Spectral Feature-aided Multi-Target Multi-Sensor Passive Sonar Tracking

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ABSTRACT

Passive sonar systems typically provide target bearing estimates that are a nonlinear function of the target state. Multiple state Gaussian Sum extended Kalman filter (EKF) and particle filter (PF) approaches have been combined in a previous multiple hypothesis tracking (MHT) architecture to improve state estimation on bearings-only data. Bearing-only measurements also introduce difficulties in data association as a result of uncertainties, ambiguities, and multiplicity of contacts on a common bearing. In this paper, we shall present an approach to improving data association and filtering by exploiting passive narrowband (PNB) spectral features. The paper identifies the PNB spectral feature target state extension, introduces a Gaussian sum frequency state model, and defines extensions to the likelihood calculations needed for improved data fusion across both kinematic and frequency domains. The track likelihood is based on both kinematic and spectral feature likelihoods. Frequency domain fusion extensions are shown to fit seamlessly into a current MHT architecture.[1]

1. INTRODUCTION

Navy objectives for the Acoustic Rapid COTS Insertion (ARCI) Advanced Processor Build (Tactical) (APB(T)) Program, as well as the future Common Undersea Picture (CUP) and related initiatives, require significantly improved data fusion and scene understanding capabilities relative to the current, primarily single algorithm kinematic approaches. Significant opportunities exist to improve data fusion through exploitation of additional types of contact information. One such example is an acoustic contact's frequency signature characteristics.

This research identifies an approach to joint fusion of kinematic and frequency information from acoustic contacts. This paper defines a parametric model for a contact's passive narrowband (PNB) spectra and develops the extensions of the track state to include PNB frequency data. The frequency state is shown to be appropriately represented via a Gaussian sum statistical model. Appropriate expressions are defined to associate contact PNB data to tracks and to updated tracks as a result of these data associations. The developed approach can be extended to non-parametric spectral representations, and fits seamlessly into an existing Multiple Hypothesis Tracking (MHT) architecture.

This paper is outlined as follows. Section 2 describes an existing MHT fusion architecture, and shows how complementary non-kinematic domain information can be fused with kinematic data within an open architecture. Section 3 characterizes PNB frequency domain data streams as a key non-kinematic domain for passive sonar fusion applications. Typical frequency data types are identified and related to the fusion architecture. Section 4 defines the statistical model applicable to the frequency domain state representation, and derives expressions for associating frequency measurements to tracks and for updating frequency domain state variables. Section 6 summarizes the research.

2. A MULTI-HYPOTHESIS, MULTI-DOMAIN FUSION ARCHITECTURE

A current implementation [1,2] of a multi-hypothesis, multi-domain fusion architecture, designated as MultiStar, is represented in Figure 1. The Fusion Input Processor (*FIP*) provides measurement scans from the reporting sensors. The Fusion Output Processor (*FOP*) provides the interface to output tracks, e.g., to the display system. The heart of the MHT architecture is the multiple-hypothesis fusion engine (*MHFE*), which performs the fusion management functions within the multiple-hypothesis environment, including: contact data clustering, hypothesis generation and management, and direction of evaluator processing.

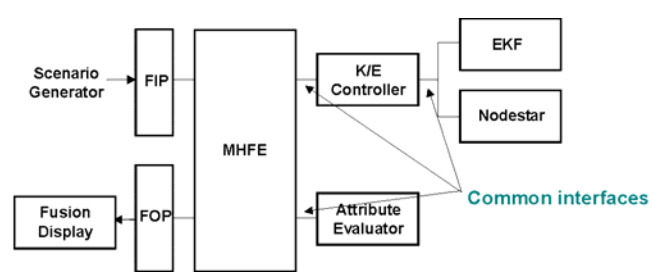

Figure 1. MultiStar Open Architecture

[1] This work was sponsored by the Office of Naval Research under contract # N00014-03-M-0025.

To the right of the MHFE are the modules forming a suite of data domain level *evaluators*. Evaluators perform domain-dependent state estimation and track-measurement association scoring, as directed by the MHFE. To allow for relatively easy incorporation of various evaluator algorithms from multiple suppliers, a standardized interface was established between the MHFE and the evaluators. This flexible design allows the addition of other state estimation and correlation domains into the system. The Kinematic Evaluator (KE) function estimates each track's kinematic state (position and velocity) and computes kinematic association scores among measurements and tracks. The MHT architecture permits multiple KE algorithms to be implemented and executed when appropriate. The multiple algorithm capability has been demonstrated via two kinematic fusion algorithms, as shown in Figure 1. The EKF module models target state probability densities with Gaussian sums, and estimates the kinematic state with an Iterated Extended Kalman Filter (IEKF). The Nodestar module, supplied by Metron, Inc. [3], employs a non-linear particle filter tracking paradigm. The KE Controller determines which of the two kinematic fusion algorithms are executed based on characteristics of the measurements and tracks potentially being fused.

In a similar manner, the architecture allows for easy addition of new domain Attribute Evaluator algorithms. Attribute Evaluators exploit additional domain measurement information in a manner different from the KE. Examples of possible Attribute Evaluators might include a

- Classification Evaluator – assess consistency of track and measurement classifications (when provided), and estimate the fused track classification state,
- Environmental Evaluator – assess the likelihood of a passive sonar return on a given bearing based on bathymetric or environmental noise characteristics,
- Frequency Evaluator – assess the consistency of the measurement frequency content with the estimated track spectral state, and estimate the fused track frequency state.

The Frequency Evaluator concept is of primary interest to the current research. Development of new evaluators is straight-forward in the above architecture in that the evaluators are only required to perform two functions (with well-defined application programming interfaces (APIs)):

- Determine the association likelihood that a given track gave rise to the given measurement based on attributes from this domain, and
- Estimate/update the subset of track state components observed via attribute measurements within this domain, given the new measurement has been associated with the given track.

3. FREQUENCY ATTRIBUTES FOR ENHANCED DATA FUSION.

Frequency estimates have been successfully exploited within the Advanced Deployable System (ADS) Field Processor to reduce the complexity of the measurement-to-track association problem. For example, Figure 2 illustrates the reduction in the number of lines of bearings (green lines) that must be considered as possible associations with an existing track when only bearing measurements are considered (top), and when both bearing and frequency measurements are considered (bottom).

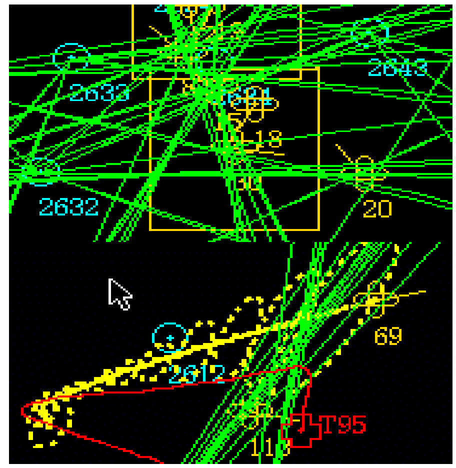

Figure 2. The ADS Field Processor demonstrated the reduction in association complexity through use of narrowband frequency consistency. Top: possible bearings-only associations; Bottom: possible associations based on bearing and frequency attributes, with resulting track (yellow).

The ADS implementation only exploited frequency information as a means of gating measurement-to-track associations across sensors. The track states did not include frequency components estimated over time. Further, association gating was simply based on a narrowband frequency component M-of-N matching test – the N_m components forming a "platform set" and representing a new measurement were compared with the N_t components in other sets already associated with the track under test. Despite its simplicity, this "evaluator" approach resulted in significant performance improvements via simplification of the data association problem. Additional association and state estimation improvements are likely possible far beyond those demonstrated in the ADS Field Processor.

Recent improvements in ARCI Contact Follower (CF) technology, coupled with interest in Spectral Histogram Probabilistic Multi-hypothesis Tracking (Spectral H-PMHT) research applied to contact following [4], suggests that both discrete PNB frequencies as well as the complete spectrum are useful as attributes. These two processing options are shown in Figure 3. A parametric model is formed by approximating the spectral state via a fixed number of components characterized by narrowband center frequencies, bandwidths, and amplitudes. The representation uncertainty is modeled statistically via a Gaussian sum probability density representation, described below (Section 4). This spectral representation is efficient when the contact's spectrum is dominated by a few narrowband components. For broadband contact spectra, a complete sampled probability density function (PDF) representation [5] or a particle filter representation (as in [3]) is likely required. The ability of the architecture of Figure 1 to handle both Gaussian Sum and Bayesian PDF representations permits appropriate representation of frequency information in the track state regardless of the method of generation of the frequency attributes. Note, however, that we will only address the Gaussian Sum parametric representation in this paper.

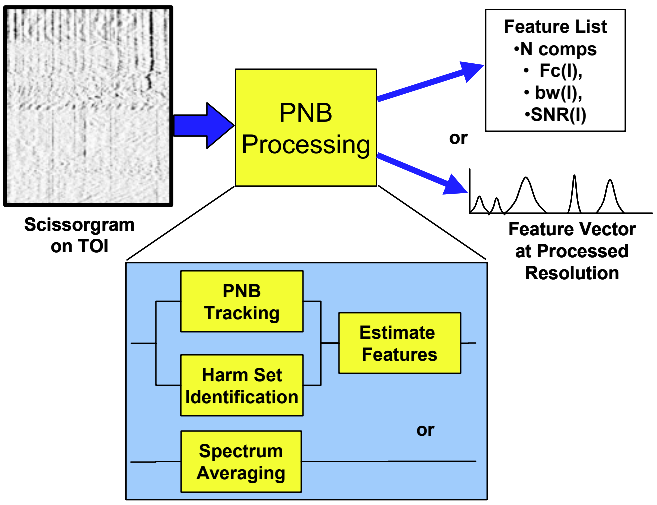

Figure 3. Generation of Contact Frequency Attributes.

The kinematic and frequency attribute information can be easily combined under an independence assumption to make appropriate joint-likelihood-based data association decisions. The likelihood $\Lambda(\mathbf{z}|\mathbf{x})$ of measurement $\mathbf{z}$ conditioned on state $\mathbf{x}$ can be partitioned into kinematic and frequency components, that is,

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_f \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{z}_k \\ \mathbf{z}_f \end{bmatrix}, \tag{1}$$

where $\mathbf{x}_k$ is the kinematic state vector component, $\mathbf{x}_f$ is the frequency state vector component, $\mathbf{z}_k$ is the kinematic measurement component, and $\mathbf{z}_f$ is the frequency measurement component. If the kinematic and frequency components are assumed to be statistically independent, then

$$\begin{aligned} \Lambda(\mathbf{z}\,|\,\mathbf{x}) \propto p(\mathbf{z}_k, \mathbf{z}_f \,|\, \mathbf{x}_k, \mathbf{x}_f) &= p(\mathbf{z}_k \,|\, \mathbf{x}_k)\, p(\mathbf{z}_f \,|\, \mathbf{x}_f) \\ &= \Lambda(\mathbf{z}_k \,|\, \mathbf{x}_k)\,\Lambda(\mathbf{z}_f \,|\, \mathbf{x}_f) \end{aligned}. \tag{2}$$

Thus, the optimal measurement-to-track joint kinematic and frequency association likelihood score is simply the product of the individual kinematic and frequency domain likelihood scores. This is represented conceptually in Figure 4. Relaxation of the independence assumption between the frequency and kinematic domains is the subject of on-going research.

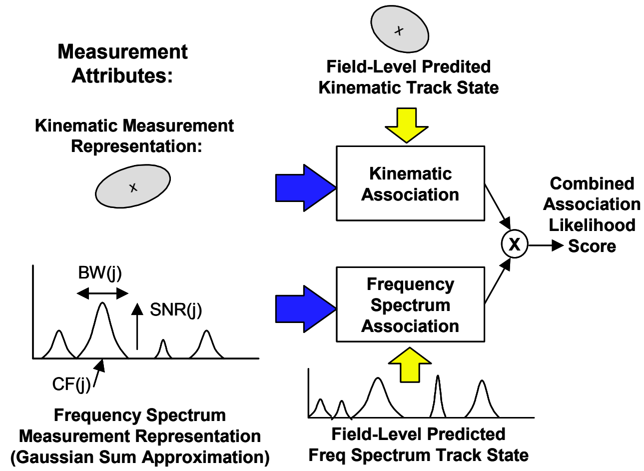

Figure 4. Combination of Kinematic and Frequency Attribute Information.

4. FREQUENCY STATE REPRESENTATION VIA GAUSSIAN SUMS

We now define the frequency domain expansion of the state vector and motivate its statistical representation. Assume a track state vector $\mathbf{x}$ and measurement vector $\mathbf{z}$ as in (1). The track state can be estimated over time based on observations according to the usual state variable and observation models; that is, at time t_k,

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{\Phi}(k)\mathbf{x}(k-1) + \mathbf{w}(k) + \mathbf{\Psi}(k)\mathbf{u}(k), & \mathbf{w}(k) &\approx N\big(\mathbf{0}, \mathbf{Q}(k)\big) \\ \mathbf{z}(k) &= \mathbf{h}\big[\mathbf{x}(k)\big] + \mathbf{v}(k), & \mathbf{v}(k) &\approx N\big(\mathbf{0}, \mathbf{R}(k)\big) \end{aligned}. \tag{3}$$

Here, $N(\boldsymbol{\mu},\Sigma)$ signifies a Gaussian-distributed random vector with mean $\boldsymbol{\mu}$ and covariance matrix Σ.

The state vector $\mathbf{x}$ is a random vector. The *a posteriori* density $p(\mathbf{x}(k)|\mathbf{z}(i), i=1,\ldots k) = p(\mathbf{x}(k)|\mathbf{Z}(k))$ provides the most complete description of the state vector [3,6], and leads to maximum likelihood association and Bayesian filter formulations. This density is determined recursively [6] from the relationships

$$p(\mathbf{x}(k)\,|\,\mathbf{Z}(k)) = c_k \;\; p(\mathbf{x}(k)\,|\,\mathbf{Z}(k-1)) \;\; p(\mathbf{z}_k \,|\, \mathbf{x}_k) \tag{4a}$$

and

$$\begin{aligned} & p(\mathbf{x}(k)\,|\,\mathbf{Z}(k-1)) \\ &\quad = \int p(\mathbf{x}(k-1)\,|\,\mathbf{Z}(k-1))\, p(\mathbf{x}(k)\,|\,\mathbf{x}(k-1))\, d\mathbf{x}(k-1) \end{aligned}, \tag{4b}$$

where the normalizing constant c_k is given by

$$\begin{aligned} (c_k)^{-1} &= p(\mathbf{z}(k)\,|\,\mathbf{Z}(k-1)) \\ &= \int p(\mathbf{x}(k)\,|\,\mathbf{Z}(k-1))\, p(\mathbf{z}(k)\,|\,\mathbf{x}(k))\, d\mathbf{x}(k) \end{aligned}. \tag{4c}$$

Under Gaussian assumptions used in the Kalman Filter formulation, both $p(\mathbf{x}(k)|\mathbf{Z}(k-1))$ and $p(\mathbf{z}_k|\mathbf{x}_k)=p(\mathbf{z}(k)|\mathbf{x}(k))$ are Gaussian due to Gaussian process and observation noise, linear measurement assumptions, and a Gaussian assumption on the initial state vector. Under more general assumptions such as that of the Nodestar particle filter approach, $p(\mathbf{x}(k)|\mathbf{Z}(k-1))$ and $p(\mathbf{z}(k)|\mathbf{x}(k))$ are arbitrary distributions.

The target state space can be extended to include frequencies associated with the target's passive acoustic spectra in several ways. We consider a parametric approach assuming a small set of narrowband components, that is, we can assume the target frequency state will consist of a small set of observable tones with observation center frequencies f_n, bandwidths b_n, and signal-to-noise ratios (SNRs) γ_n. Let the frequency state space consist of a number of narrowband center frequencies

$$\mathbf{x_f} = [f_1, \ldots, f_N]^T. \tag{5}$$

Each center frequency is accompanied by a set of auxiliary parameters describing additional known (or estimated) attributes of the frequency states. Let b_n and γ_n represent the bandwidth and the signal-to-noise ratio, respectively, of the n^{th} frequency state component. We then define a frequency attribute parameter vector

$$\boldsymbol{\theta}_\mathbf{f} = [b_1, \ldots, b_N, \gamma_1, \ldots, \gamma_N]^T. \tag{6}$$

The frequency measurement model is a special case of that in (3),

$$\mathbf{z_f}(k) = \mathbf{H_f}(k)\mathbf{x_f}(k) + \mathbf{v_f}(k), \tag{7a}$$

and is assumed linear with Gaussian measurement noise $\mathbf{v_f}$. We assume that measurement errors for any two frequency components are uncorrelated, that is,

$$E\{v_{\mathbf{f},i}\} = 0, \quad E\{v_{\mathbf{f},i}v_{\mathbf{f},j}\} = \delta_{ij}\sigma^2_{\mathbf{f},i} \qquad i = 1,2,\ldots,N \tag{7b}$$

where $\sigma^2_{\mathbf{f},i}$ is the variance of measurement error for the i^{th} frequency component. In practice, we will have observed values of b_n and γ_n as well, and we can easily extend the model above for these observations.

Dynamics of the center frequency components can be modeled in the state update equation either via (a) a time-dependent state transition matrix, (b) process noise, or (c) a time-dependent forcing function describing deterministic changes (such as a speed change or equipment status change at time index k). We will assume that the center frequency is constant initially, and that the uncertainty in the observed center frequency is represented by the component bandwidth (ie, the standard deviation of the independent additive noise process $\mathbf{v_f}$ is taken to be the component bandwidth measurement b_n).

<u>The State Transformation Matrix, $\mathbf{H_f}$</u>. For a constant center frequency state, the state transformation matrix $\mathbf{H_f}$ will model only whether a particular state component f_n is observed or not. The observability assumption is significant – the observed frequencies result from scan-to-scan detections of variable strength, potentially fading narrowband components, and will not necessarily be of the same dimension as the frequency state. For example, frequency selective or orientation-dependent fading in the ocean medium implies that measurements $\mathbf{z_f}$ may consist of a subset (or even superset) of the frequencies currently representing the track state. The components of $\mathbf{H_f}$, denoted by $h_\mathbf{f}(j,n)$, are given by

$$h_f(j,n) = \begin{cases} 1, \text{ if the } j^{th} \text{ observed component} \\ \quad \text{corresponds to } f_n \\ 0, \text{ otherwise .} \end{cases} \tag{8a}$$

$\mathbf{H_f}$ is therefore a JxN matrix, where J frequencies are observed, and N is the current dimension of the frequency state vector. Note that J may be greater than N, so that in general, N+J track frequency components may be assumed with the additional J frequency states representing components not yet observed. Let A_n represent the event {the n^{th} frequency component of the track state is observed}, and let the probability of the event A_n be given by α_n, that is,

$$\Pr\{h_f(j,n)=1 \text{ for some } j\} = \Pr\{A_n\} = \alpha_n . \tag{8b}$$

Further, let $\eta(j)=n$ represent the event {the j^{th} observation corresponds to the n^{th} component of the track state}. In practice, the assumption that we know that the j^{th} observation corresponds to the n^{th} component of the track state is unreasonable. This can be alleviated by assuming correspondence as a result of a gating of measurements to tracks. However, we will instead assume an unknown correspondence and will show that assumptions on correspondence can be removed through an application of Bayes law. The probability of the j^{th} observation conditioned on it's corresponding to the nth component of the track state and on the observability of that component is defined by the observation model, and is therefore Gaussian,

$$\begin{aligned} p(z_{\mathbf{f},j}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f}, \eta(j), A_n) \\ = N(z_{\mathbf{f},j}(k) - f_{\boldsymbol{\eta}(j)}, \sigma_j^2) \end{aligned}, \tag{9}$$

where $z_{\mathbf{f},j}(k)$ is the observed frequency value at time index k, $f_{\boldsymbol{\eta}(j)}$ is the $\boldsymbol{\eta}(j)^{th}$ state frequency value, and σ_j^2 is the variance of the j^{th} measurement. For J observations, let the vector $\boldsymbol{\eta}$ represent the event {the J observations correspond to the $\boldsymbol{\eta}(j)^{th}$ components of the track state}. The vector $\boldsymbol{\eta}$ thus represents the correspondence of the vector of observations to components of the track state. Additionally, let $\mathbf{A}$ represent the vector of length J of components $\mathbf{A}(j)=A_n$. When conditioned on the vectors $\boldsymbol{\eta}$ and $\mathbf{A}$, the joint density function of the observation can be written as the product of densities (due to assumed independence of the observation noise and observability of the track state components, that is,

$$\begin{aligned} p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f}, \boldsymbol{\eta}, \mathbf{A}) \\ = \prod_{j=1}^{J} N(z_{\mathbf{f},j}(k) - f_{\boldsymbol{\eta}(j)}, \sigma_j^2) \end{aligned}. \tag{10}$$

From the Total Probability theorem,

$$p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f}) = \sum_{\Omega_\eta} \sum_{\Omega_\mathbf{A}} \left(\prod_{j=1}^{J} N(z_{\mathbf{f},j}(k) - f_{\boldsymbol{\eta}(j)}, \sigma_j^2) \right) p(\boldsymbol{\eta}, \mathbf{A}) \quad (11)$$

Applying Bayes theorem to p(η,**A**) above,

$$p(\boldsymbol{\eta}, \mathbf{A}) = \frac{p(\boldsymbol{\eta} \mid \mathbf{A}) p(\mathbf{A})}{\sum_{\Omega_\mathbf{A}} p(\boldsymbol{\eta} \mid \mathbf{A}) p(\mathbf{A})}. \quad (12)$$

From our initial assumptions on A_n, the probability of the observed states is the product of the individual probabilities of each frequency component being observed, that is,

$$\mathrm{p}\{\mathbf{A}\} = \prod_{\mathrm{j}=1}^{\mathrm{J}} \alpha_{\boldsymbol{\eta}(\mathrm{j})} . \quad (13a)$$

Since most of the applications of interest will involve association of partial data with the track frequency state, the probability of not observing a frequency measurement is not included in (10).

We will assume an unknown and equally likely correspondence across observations, that is, the ordering of the observed components is unknown and uniformly distributed. Therefore,

$$p(\boldsymbol{\eta} \mid \mathbf{A}) = \frac{1}{J!} \quad (13b)$$

since there are J! (J factorial) possible correspondence orderings of observations to observed states given the J states observed. The term in the denominator is p(η), and can be viewed as a normalizing constant that is not a function of the specific correspondence ordering in the total probability representation.

<u>J=1</u>. For a single PNB frequency observation, (12) - (13) can be substituted back into (11) to obtain

$$\begin{aligned} p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f})\big|_{J=1} &= c \sum_{n=1}^{N} \alpha_n N(z_{\mathbf{f},j}(k) - f_n, \sigma_j^2) \\ &= \left(\sum_{n=1}^{N} \alpha_n \right)^{-1} \sum_{n=1}^{N} \alpha_n N(z_\mathbf{f}(k) - f_n, \sigma_j^2). \end{aligned} \quad (14)$$

Note that the above makes no assertion on measurement-to-frequency state component correspondence (η(j) can be reindexed and simply indicated by index n). For narrowband components separated in frequency by much more than the component bandwidth, this is approximately

$$p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f})\big|_{J=1} \approx \frac{\alpha_n}{\sum_{m=1}^{N} \alpha_m} N(z_{\mathbf{f},j}(k) - f_n, \sigma_j^2), \quad \text{for } n = \min_{\mathrm{n}} \left\{ z_{\mathbf{f},j}(k) - f_n \right\} \quad (15)$$

which indicates the appropriate interpretation when gating can be applied to the measurements.

***<u>Key observation</u>*:** Equation (14) indicates that measurement probability density function conditioned on the track state $p(\mathbf{z}_k|\mathbf{x}_k)$ can be appropriately modeled by a **Gaussian Sum** density function [6] with Gaussian center frequencies f_n (the track state center frequencies), common variance σ_j^2 (the measurement variance, with j=1), and amplitude weightings α_n (the probabilities of observing the n[th] frequency state).

<u>J>1</u>. When J>1, multiple PNB measurements are received from the same contact by definition. This condition implies some additional knowledge or processing that specifically identifies J PNB measurements as belonging to a single contact. This knowledge is typically not available, though "Source Set" processing may attempt to assert this information based on operator input or on harmonic set processing input, for example.

For multiple observations corresponding to distinct frequency state components, p(**A**) in (13a) is restricted to a product of probabilities from distinct components. This complicates the extension to the J>1 case in that the product of summations does not include the terms representing multiple observation-to-single state correspondences. For narrowband components separated in frequency by much more than the component bandwidth, however, these terms are negligible, and the result is approximately

$$p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f}) \approx \prod_{j=1}^{J} \sum_{n=1}^{N} \left(\sum_{m=1}^{N} \alpha_m \right)^{-1} \alpha_n N(z_{\mathbf{f},j}(k) - f_n, \sigma_j^2), \quad (16)$$

where no assertion on measurement-to-frequency state component correspondence is asserted other than that of distinct components. For narrowband components separated in frequency by much more than the component bandwidth, this is further approximated by

$$p(\mathbf{z_f}(k) \mid \mathbf{x_f}(k), \boldsymbol{\theta}_\mathbf{f}) \approx \prod_{j=1}^{J} \frac{\alpha_{n(j)}}{\sum_{n=1}^{N} \alpha_n} N(z_{\mathbf{f},j}(k) - f_{n(j)}, \sigma_j^2), \quad \text{for } n(j) = \min_{\mathrm{n}} \left\{ z_{\mathbf{f},j}(k) - f_n \right\}. \quad (17)$$

<u>Key observation</u>: For J>1, the measurement probability density function conditioned on the track state can again be modeled as (in this case) a **vector Gaussian Sum** density function, with parameters defined as for the J=1 case.

Form of the Prior/Posterior Density. As with the kinematic association problem, the densities above must be combined with the prior state density $p(\mathbf{x}(k-1)|\mathbf{Z}(k-1))$ via $p(\mathbf{x}(k)|\mathbf{Z}(k-1))$ to achieve the posterior density $p(\mathbf{x}(k)|\mathbf{Z}(k))$ in (4). Interpreting the frequency state as a collection of independent frequency components initialized with a Gaussian sum density and recursively estimated from measurements with a Gaussian sum density $p(\mathbf{z}(k)|\mathbf{x}(k))$ justifies a track frequency state representation via a Gaussian sum representation. Thus, we assume a Gaussian sum track state density representation of the form

$$p(\mathbf{x_f}(k)\,|\,\mathbf{Z_f}(k),\boldsymbol{\theta}_\mathbf{f}) = \sum_{\eta=1}^{N_k} \frac{\gamma_\eta}{\sum_{m=1}^{N_k}\gamma_m} N(\mathbf{x}_{\mathbf{f},\eta}(k)-f_\eta,\ b_\eta^2)\,, \tag{18}$$

where the state parameters from (5) and (6) are invoked. We note that the Gaussian sum consists of N_k terms at time index k, and that the number of terms should be expected to change over time as new measurement data is available.

State transitions from time index k to time index k=1 are controlled by the state transition equation in (3). We assume a unity state transition matrix and ignore forcing function inputs. Therefore, the prediction (or diffusion) expression in (4b) yields

$$p(\mathbf{x_f}(k+1)\,|\,\mathbf{Z_f}(k),\boldsymbol{\theta}_\mathbf{f}) = \sum_{\eta=1}^{N_k} \frac{\gamma_\eta}{\sum_{m=1}^{N_k}\gamma_m} N(\mathbf{x}_{\mathbf{f},\eta}(k)-f_\eta,\ P_\eta)\,, \tag{19a}$$

where the inflated state variance is the sum of the frequency state variance b_η^2 at the previous update time and the process noise q_η^2, that is,

$$P_\eta = b_\eta^2 + q_\eta^2\,. \tag{19b}$$

Note that the process noise model can be expanded to include dependencies on time since the last measurement, but this is not considered here. For initialization, we assume that p(x(0)) is given by a single term density approximating a uniform density, that is,

$$p(\mathbf{x_f}(0)) = N\left(\mathbf{x}_{\mathbf{f},\eta}(k)-\left(\frac{F_{Max}-F_{Min}}{2}\right),\left(F_{Max}-F_{Min}\right)^2\right), \tag{20}$$

with N_0=1 and γ_n=1.

From expressions in (14) (or (16)), (18), and (19), the posterior density $p(\mathbf{x}(k)|\mathbf{Z}(k))$ will also be a Gaussian Sum density, given by

$$p(\mathbf{x_f}(k)\,|\,\mathbf{Z_f}(k),\boldsymbol{\theta}_\mathbf{f}) = \sum_{n=1}^{N_k}\sum_{\eta=1}^{N_k} \frac{\alpha_n}{\sum_{i=1}^{N_k}\alpha_i}\frac{\gamma_\eta}{\sum_{j=1}^{N_k}\gamma_j} {}^*N(\mathbf{x}_{\mathbf{f},\eta}(k)-f_\eta,\ b_\eta^2+q_\eta^2)N(z_{\mathbf{f},j}(k)-f_n,\sigma_j^2) \tag{21}$$

Strictly, however, based on our observation model in (8a), the above is only valid when the measurement is associated with a specific component, that is, for n=η, since the observation matrix is zero otherwise. This suggests that (21) be written as

$$p(\mathbf{x_f}(k)\,|\,\mathbf{Z_f}(k),\boldsymbol{\theta}_\mathbf{f}) = \sum_{n=1}^{N_k+1} \frac{\alpha_n\delta_{n,\eta}}{\sum_{i=1}^{N_k}\alpha_i}\frac{\gamma_\eta}{\sum_{j=1}^{N_k}\gamma_j} {}^*N(\mathbf{x}_{\mathbf{f},n}(k)-f_n,\ b_n^2+q_n^2)N(z_{\mathbf{f},j}(k)-f_n,\sigma_j^2) \tag{22}$$

where J=1 observed components is assumed. The order of the Gaussian sum representation is increased to account for the possibility that the measurement represents a new narrowband component not previously incorporated into the track state. Because of the form of the observation matrix, the number of terms in the posterior density will increase by at most J components, where J is the number of components in the measurement observation.

Recursive State Update. The frequency state components f_n can be updated recursively (ie, via a Kalman Filter formulation) based on measurements $\mathbf{z}_{\mathbf{f},j}$ (received with uncertainty σ_j^2) that are associated with the track. Following the Kalman Filter formulation for Gaussian sum densities [6], the update equations are, for J=1,

$$\begin{aligned}
r_n(k) &= z_{\mathbf{f},j}(k)-f_n(k-1)\\
b_n^2(k|k-1) &= b_n^2(k-1)+q_n^2\\
C_n(k) &= b_n^2(k|k-1)+\sigma_j^2(k)\\
K_n(k) &= b_n^2(k|k-1)C_n^{-1}(k)\\
f_n(k) &= f_n(k-1)+K_n(k)r_n(k)\\
b_n^2(k) &= \left(I-K_n(k)\right)b_n^2(k|k-1)
\end{aligned}\ . \tag{23}$$

given state mean f_n(k) and variance b_n^2(k) and the process noise q_n^2, and measurement $\mathbf{z}_{\mathbf{f},j}$(k) with variance/uncertainty σ_j^2, and assuming that the update is performed for the one component of the state whose normalized residual is smallest, that is, for which

$$r_{\min} = \min_n\left(r_n^{\,T}C_n^{\,-1}r_n\right) \tag{24}$$

with C_n and r_n as computed in (23). Condition (24) indicates that frequency component n is most likely to have generated the measurement.

For J>1, the product in (16) complicates computation of the posterior density function. The condition J>1 implies that two components are received simultaneously, and that association with a track must be considered jointly. As an approximation, we will initially ignore this implication, and treat each narrowband component separately, relying on a product of individual association likelihoods via (22) and on sequential recursive updates of the track state via (23).

The posterior density in (22) has an increasing number of terms (even though it limits the growth of terms to one term per update). The growing number of terms would seriously limit the practical utility of this approach if not for a natural approach to combining and eliminating terms, as indicated in [6], which will not seriously affect the posterior density approximation.

The remaining quantity to be specified in the Gaussian Sum formulation are the mixing proportions α_n and the recursion for γ_n. As suggested by (22) and defined in [6], updates can be computed as the normalized product

$$\gamma_n\left(k+1\right)=c\ \alpha_n\left(k+1\right)\gamma_n\left(k+1\right) \tag{25a}$$

with normalizing constant c computed over all n after the k+1[st] iteration. A second approach is to set α_n equal to a smoothed version of the input SNR, that is,

$$\begin{aligned}\alpha_n'\left(k\right)&=\left(1-\beta\right)\alpha_n\left(k-1\right)+\beta\gamma_n\left(k\right)\\ \alpha_n\left(k\right)&=\left(\sum_{n=1}^{N}\alpha_n'\left(k\right)\right)^{-1}\alpha_n'\left(k\right)\end{aligned} \tag{25b}$$

where β is an exponential smoothing constant. This update can easily reflect (a) the addition of new components to the frequency state vector if a new frequency measurement does not sufficiently associate with an existing state frequency component, and (b) the removal of the n[th] frequency components when α_n drops below a predetermined threshold. However, this formulation does not take into account how often frequency components are updated (frequently updated frequencies are more characteristic of the target state), and is susceptible to domination by high SNR components. Since α_n is a "probability of observation" term, a histogram-based approach may also be justified and would be independent of SNR. A limiting (or squashing) function $\phi(\gamma_n)$ can be introduced in place of γ_n to address the second issue if SNR domination.

6. SUMMARY

This paper has provided the initial groundwork for including narrowband frequency component data in the passive sonar data fusion problem. PNB frequency domain data streams have been identified as components of a non-kinematic domain. We then defined a statistical model applicable to the frequency domain state representation, and derived expressions for associating frequency measurements to tracks and for updating frequency domain state variables. The multiple domain data can be readily fused in an MHT fusion architecture toward improving the overall data fusion system performance. The performance of this joint kinematic and frequency domain data fusion system is currently under evaluation.

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