Model supported AUV freshwater distribution surveys in the upper layers of deep fjords

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Abstract: The Trondheim Fjord is characterized by a high degree of stratification, considerable density variations and freshwater fronts important to the vertical mixing of freshwater and nourishments, and thus significant to the fjord system. These conditions are modelled in a numerical model. Two AUV surveys were conducted in the Trondheim Fjord, Norway in June 2001 and April 2002 to complement and validate the numerical simulations.

The Danish AUV Maridan 150 was used in the operations. Salinity, temperature, and current data were necessary to investigate the validity of the numerical models. A CTD unit and ADCPs pointing in both vertical directions formed the AUV payload. In the first survey the vehicle traversed the fjord to produce horizontal density and current profiles. Observations made during these runs placed focus on fresh water front structures, which were investigated more closely in the subsequent survey. The paper presents the survey set-up and execution along with experiences made underway.

I. INTRODUCTION

A. The MODTEQ programme

To bridge the gap between marine science and marine engineering, the Norwegian University of Science and Technology (NTNU) in Trondheim initiated the programme; "Model-based development of advanced marine equipment" (MODTEQ) [1]. The programme aspires to establish permanent axes for marine collaboration by forming cross-disciplinary teams for exchange of marine scientific knowledge, expertise and research facilities. Researchers from the disciplines of oceanography, biology, and marine engineering participate in the programme.

B. The Trondheim Fjord

The physical oceanography of the Trondheim Fjord is a central research subject of the MODTEQ project. It is an environment that is complex and challenging to investigate. The fjord receives a substantial amount of freshwater mainly from the six largest rivers; Orkla, Gaula, the Nidelva, the Stjørdalselva, the Verdalselva and Snåsavassdraget, see Fig 1. Totally these rivers have a mean annual runoff of 725 m^3s^{-1} and a spring flood maximum of up to 6,430 m^3s^{-1} [2]. There are roughly two tidal cycles each day with a mean difference in water level of 1.6 m.

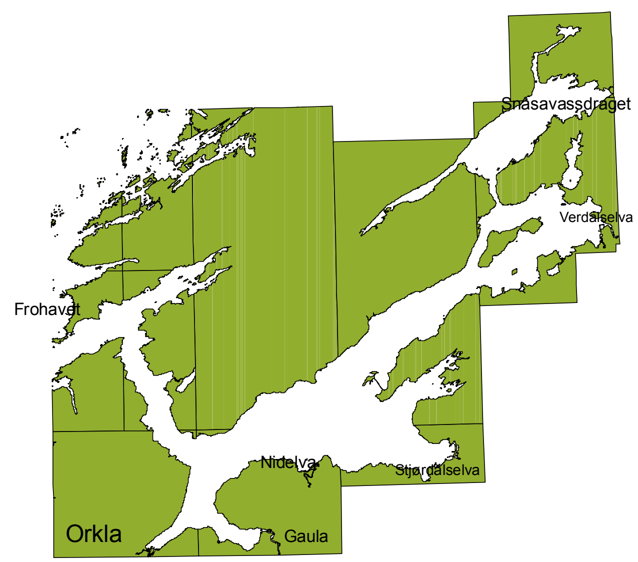

Fig. 1 Map showing the Trondheim Fjord, the six main rivers and Frohavet.

Primary production is highest in the few upper meters of the fjord and is thus affected by river discharge. The rivers bring humus and silt which absorbs light and therefore makes the photic zone thinner and reduces the production rate. Furthermore, the stratification due to river discharges increases vertical stability. The consequently reduced vertical mixing causes a bloom of phytoplankton in early spring when the light is a limiting factor. However, the reduced vertical transport of nutrients will limit the primary production.

The area including the Orkdal Fjord, the Cross Fjord and Gaulosen, see Fig. 2, was chosen to restrict our research area, and because it is a highly interesting region considering river discharges. This part of the fjord is dominated by two rivers; Orkla and Gaula. The discharge from these rivers forms distinct river plumes that meet in the Cross Fjord.

There exist several studies of the Trondheim Fjord. A summarize of the insight into the physical oceanography in the Trondheim Fjord gained can be found in [3] and [4].

With exception of the studies in the Orkdal Fjord there are a few studies in the Trondheim Fjord that focus on the distribution of discharged freshwater from rivers. Our aim was therefore to study this further, using high-resolution numerical simulations combined with *in-situ* measurements. Modellers in the MODTEQ team have thus established a numerical model of the fjord, using realistic topography, tides, wind, river discharge and heat flux. The model used is The Navy Coastal Ocean Model (NCOM), ([4] and [5]).The data obtained during the surveys provides useful understanding about the dynamics of the upper layer in an area.

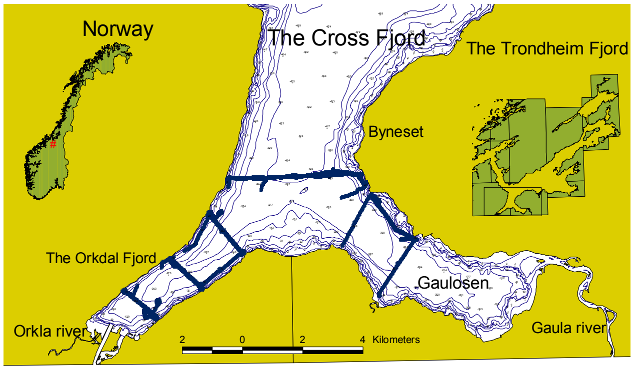

Fig. 2 The Cross Fjord with the Orkdal Fjord and Gaulosen.

C. AUVs in physical oceanography

AUVs has been utilised in physical oceanography surveys in other projects known from literature, such as the Haro Strait Experiment [6] where an Odyssey class AUV was used to map coastal fronts in the Haro Strait, BC. The interdisciplinary Cape Cod Bay project [7] conducted by Woods Hole Oceanographic Institution aimed at chart the biomass patchiness in Cape Cod Bay. The Autosub team has performed a survey of the current regime in the Sicily Strait [8]. The MODTEQ team wanted to find out if the AUV technology had matured enough to be an efficient tool for surveying complex conditions such as the Cross Fjord.

II. EQUIPMENTS AND METHODS

A. NCOM

The Navy Coastal Ocean Model (NCOM) model is a parallelized 3-D, hydrostatic, primitive equation model developed at the U.S. Naval Research Laboratory ([4] and [5]) and was used for simulations of the Trondheim Fjord area. The model uses hybrid vertical coordinates with sigma layers near the surface and z- level below a specified depth. This way the NCOM can be configured for the Trondheim Fjord with high vertical resolution in the surface and at the same time avoid problems that may occur when sigma layers are used in combination with steep bathymetry. The model is thus set up with 15 sigma layers down to a depth of 6 m. The upper layer is approximately 4 cm and the layer thickness increases linearly to 1.5 m. From 6 m to a maximum depth of 614 m, 25 z-levels are employed. The model is further configured to cover the whole Trondheim Fjord and the horizontal grid spacing is 100 m. This gives a total numbers of degrees of freedom of order $O(10^9)$ and 16 processors at a SGI Origin 3800L are required to run the model.

The Trondheim Fjord model has an open boundary towards the Frohavet. At this boundary five tidal components are specified to simulate the tides in the fjord. The model is further forced by wind, heat-flux and a total of six river discharges.

B. AUV

An AUV was employed to perform high-resolution in-situ measurements of the physical parameters in the Cross Fjord area. The aim of the AUV measurements was to complement and validate the numerical simulation results.

The AUV measurements were conducted from the Danish AUV M-150, rented from Maridan A/S. The vehicle is about 4.5 m long and weighs about 1000 kg dry. Storing 10 kWh of energy the vehicle is capable of 2 ms^{-1}. This normally allows 6 – 8 hours of operation depending on the intensity of the operation. Navigation is conducted using Maridan's own system MARPOS [9] based on Kearfott SEANAV [10] inertial navigation plattform and an 1200 kHz RD Instruments Doppler velocity log. The AUV contains a payload bay of 40 cm x 60 cm x 60 cm where the ADCP sensors were fitted during these operations.

Underwater the vehicle is completely autonomous and there is no communication with the surface. The vehicle acts according to a pre-programmed survey plan file. On deck communication with the vehicle is performed using an Ethernet cable, and in surface position survey plan files [11] and AUV log files can be downloaded and uploaded using wireless Ethernet. Manoeuvring can be controlled using a control box on the surface for e.g. launch and recovery situations.

C. ADCP

The AUV was in 2001 equipped with two ADCPs pointing in both vertical directions: a 300 kHz RDI Workhorse pointing downwards and a 500 kHz NORTEK pointing upwards, see Fig. 3. In 2002 only one RDI 1200 kHz pointing upwards was mounted.

The 300 kHz unit collected data in 2 m vertically bins underneath the vehicle while a 500 kHz unit profiled the volume between the vehicle and the surface in 1 m bins. In April 2002 the AUV had only one RDI 1200 kHz Workhorse ADCP looking upward through 25 cm bins as the vehicle was manoeuvred underneath density fronts. Special software was designed to transform the measured current velocities into an Earth-bound frame of reference. The 300 kHz unit were e.g. set to ping rates and averaging intervals that produced an along-track resolution of about 27 and 80 meters at 30 and 100 meters depth respectively, at an average vehicle speed of 1.5 ms^{-1}.

Fig. 3 The M-150 AUV with two ADCP and the CTD sticking out of the hull. The NORTEK ADCP is on top of the AUV, while the RDI unit can be seen in the payload bay.

D. CTD

The internal CTD of the vehicle (a Falmouth 2" mini-CTD) was used as payload in the operations. To ensure the most reliable results the unit was mounted with the sensor head pointing out of the vehicle.

E. Survey execution

In June 2001 the first AUV cruise was organised in the Gaulosen, the Cross Fjord and the Orkdal Fjord. The vehicle was programmed to obtain cross-sectional transects on 0, 2, 5, and 10 meters depth of the area, see Fig. 2. The design of the survey plans were based both on the numerical simulations and previous observations, but also on recently downloaded AUV CTD data, readings from the ship-mounted CTD and visual observations of wind, tides and currents. During the cruise in April 2002 the vehicle was programmed to zigzag across density fronts. These fronts were located with a ship-mounted CTD and by visual observations. The vehicle executed survey plans based on geographic fixed way-points. Regular trips to the surface were necessary to update the inertial navigation system with a GPS fix.

The operations were organised from a 60 foot twin hull vessel, M/S Vita. The ship was equipped with a small hydraulic crane for launch and recovery. A ship mounted CTD was installed to be able to locate and confirm density fronts before the AUV was launched. During the 2002 survey, an ORE LXT USBL system was used to track the AUV during operations, for safety and operational efficiency as a guiding instrument. The USBL measurements were, however, too noisy to be of any value to the AUV navigation.

F. Remote sensing

To be able to get an overview of the fronts in the fjords, aerial pictures were taken from a small plane. Foam strikes created by density fronts could easily be spotted from an aerial photo. In some instances we could also observe differences in colour for fresh- and seawater. On a nearby hill a surveillance camera were placed at 300 meters above sea level to provide time series of the situation. The camera took pictures every three minutes for several weeks to identify surface signature of freshwater fronts. These pictures were assembled to movies in order to reveal trends in the surface signature of the oceanographic structures investigated.

III. RESULTS

A. The effect of the Coriolis force in the Orkdal Fjord

The AUV cruises altogether resulted in CTD- and ADCP data from about 85 km of survey trajectory. The CTD data revealed large gradients in salinity and hence density, both vertically and horizontally. These gradients were related to frontal boundaries separating different water masses. Generally, the surface layer in this region has three distinct water masses, the brackish water, the Orkla River plume and the Gaula River Plume. In the Orkdal Fjord we found salinity differences of 3-4 units at 1 m depth over a distance less than 100 m, see Fig. 5. The vertical profile reveals a highly layered structure. There were two sharp haloclines approximately at 1 m and 3 m depth. Across the latter, the salinity was found to increase by 6-8 units over a vertical distance less than 1 m. The shallower layer at 1 m depth is not found along Section 2. Furthermore, the upper layer along this section was well mixed, and its thickness was greater at its south -eastern side. The plume was thicker at this side, which was also seen from the horizontal salinity profile taken at 1 m depth in Section 1. This indicates a turning to the right of the plume, as would be expected according to the Coriolis forcing of the plume.

The shallow fresh layer found in Section 1 indicates a discharge of fresh river water that outflows over a more dense mixture of previously discharged river water and the more saline fjord water. The horizontal profile taken along Section 1 at 1 m is thus taken at the interface between these two layers. This explains the high variation in salinity across this profile, along in which water from both layers was observed. Variation can thus be due to both variations of plume thickness and interfacial waves.

The freshwater fronts were further characterised by strong convergence currents forming bands of foam and debris on the surface, see Fig. 5. An approximately position of this front is indicated by the grey line. The photo of the front was taken at the same time as the horizontal profile along Section 2 was taken.

The extension and structure of the plume naturally depend on external forcing present at the time of observation, like: tide, wind, flow rate in the river etc in addition to the Coriolis force. During the survey in 2001, wind was believed to be of importance. This was also indicated by the well mixed layer of 2-3 m depth observed in the Orkdal Fjord.

The effect of the Earth's rotation on the outflow of fresh water from the Orkla River in the Orkdal Fjord was also found by McClimans, [12]. To investigate these findings further, experiments were performed in a 5 m diameter rotating tank, ([13] and [14]). Results from the latter work have been successfully compared qualitatively with a similar setup using NCOM. Both methods showed that under influence of rotation, the jet issuing from the river mouth diverts to the right, i.e. it crosses over from one side to the other of the Orkdal Fjord, consistent with the results presented in Fig. 4.

The diverted jet leaves a backwater of anti-cyclonic circulation. The plume further progresses as a density driven coastal current in the direction of Kelvin Wave propagation. The Coriolis force constrains this current to the coast and its width is found to be comparable with the baroclinic Rossby radius and the jet will turn according to the inertial radius ([13] and [14]).

Another baroclinic coastal current is generated in Gaulosen. In the Cross Fjord we then have the situation where two such buoyancy driven coastal currents meet. The coastal current issuing from the Orkdal Fjord is prevented to continue its propagation into Gaulosen. The coastal current generated in Gaulosen is generally less dense and forces the Orkla plume current to submerge. These two then create the coastal current that follows the east side of the fjord, see Fig 2. As this current progress, growth of cyclonic frontal eddies is observed, see Fig. 4. Cyclonic eddies in this part of the fjord were previously found by Jacobson [3]. Studies of this type of instability of buoyancy driven coastal currents can be found in [15]

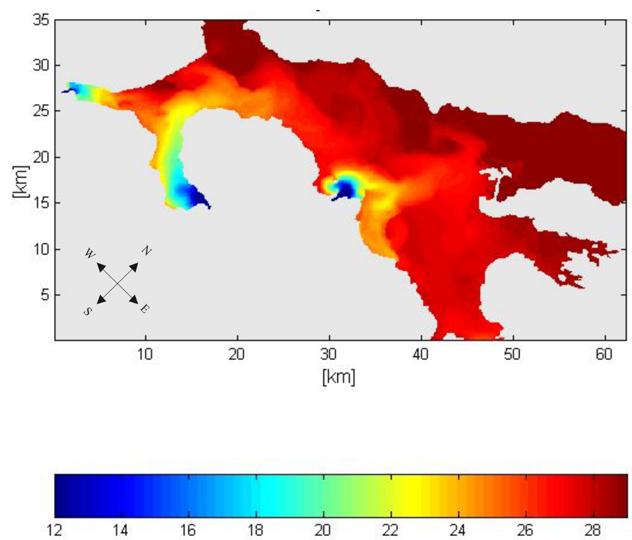

Fig. 4 Surface salinity in the central basin of the Trondheim Fjord in April 2002 simulated with NCOM. The Snap shot is taken at ebb.

B. Influence of tides on the outflow of fresh water from Gaulosen and the Orkdal Fjord

Based on previous observations from the area including Gaulosen, the Orkdal Fjord and the Cross Fjord, we anticipated that the tides were of minor importance to the outflow of freshwater in the region [3]. This was also seen from model results in the case of zero vertical density gradient. The barotropic velocities were only up to 10% of the velocity of the density driven river plume. (This was found applying mean river discharges of Q =100 m^3s^{-1}). However, results from the two surveys in 2001 and 2002 showed that the tides did influence the outflow of fresh river water. The tidal effect also became evident in model results when a density gradient presentable for the situation in April 2002 was introduced in the model setup. These results revealed an internal Kelvin wave propagating around the central fjord basin. The signal in surface elevation of this internal wave is shown in Fig. 5. The phase speed of the internal wave was c_i = 0.74 ms^{-1} and the baroclinic Rossby radius, r_o = 4 km. The period of the wave was approximately 12 h.

Note the resonance of this wave in the Orkdal Fjord. The resonance phenomenon seen here enhances the surface currents in the fjord. This effect puzzled us for a while. Incidentally, in one of our simulations the phase of this internal wave enhanced outflow in the surface during rising tides, which was seemingly a paradox, considering the inflowing barotropic tide. The opposite situation was observed during ebb. The pile up of freshwater in the Orkdal Fjord is seen in Fig. 4.

C. Frontal structures in the Cross Fjord

Visual observations, such as photographs taken from a plane and from the time-series surveillance camera at Byneset, showed the complexity of the surface layer in the area. For instance, lines of convergence that may be attributed to internal bores was observed. Convergences along density fronts were also identified by the formation of cusps that point toward the less dense water along the frontal line. We will here show an observation of a feature from air photography that has been reproduced in model simulations. It consists of two crossing convergence lines. It actually resembles the shape of a cross, and it is observed in The Cross Fjord[1] see Fig. 4a). The cross feature is found in the middle. A characteristic freshwater front with cusps related to the frontal boundary of the baroclinic coastal current around Byneset described earlier, can also bee seen. Fig. 4 b) shows the horizontal divergence calculated from simulation results for the surface. Negative values indicate here surface convergence. There are further complex patterns of divergence and convergence that are due to wave interactions. The generation of a cross is due to the interaction between the Orkla River plume and the Gaula River plume. Three of the convergence lines forming the cross are related to horizontal density fronts. The fourth line, the one in the lower left corner of the two pictures, is in connection to an internal bore generated when the two currents collide. This internal bore will eventually propagate away from the area, leaving behind a Y-shaped structure. The phenomenon is observed both from visual observations and in model simulations.

During a survey with a 28 foot boat, Havfruen, in October 2001, another frontal structure in the Cross Fjord was traced by measuring salinity across two observed foam fronts at 0.5 m depth. The shape and location of the two fronts are again found to be consistent with model results.

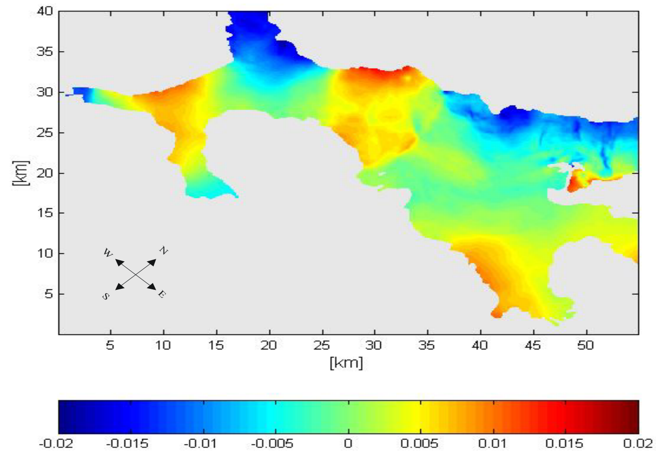

Fig. 5 An internal wave in the Trondheim Fjord shown from signal in elevation from simulations with NCOM at ebb in April 2002. The mean elevation is subtracted and the colour scale is in [m].

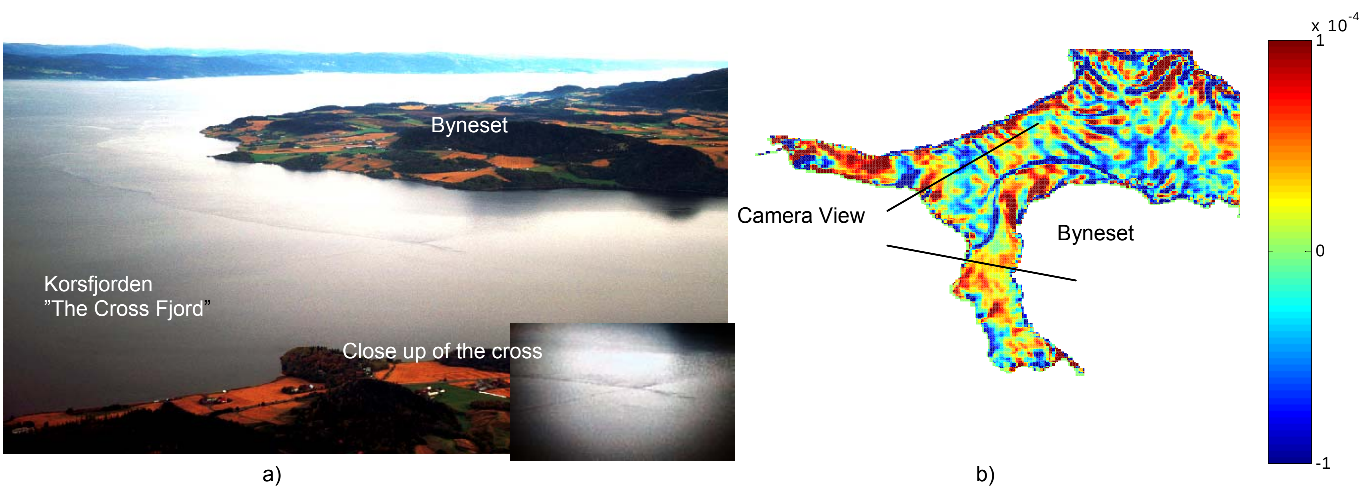

Fig. 4 a) Photography from the area covering parts of the Cross Fjord. The mouth of Gaulosen is to the right. b) Horizontal divergence calculated from simulation results. The cross is seen further out in the Cross Fjord than in the picture displayed in a).

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[1] The name of the Fjord is literally translated from Norwegian, i.e. it is not named after our observation

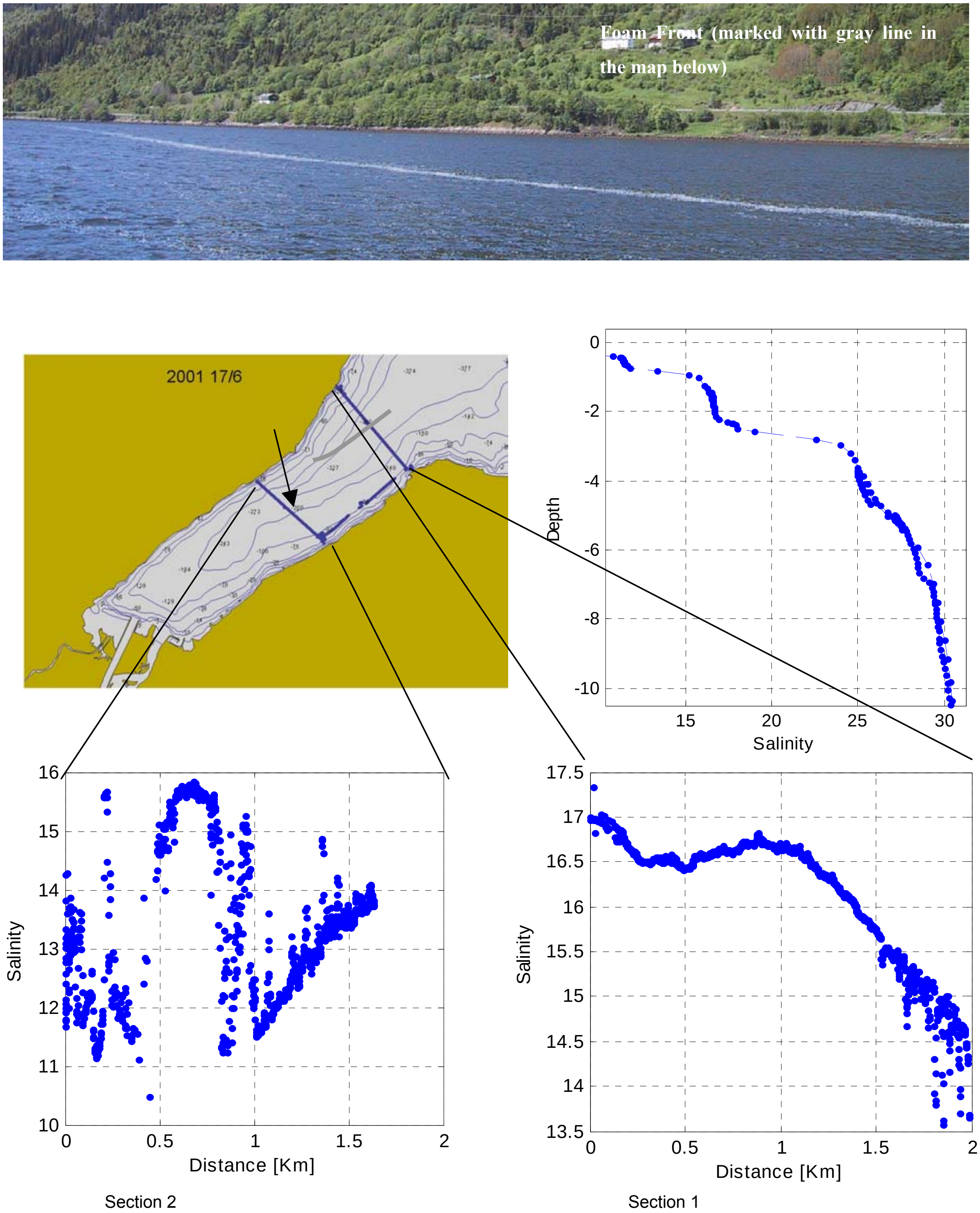

Fig. 5 The figure show results from the AUV survey in the Orkdal Fjord in June 2001. The two horizontal profiles of salinity are taken at 1m depth along the lines given in the map and within the positions given by the black lines

IV. DISCUSSION

A. Quality and reliability of AUV-data

A fundamental compromise in all marine surveys is coverage versus resolution. In surveys of dynamic oceanographic processes, timing and survey endurance becomes important parameters. The survey endurance should be short relative to the time constants of the process investigated and the survey track should be as long as possible to provide resolution. AUVs have a finite amount of energy stored, and as the drag resistance of a vehicle increases with the square of speed, high speed and long range is hard to combine. The requirement of synopticity must thus be relaxed using the present available AUV technology. Data points spread out in time induces inaccuracies in the sampled data-set [16]. In our operations this reduced the quality of the data. The data had to be viewed as sub sets of information due to the duration of the operation. The error due to this kind of latency in the data set is not yet quantified.

In our surveys the requirements for accuracy in positioning of the data points were not as strict as in most seabed surveys due to the 100 m horizontal resolution of the NCOM simulations. For absolute positioning the MARPOS navigation system used was sufficient, but the ADCP posed important requirements for the navigation system. The quality of the gathered current profiles was directly dependent on the quality of the velocity and attitude information for the AUV. During our operations the AUV had to operate in "water track" mode, clearly posing inaccuracies to the DVL measurements due to the dynamic water masses. Together with the drift in the inertial platform, this posed inaccuracies in heading and velocity measurements, which further reduced the quality of the resulting the ADCP profiles.

Compensation for INS-drift in the MARPOS system could have improved the accuracy in our operation. A low-frequency ADCP/DVL capable of obtaining bottom track in the deep Cross Fjord could have been used for this purpose. Another alternative is tracking the vehicle with an acoustic baseline system as described in [17].

To be able to quantitatively validate the model from the AUV data set, the modeller has to be able to recognise oceanographic features and quantities in the AUV data set. Even though the total data collected during the 2001 survey is large, it is widely spread in time. The data therefore consist of subsets that can be difficult to interpret. Furthermore, during the 2001 survey, wind was an important factor. We did not register wind data from the area that later turned out to be necessary for a realistic simulation. In 2002 wind gauges were put out in the area, but during the three weeks that these were out, there were not wind. We are currently working with simulations for this period, and the results are not yet ready.

However, qualitatively we have observed similar features in both the AUV data set and in the results from the numerical simulations. These features were also observed from the time-series surveillance camera and from aerial photos. When a phenomenon was identified from both observations and simulations, the latter was used for further investigations. Totally, this has provided further insight to the complex nature of the Cross Fjord area. In future surveys of this or similar areas, the new knowledge may enable us to improve survey plans, and then more efficiently produce data that are more suitable for quantitative model validation.

B. Improvement of AUV-survey efficiency

An oceanographic survey in such complex and dynamic environment has a demanding requirement; the measurement should be as synoptic as possible. To perform an efficient investigation of density fronts and river plumes with an AUV one has to design the AUV-trajectory with care. It is important to use the available run-time in the most interesting areas.

Survey plans based on geographic waypoints is inefficient. Oceanographic structures move continuously, and the survey plans became obsolete quickly. Adaptive survey plans would thus be beneficial. An algorithm could e.g. adjust the survey track to follow a density front. This will considerably increase the overall survey efficiency. Adaptive survey plans have been used with success in surveys of a scalar chemical field [18].

To increase efficiency a survey should be organised in few long survey plans. Each time a plan was completed valuable time is lost to download results and a new plan and to check the system. But considering the changing environment it may be risky to use long plans as they may be obsolete, and the survey may be missing the process of interest.

There were also operational issues decreasing survey efficiency. It proved challenging to navigate the AUV close to the surface in highly varying density. When the AUV crossed a density front the buoyancy of the vehicle changed. A density front normally sets up a hydrodynamic rotor close to the convergence line which implies change in vertical velocity when crossing the front with an AUV. This rotor affects the vertical stability of the vehicle. In some instances the rotor made our AUV oscillate in the vertical plane. Robustness of vertical control is thus important in this kind of survey.

V. CONCLUSION

The measurements obtained from the AUV survey showed qualitative consensus with the numerical simulations. The patterns observed in the AUV data-sets were consistent with pictures of the Cross Fjord area.

With the described surveys, we have shown that AUV systems have a large potential in operational oceanography. To be cost efficient in this type of operations, the technology has to be specifically adapted to meet the strict time constraints and the dynamics of the water masses. As discussed, AUV-mounted ADCPs set certain requirements to the navigational system of the AUV. Adaptive survey plans is believed to increase the efficiency in mapping of oceanographic structures considerably. Density gradients and convergent water masses sets strict requirements to the vertical robustness of the control system applied in the AUV.

The modifications necessary to increase the cost efficiency considerably in the described operations are small. By refining the present AUV-technology, efficiency in certain types of operational oceanography could be significantly improved, which would make AUVs an important tool in this type of research.

ACKNOWLEDGMENTS

The authors want to thank S. L. Morey and J. J. O'Brien at COAPS, Florida State University, for introduction to and assistance with NCOM, and P. J. Martin at U.S. Naval Research Laboratory for the development of the model. Part of the configuration of NCOM for the Trondheim Fjord was performed at COAPS during spring 2002. The authors also want to thank Bo Krogh in Maridan for his efforts during the cruises. The project is financed through The Norwegian Research Council.

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