{l1} y_{l2}] &= \mathbf{W}^H E[\mathbf{X}_{l1}\mathbf{X}_{l1}^H]\mathbf{W}\,\mathbf{W}^H E[\mathbf{X}_{l2}\mathbf{X}_{l2}^H]\mathbf{W} \\ &= E[y_{l1}]E[y_{l2}] \end{aligned}$$

We normalize the observation $\mathbf{Y}$ with v so that

$$\overline{\mathbf{Y}} = \begin{bmatrix} \bar{y}_0 & \bar{y}_1 & \cdots & \bar{y}_{L-1} \end{bmatrix}^T = \mathbf{Y}/v \tag{3.16}$$

The distribution of $\bar{y}_l$ is normalized chi-square or non-central chi-square distribution as

$$\begin{aligned} H_0: &\quad \bar{y}_l \sim \chi_{2N}^2(0,1) \\ H_1: &\quad \bar{y}_l \sim \chi_{2N}'^2(\bar{\lambda},1) \end{aligned} \tag{3.17}$$

where $\bar{\lambda} = \lambda/v = NM^2A^2/v$ is the non-centrality parameter. Then the PDFs of $\overline{\mathbf{Y}}$ are given by

$$\begin{aligned} p_{\overline{\mathbf{Y}}|H_0}(\overline{\mathbf{Y}}|H_0) &= \prod_{l=0}^{L-1} p_{\bar{y}_l|H_0}(\bar{y}_l) \\ &= \left(\frac{1}{2^N\Gamma(N)}\right)^L \left(\prod_{l=0}^{L-1} \bar{y}_l\right)^{N-1} \exp\left(-\frac{\sum_{l=0}^{L-1}\bar{y}_l}{2}\right) \end{aligned} \tag{3.18}$$

$$\begin{aligned} p_{\overline{\mathbf{Y}}|H_1}(\overline{\mathbf{Y}}|H_1) &= \prod_{l=0}^{L-1} p_{\bar{y}_l|H_1}(\bar{y}_l) \\ &= \left(\frac{1}{2}\right)^L \left(\frac{1}{\bar{\lambda}}\right)^{\frac{L(N-1)}{2}} \left(\prod_{l=0}^{L-1} \bar{y}_l\right)^{\frac{N-1}{2}} \prod_{l=0}^{L-1} I_{N-1}\left(\sqrt{\bar{\lambda}\,\bar{y}_l}\right) \\ &\quad \cdot \exp\left(-\frac{L\bar{\lambda} + \sum_{l=0}^{L-1}\bar{y}_l}{2}\right) \end{aligned} \tag{3.19}$$

Let $\zeta = [\lambda \quad v]$, the generalized likelihood ratio is given by

$$\Lambda(\mathbf{Y}) = \frac{\max_{\zeta} p_{\mathbf{Y}|H_1}(\mathbf{Y}|H_1,\zeta)}{\max_{\zeta} p_{\mathbf{Y}|H_0}(\mathbf{Y}|H_0,\zeta)}$$

The logarithm of the likelihood ratio is

$$\ln \Lambda(\mathbf{Y}) = \ln p_{\mathbf{Y}|H_1}(\mathbf{Y}|H_1,\lambda,v_1) - \ln p_{\mathbf{Y}|H_0}(\mathbf{Y}|H_0,v_0)$$

where λ, v_1 and v_0 are the value that maximizes the PDF under H_1 and H_0 respectively. The subscript "1" means the value under H_1 and "0" means the value under H_0. Since we use the normalized variable $\bar{y}_l$ as the observation, it only needs to estimate v_1 and λ. Then the corresponding logarithm of the likelihood ratio can be obtained as

$$\ln \Lambda(\overline{\mathbf{Y}}) = \ln p_{\overline{\mathbf{Y}}|H_1}(\overline{\mathbf{Y}}|H_1,\lambda/v_1) - \ln p_{\overline{\mathbf{Y}}|H_0}(\overline{\mathbf{Y}}|H_0) \quad (3.20)$$

C. Parameters Estimation

λ and v_1 are the unknown parameters of the log likelihood ratio. They can be obtained by maximum likelihood estimation. However, the procedure needs complicated nonlinear computation. Here we use the simple moment estimation to derive the unknown parameters.

For the random variable y_l with $\chi'^2_{2N}(\lambda)$ under H_1, the mean and the variance are

$$E[y_l] = \lambda + 2Nv_1 \quad (3.21)$$

$$Var[y_l] = 4\lambda v_1 + 4Nv_1^2 \quad (3.22)$$

We define new variables m_y and σ_y^2. Let $m_y = E[y_l] = \frac{1}{L}\sum_{l=0}^{L-1} y_l$ and $\sigma_y^2 = Var[y_l] = \frac{1}{L-1}\sum_{l=0}^{L-1}(y_l - m_y)^2$, the estimated parameters are derived from above equations as

$$\hat{\lambda} = \sqrt{m_y^2 - N\sigma_y^2} \quad (3.23)$$

$$\hat{v}_1 = \frac{m_y - \hat{\lambda}}{2N} \quad (3.24)$$

D. Normalized Chi-GLRT Detector

Now we design the GLRT detector. Since the observation $\bar{y}_l$ follows normalized chi-square distribution, we denote the detector as normalized Chi-GLRT detector. The direction of arrival ψ_0 is unknown. The variable $\bar{y}_l$ is the function of ψ. When $\psi = \psi_0$, the test statistic shows a peak at the corresponding angle.

The log generalized likelihood ratio has been derived in (3.20). By substituting the unknown parameters with the estimated values, the detector can be obtained as

$$\ln \Lambda(\overline{\mathbf{Y}}) = -L\ln 2 - \frac{(N-1)L}{2}\ln\bar{\lambda} - \frac{N-1}{2}\sum_{l=0}^{L-1}\ln \bar{y}_l + \sum_{l=0}^{L-1}\ln I_{N-1}\left(\sqrt{\bar{\lambda}\,\bar{y}_l}\right) - \frac{L\bar{\lambda}}{2} + L\ln\left(2^N\Gamma(N)\right) \quad (3.25)$$

where $\bar{\lambda} = \hat{\lambda}/\hat{v}_1$, $\hat{\lambda}$ and $\hat{v}_1$ have been estimated by (3.23) and (3.24).

Hence, the new detector denoted as normalized Chi-GLRT is

$$T(\psi) = \ln \Lambda(\overline{\mathbf{Y}}(\psi)) \quad (3.26)$$

We search ψ from $[-\pi/2, \pi/2]$. The smaller the search interval, the higher the resolution of ψ. For $T(\psi) > \gamma$ (threshold), its peak corresponds to the direction of arrival. The processor can also be used to detect multiple signals.

IV. SIMULATIONS

In this section, we present the simulation results for the processor we have developed.

Consider a uniform linear array with M = 5 sensors spaced half-wavelength apart $d/\lambda_w = 1/2$, λ_w is the wavelength of the propagation. The number of data for each sensor is Q = 2000, the number of data for each segment is N = 20. There is no overlapping between different segments. The direction of arrival is $\psi_0 = 30^\circ$. The signal to noise ratio is defined as $SNR = A^2/\sigma^2$.

A. Isotropic Noise Environment

When the sensor spacing is equal to multiples of half the wavelength λ_w, the isotropic noise is not correlated from sensor to sensor. Fig. 3 shows the simulation results of using the conventional beamforming and the normalized Chi-GLRT as given in (3.25). The top panel is the result from the conventional beamforming while the result from the normalized Chi-GLRT is presented in the bottom panel. It is shown that the new algorithm produces a sharper peak at the target DOA and a smoother background.

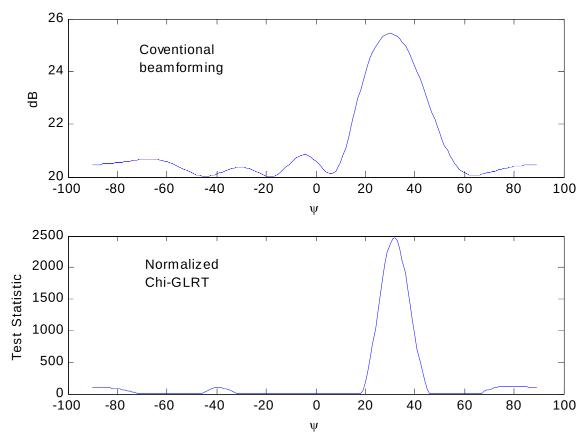

Fig.3 Steered responses by conventional beamforming and normalized Chi-GLRT, SNR=0.5

B. Multiple Signals

Suppose that there are two uncorrelated signals with the same amplitude impinging on the array from $\psi_1 = 10^\circ$ and $\psi_2 = -20^\circ$ respectively. The noises from sensor to senor are uncorrelated. Fig. 4 shows the steered responses for this situation. It is shown that the new normalized Chi-GLRT processor can be used for multiple signals situation.

C. Noise with Directionality

In the above simulations, it is assumed that the noise has no directionality and is spatially white. That is, the noises are uncorrelated from sensor to sensor. When the noise has directionality, the noises between sensors are correlated. When the *SNR* is low, the conventional beamforming algorithm will be affected greatly by the directionality of noise, while the normalized Chi-GLRT processor can work well.

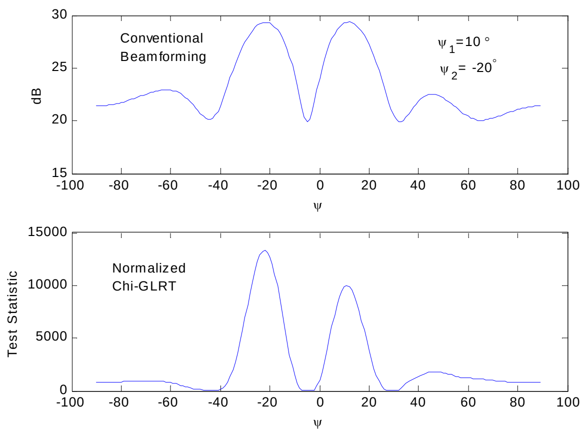

Fig. 4 Steered responses for two uncorrelated signals

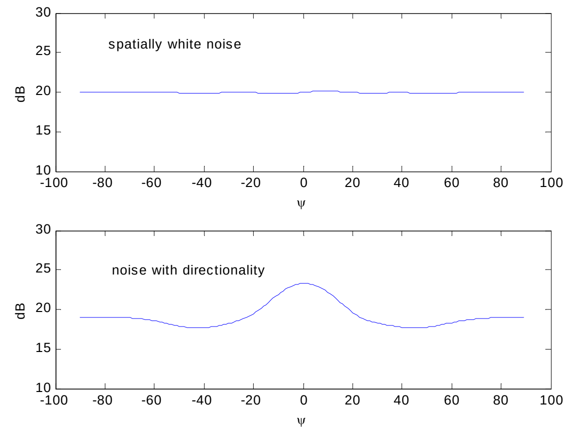

Fig. 5 Steered responses when only noise exists

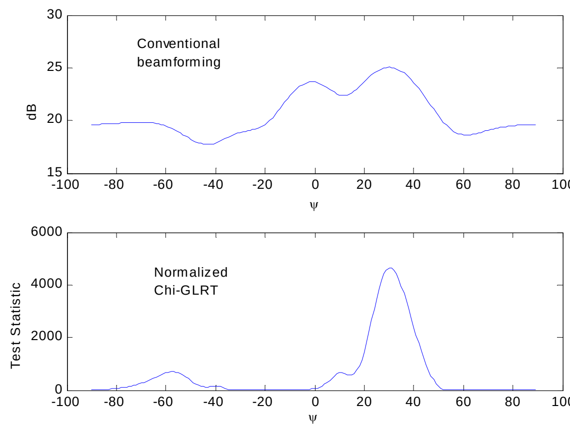

Fig.6 Steered responses when noise has directionality

The steered responses of conventional beamforming are plotted in Fig. 5 for the case that only noise exists. The top panel is the result in the case of spatially white noise while the bottom one is the case of noise with directionality. When the noise is spatially white, the output of beamforming is almost a straight line. However, when the noise has directionality, the output shows slow waving with the change of angle.

Fig. 6 shows the simulation results under the directional noise. From the figure, we observe that in the top panel, the conventional beamforming method generate two peaks, one is near the true angle of direction of arrival, the other is near 0°, which is caused by the directionality of noise. For the result from normalized Chi-GLRT, it seems that the influence by noise directionality is little. Therefore, the Chi-GLRT method is nearly robust to the noise directionality.

V. CONCLUSION

In this paper, we develop a novel detection algorithm, denoted as normalized Chi-GLRT, for target detection. Contrasted to the conventional beamforming method, it exploits the statistical properties of observations. Unlike the simple integration of observations by summing, the new algorithm utilizes the probability distributions of the observations. The Chi-GLRT method can normalize the background and show the property like CFAR. When *SNR* is low, the noise directionality will affect the conventional beamforming largely, but for the new algorithm, it is effective in this situation. It can be generalized to multiple directions of arrival.

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