FRACTIONAL FOURIER TRANSFORM TECHNIQUES APPLIED TO ACTIVE SONAR

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Abstract – The performance of the fractional Fourier transform is studied for two different applications, as an alternative to matched filtering and as a method of producing time/frequency surfaces. The fractional Fourier transform has shown several advantages over matched filtering regarding robustness against distortion of the received signal. The matched filter is only optimal in the case of the returning echo maintaining the original properties of the emitted pulse, whereas the fractional Fourier transform is still optimal as long as the chirp rate of the original signal is unaffected. Due to the fractional Fourier transform taking advantage of the known transmitted pulse in active sonar this gives an advantage over the traditional Fourier transform for making time/frequency surfaces. Time/frequency surfaces made with the traditional spectrogram are compared to surfaces made using the short time fractional Fourier transform (STFrFT). Both of these applications are studied using real data collected at two different sea trials.

I. INTRODUCTION

The analysis of sonar data has traditionally been done by conventional time-series techniques or in the frequency domain. Over the last ten years or more, interest in the use of Time-Frequency tools has increased. One popular technique in the radar and sonar context has been the Wigner distribution [1]. Moreover in the sonar context, or from a sonar operator point of view the traditional lofargram is the main tool for data analysis. In this paper a comparison is made between the traditional spectrogram [2] and the Short-Time Fractional Fourier Transform (STFrFT) [3], [4], for the time/frequency analysis of the sonar data. Furthermore the performance of the Fractional Fourier Transform is compared to matched filtering as an alternative for detection purposes, using both real and synthetic data. The FrFT is a generalization of the Fourier Transform (FT) developed within the optics community which uses linear chirps as the basis functions. In this sense it is perhaps ideally suited to sonar applications. The outline of this paper is as follows.
A brief introduction to the FrFT is given. It is followed by a description of the analysis where the FrFT is compared to matched filtering followed the time/frequency analysis using the spectrogram and the STFrFT. Then a short description of the data set is given and it is followed by the actual analysis which includes a measure of the quality of the time/frequency surfaces is presented. In the final section the conclusions are drawn and are followed by a brief discussion about the possibilities with the FrFT and the STFrFT.

II. PRELIMINARIES

In this section some definitions and a brief discussion of the spectrogram and STFrFT. For a more complete description we refer to [2], and [5].

The recorded sonar signal $x(t)$ (t=1,…, K) is assumed to be a real periodic discrete time series divided into L blocks each with N samples. The Fourier spectrum $X^n(f)$ of the time series $x(t)$ for block n is,

$$X^n(f) = \sum_{t=1}^{N} x(t) e^{-j2\pi ft/N} . \tag{2.1}$$

The power spectrum is formed as an average over all blocks L as

$$P(f) = \frac{1}{L}\sum_{n=1}^{L} X^n(f) X^{n*}(f) , \tag{2.2}$$

where * denotes complex conjugate. Only the positive part of the power spectrum is estimated, due to symmetry, i.e. $P(f) = P(-f)$.

The spectrogram is the surface built out of the L blocks placed adjacent to one and other, the *i:th* part of the spectrogram is defined as $P_i(f) = X^n(f)\ X^{n*}(f)$. So the estimated power spectrum is the average over the spectrogram surface.

The FrFT is defined as:

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$$F^{\alpha} f(x)=\frac{\exp(-j(\frac{1}{4}\pi-\frac{1}{2}\alpha))}{(2\pi|\sin\alpha|)}\exp(\frac{1}{2}jx^{2}\cot\alpha)\ldots \qquad (2.3)$$
$$\int_{-\infty}^{\infty}\exp(\frac{-jxy}{\sin\alpha}+\frac{1}{2}jx^{2}\cot\alpha)f(x)dx$$

where α is the transform order as

$$\alpha=\frac{2}{\pi}\tan^{-1}(\frac{fs^{2}/N}{2a}) \qquad (2.4)$$

Where N is the number of samples in a the segment and fs is the sampling frequency. The variable a is the rate of change from the general description of a linear chirp, as in $\exp(j(at^2+bt+c))$, and it has the instantaneous frequency $\phi' = 2a+b$. For a transform order equal to α =1, the FrFT is equal to the traditional FT [5], [6].

The STFrFT is defined very much like the STFT or the Spectrogram. The time series is segmented into L records and to form the time/frequency surface these segments are placed next to one and other.

For completeness, the definition of a matched filter is repeated.

$$y(t)=\sum_{i=0}^{K-1}h(i)x(t-i)=\sum_{i=0}^{K-1}x(K-i-1)x(t-i). \qquad (2.5)$$

where x(t) are the data segments mentioned before equation (2.1).

Then we define the quality measures *Ad and D*, where *Ad* is the average distance between two matrices and *D* is the dynamics in the matrix, [7].

$$Ad=\frac{\sum_i\sum_j|R_{ij}-S_{ij}|}{Ncols\cdot Nrows}. \qquad (2.6)$$

$$D=10\log_{10}\left\{\frac{\max|R|}{\min|R|}\right\} \qquad (2.7)$$

Where $i = 1...Ncols$, and $j = 1...Nrows$. The matrices are equally sized *Ncols x Nrows*. *R* is the surface made with the STFrFT or Spectrogram on real data and *S* is the STFrFT or Spectrogrram on synthetic data.

The synthetic data is made out of synthetic chirps with arrival times and amplitudes matching the real chirps in the real data, surrounded by zeros, so the synthetic data is undistorted and free of noise.

The data analysis falls into two different parts, the first one being a comparison of the performance of the FrFT versus the matched filter. The second part is the time/frequency part of the analysis where a measure for the quality of the images is presented. First the two methods for detection (matched filtering and FrFT) are compared using both real and synthetic data. The ability of both methods to work both within and slightly outside their intended frequency range is investigated.

Then in the second part of the analysis the STFrFT is compared to the more traditional spectrogram.

III. EXPERIMENTAL AND SYNTHETIC DATA SETUP

For the first part of the analysis the FrFT and matched filter are tested using synthetic chirps with slightly different characteristics. This simulation assumes that the chirp that is transmitted is a chirp with a center frequency of 1.5kHz, a bandwidth of 1kHz, and a duration of 0.1sec. This chirp is thereafter distorted in a number of ways to test the methods. See Table (3.1) where the first row describes the original transmitted chirp.

Center frequency	Bandwidth	Duration
1.5 kHz	1 kHz	0.1 s
1.6 kHz	1 kHz	0.1 s
1.5 kHz	1 kHz, LP filtered	0.1 s
1.5 kHz	1 kHz, HP filtered	0.1 s
2 kHz	2 kHz	0.2 s
1.25 kHz	0.5 kHz	0.05 s
3 kHz	1 kHz	0.1 s
1.5 kHz	1.2 kHz	0.1 s
1.5 kHz	0.8 kHz	0.1 s

Table 3.1 Synthetic chirps used in the analysis.

The chirps in rows 1-7 are chirps that have the same rate of change, whereas the chirps in rows 8 and 9 have different rates of change.

For the comparison of the two methods using real data, two examples from sea trials conducted in the archipelago of Stockholm in Baltic Sea, are analyzed. The real data consists of two different chirps, the first one with a center frequency of 1.5kHz, a bandwidth of 1kHz, and a duration of 0.2sec, the second chirp is one with a center frequency of 18kHz, a bandwidth of 4kHz, and a duration of 0.2 sec. Both sea trials were previously described in [6].

The time frequency analysis is carried out for the two data sets in the manner described above.

IV. DATA ANALYSIS AND RESULTS

The first part of the analysis is the comparison between the matched filter and the FrFT. The synthetic data presented in Table 3.1 is treated with both methods. Both levels for the matched filter and the FrFT are normalized so that the maximum level is equal to one for both. The top panel in Figure 4.1 is the matched filter and FrFT applied to the transmitted chirp, so naturally the level is equal to one for the matched filter case.

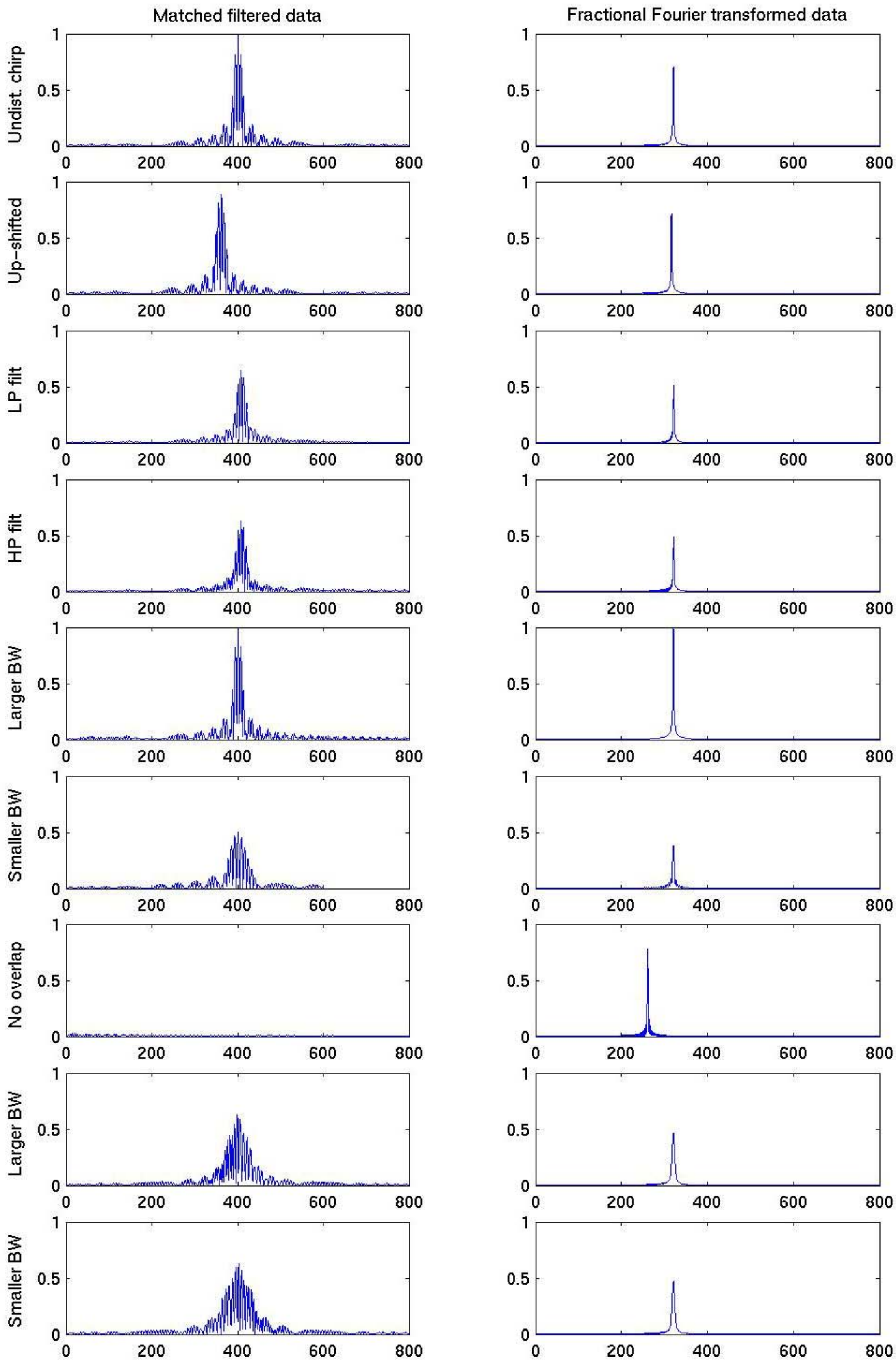

Figure 4.1 Synthetic chirps analyzed with the matched filter and FrFT.

The reason for the FrFT having a smaller value is that in the fifth panel from the top the bandwidth is larger and the FrFT takes advantage of this and produces its largest value here, but since all other values are normalized to this value the peaks are slightly lower. Notice that the matched filter still has one as its value in this panel.

The panel, second from the top displays a chirp that has been shifted up in frequency, moving the center frequency with a 100Hz. Since this is a small shift frequency neither of the methods are affected very much, but as can be expected the levels are slightly lowered and pulse width slightly widened.

In the following two panels (3 and 4, from the top) the chirp has been either low pass or high pass filtered. The filter frequency was set to 1.5kHz. The filter was a Butterworth filter using 4 taps. Both low pass and high pass filtering has about the same effect on the result, lower peaks.

In panel 6 from the top where the bandwidth is only half the original bandwidth, there is a significant widening of the pulse in the matched filtering case. This is however hardly noticeable in the FrFT case.

In panel 7 from the top, we see the results from the analysis of a chirp that is entirely outside the original frequency range, which is 1-2kHz. One finds that the matched filter has a very low level so there is no peak discernable at all. Since the first seven chirps have maintained their chirp rates the FrFT shows very consistent behavior. This is one of the strengths of the FrFT, as long as the chirp rate is maintained the FrFT is optimal.

In the lower most panels, 8 and 9, there are chirps that have different chirp rates from the original. And this manifests it self as a lowering of the peak values and most importantly as a widening of the pulse width. At this point it can be found in both the match filtered and FrFT treated data.

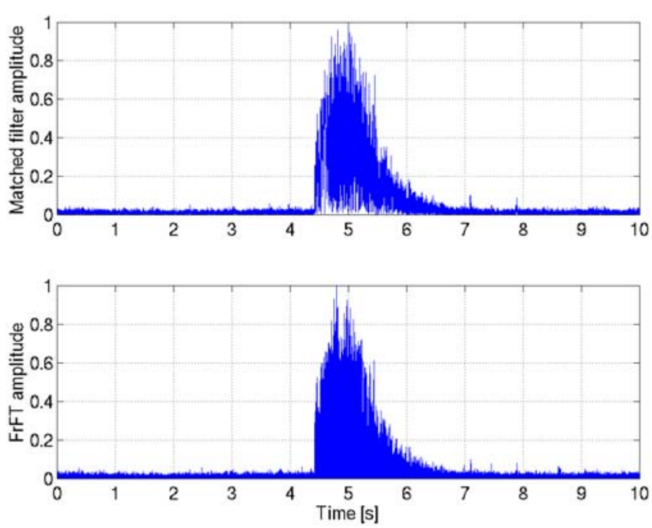

Figure 4.2 Matched filter compared to the FrFT on real chirp 1-2kHz, 0.2 sec duration.

The next step in the analysis is the comparison between the matched filter and FrFT using real data. First the chirps with a BW of 1kHz, center frequency of 1.5kHz and 0.2 sec duration. As one can see there is very little difference in performance, which after all is to be expected since both methods are matched to the transmitted chirp. And can be argued to be different implementations of a perfect filter. Although the sound propagation in the Archipelago is quite difficult to come to terms with the direct path in both cases is the by far strongest and therefore rather uninfluenced. See Figure 4.2.

The same can be found in Figure 4.3 where the results from the analysis of the 18kHz center frequency, 4kHz BW, and 0.1 sec duration chirp are presented.

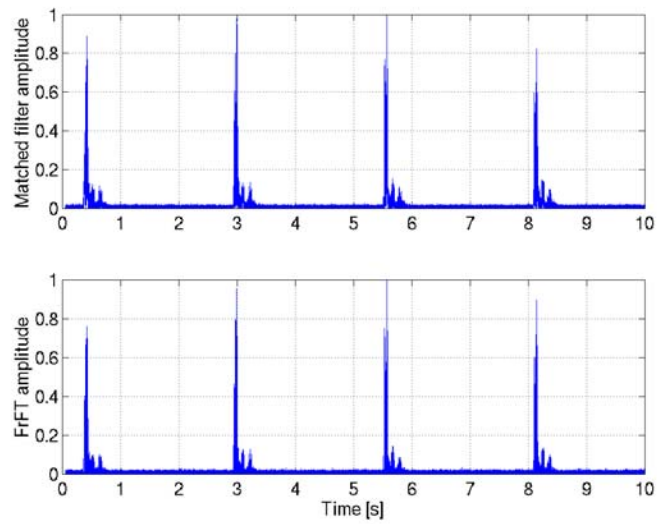

Figure 4.3 Matched filter compared to the FrFT on real chirp 16-20kHz, 0.1 sec duration.

The difference between the two methods is of course related to what scale the images are compared at. Only viewing Figure 4.3 one finds that the noise floor between the pulses is on the same level in both cases, and the top peak values are approximately the same.

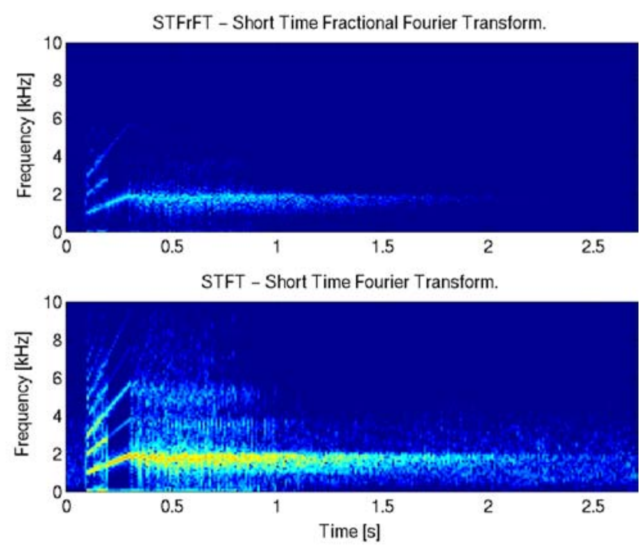

Figure 4.4 STFrFT compared to the spectrogram, 1-2kHz chirp with a duration of 0.2 sec.

The second part of the data analysis is the time/frequency analysis, where the chirps appearing in the comparison between the matched filter and the FrFT are analyzed over again but this time in order to form time/frequency surfaces.

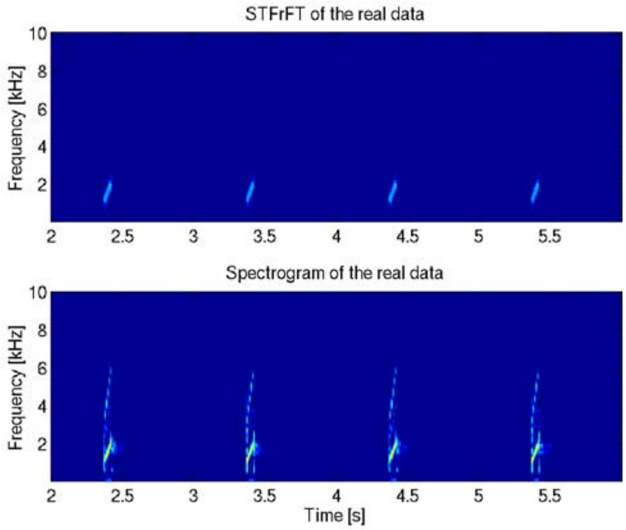

Figure 4.5 STFrFT and spectrogram of a reference signal, a linear chirp 1-2kHz with a duration of 0.05 sec. The signal was repeated once every second.

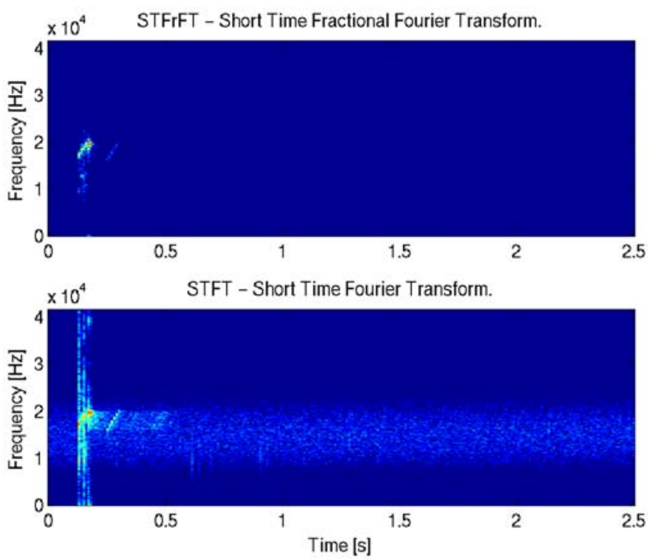

Figure 4.6 STFrFT compared to the spectrogram, 16-20kHz chirp with a duration of 0.1 sec.

Signal: LFM	*Ad Fr*	*Ad Sp*	*D Fr*	*D Sp*
1-2kHz, 0.2sec	0.9	7.4	51.8	65.3
1-2 kHz, 0.05sec	0.5	3.0	62.0	73.0
16-20 kHz, 0.1sec	0.5	5.9	65.0	68.8
16-20 kHz, 0.1sec	0.55	6.5	56.3	68.2
16-20 kHz, 0.1sec	0.53	6.4	66.1	72.8

Table 4.1 Results from the comparison between the STFrFT and the spectrogram. *Fr*and *Sp* stand for STFrFT and Spectrogram respectively, *D* is displayed in Decibel.

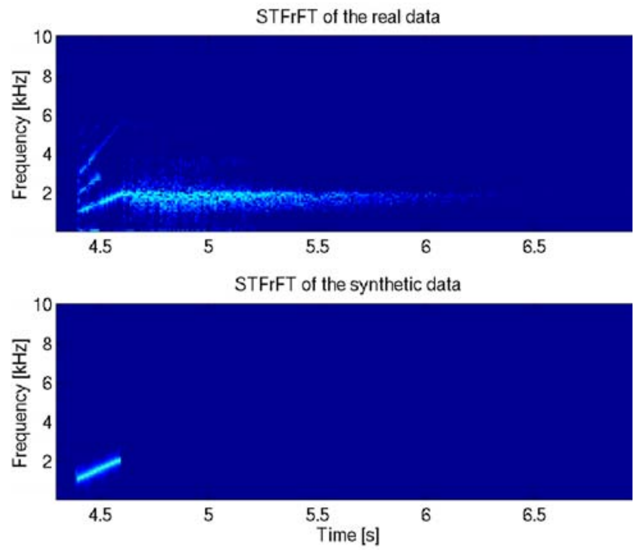

Figure 4.7 STFrFT for real and synthetic data. The pulse beginning at 4.2 seconds is a 1-2kHz chirp, with a duration of 0.2 sec.

The results from the comparison between the STFrFT and the spectrogram are presented in Table 4.1. According to the measures in equations (2.6) and (2,7) *Ad* and *D*, there is a difference between the STFrFT and spectrogram. Looking at the *Ad* measure it is in the order of 0.5 in favor of the STFrFT, on the other hand the *D* measure says that the Spectrogram has about 5-10dB better dynamics. In Figure 4.5 the STFrFT and spectrogram of a reference signal is presented, this signal was recorded to ensure that the recording system and transmitting system behave as expected. It worth noting that even in a case like this where there is very little reverberation present, the STFrFT shows a better result, that is a smaller distance between the real and simulated surfaces. Also view Figures 4.7 and 4.8 where the real data is compared to synthetic, for the 1-2kHz case.

Rows 4 and 5, in Table 4.1, show further examples of the difference between the STFrFT and the spectrogram.

By viewing the Figures one finds that the spectrogram has speckles almost all over the surface, whereas the STFrFT is nearly free of energy content especially in the higher frequencies, and the reverberation tail dies away faster in the STFrFT case. So the difference between the synthetic and real STFrFT surfaces is smaller than the difference between the synthetic and real spectrogram surfaces. A big part of this difference is due to the fact that FrFT does not describe reverberation well and that it does not react well to changes in the chirp rate so the distortions higher up in frequencies that can be seen in the spectrogram surfaces are not found in the STFrFT surfaces.

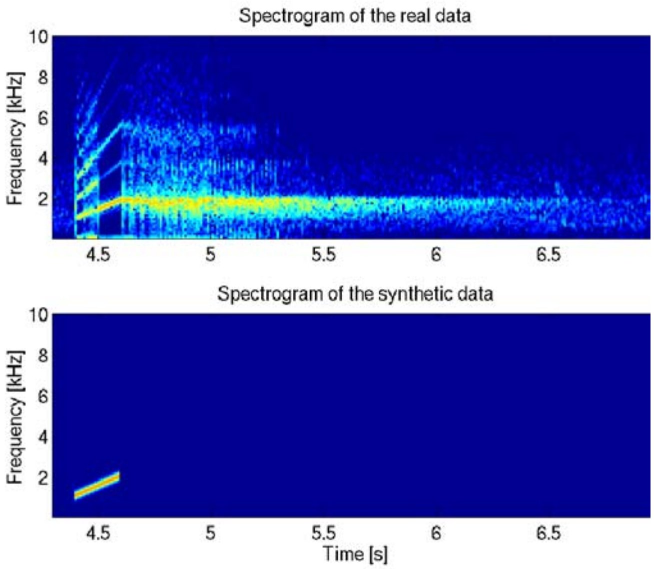

Figure 4.8 Spectrogram for real and synthetic data. The pulse beginning at 4.2 seconds is a 1-2kHz chirp, with a duration of 0.2 sec.

V. CONCLUSIONS AND DISCUSSION

For a number of cases the FrFT does outperform the traditional methods of detection and time/frequency analysis. Especially at times when there are large changes in the frequency content of the signals, but the chirp rate is unaffected. However, looking at the real data one finds that there is indeed very little difference between the two methods.

The time/frequency surfaces produced using STFrFT are in general better according to the *Ad* measure. Typically the difference between the STFrFT and spectrogram is in the order of 1 to 10 in the favor of the STFrFT. However, if the goal is solely set on producing an image with the greatest dynamic range the Spectrogram is better. The difference might seem large but in the end trying to quantify the difference between two images is a difficult task since the only thing that really matters is what it looks like. After all, the human eye is a superior tool for image processing.

Acknowledgments

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