

ARCANGELLO III

Human powered submarine

Arcangelo III team
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This paper includes general aspects of the human powered submarine Arcangelo III.

I. INTRODUCTION

The submarines of human propulsion, of Arcangelo Type have undergone modifications since their conception in 1995. These submarines have been characterized by the simplicity of the design, good stability, and low investment for the project, nevertheless the experiences that have been obtained from previous competitions, shown to us that our problems reside basically in the propulsive system.

It is for that reason that we have occurred to the task mainly of making the adaptation of this one system, with the purpose of making it, the most efficient possible.

Besides of the improvement of some security aspects, maneuverability and fixation of the pilot.

OUR GOALS

- The previous experience has taught us that when a human powered submarine is constructing, the simplicity is of extreme importance. For that reason, we will continue with this principle in mind for the accomplishment of this project
- It is of extreme importance the adaptation of the propulsive system, being the main goal to change this system.
- Another goal to follow is to refine all the systems that conforms this submarine (maneuverability, security, etc.)
- Also we hope to increase the amount of practicing hours in the water for the tests of our submarine, in order to gain valuable experience in the handling of it.
- Finally, to break the, one person propeller driven submarines world record.

THE TEAM

ARCANGELLO III team	
Members name	Position
Mariano Azzur Hernandez Contreras	support diver
Oscar Manuel Gomez Diaz	Team leader and support diver
Alondra Martinez Garcia	Design and Construction
Sergio Isrrael Dominguez Panama	Workshop chief
Carlos Alfonso Hernandez Montes	Pilot
Juan Jose Ramos Delgado	Drawings and construction

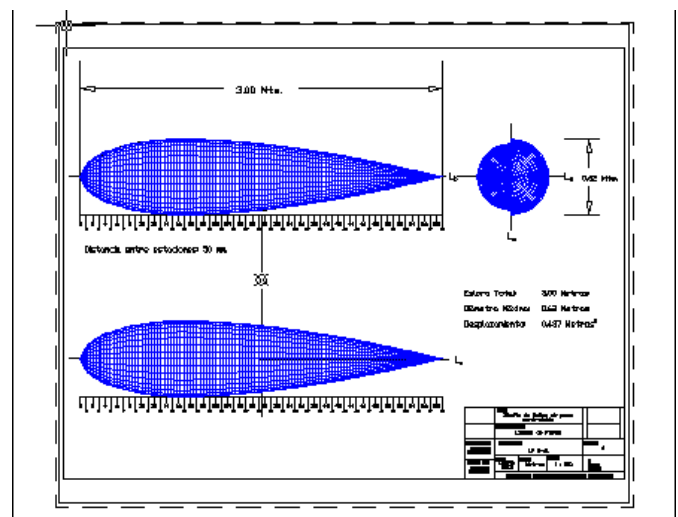
PAST ISR INVOLVEMENT

In 1995 in Bethesda Maryland, USA; the fourth international competition of submarines ISR carried out, in which our country had participated for the first time represented by the University Veracruzana Naval Engineering Faculty with the submarine Arcangelo 40 obtaining an outstanding second place.

In 1997 in the David Taylor model basin (DTRC). The University Veracruzana participated for the second time obtaining the third place. With the submarine Arcangelo II.

II. DESIGN PHILOSOPHY

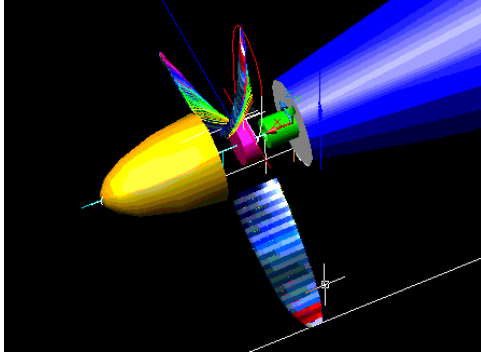
These kind of submarine must have an hydrodynamic shape, but at the same time they do not have to jeopardize the capacity of the pilot to action the propulsive system with its legs; besides to include favorable characteristics of maneuverability, stability and resistance, but fulfilling the norms of security demanded by the organizers.



THE DESIGN SPIRAL

One of the most difficult things to elaborate is the plan or the organization to follow, to carry out the project, since this is not a boat, a car, etc. where the process already has certain phases to follow.

It is for this reason, that the implementation of the design spiral results extremely necessary, in order to achieve our prime objective that is to have a human powered submarine with the greater possible speed.



Controllable pitch propeller image

III. GENERALITIES

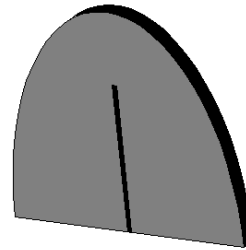
STRUCTURAL DESIGN.

The structural design is dictated by the rules of the ISR (International Submarine Races), which specify that the contestant submarine must be completely flooded with water, this facilitates in a big way the calculations and design of its structural reinforcements.

The structure of the submarine is no watertight, therefore the pressures inside and outside are of equal value, for this reason there is not need to design a hull to stand hydrostatic pressure. The submarine structure only has to support the internal material effort due to:

- The internal water mass with an implicated acceleration.
- That stresses produced by the propulsion mechanism.
- Stress concentrations in abrupt changes of section, like hatchways.
- Pilot internal movements.
- Dry manipulation and transport of the submarine.
- Towing traction force, ballast and own weight of the submarine.
- accidental impacts.

For all these reasons, aluminum frame reinforcements are placed at the major hard changes of pressure, besides longitudinal reinforcements as show in the picture below:



Internal reinforcements of Structure:

1. Wall-plates of 1.5 x 1/8" covered with 2 layers of fiber glass, doubled in form of hoops.
2. Wall-plate of 1.5 x 1/8" throughout the union perimeter that forms the hull and the longitudinal reinforcement.
3. Acrylic windows, inserted in openings practiced in the hull.

HULL

The hull is a revolution solid with a maximum diameter at 7/24 of the length from the bow and a L/D ratio of 5. This ratio was obtained after analyzing 30 submarine prototypes with different proportions but with similar shapes. The base of the design is to put inside of the submarine a pilot, the propulsion device and the breathing equipment. It has observation windows and a safety hatch for security.

The hull is constructed using fiber glass structured by 4 fiber layers.

Some geometric conditions were taken into account for the design of the hull as follows:

1. The smallest surface possible is advisable to obtain a smaller resistance.
2. In order to obtain a given displacement it is necessary to have the smaller surface with the greatest diameter and the smallest length possible.
3. The thinning towards stern must be little prominent.

4. Without parallel body:

L/D 4-10

Maximum section from the bow $0.36L - 0.52L$

$L_p - 0$

Wet surface coefficient $C_s = S/\pi$ DL $0.689 - 0.809$

C_p $0.55 - 0.70$

5. With parallel body:

L/D 0.7 – 17.5

C_p 0.685 – 0.840

S.M.P. 0.28L – 0.16L

L_p 0.285L – 0.60L

C_s 0.785 – 0.895

$L_E + L_R/D$ 5 – 7

6. The total drag of a revolution solid is smaller at a length-diameter ratio of: $L/D = 4,5$ (DSB).

7. In a solid of revolution of $L/D = 5$, at low speeds (5 knots) the flow stays laminar (JLR).

8. The resistance by form is directly proportional to the presented frontal area of the flow (PNA).

9. At greater presented frontal area to the flow the proportion of the form resistance is greater so it does the total resistance (PNA).

10. in the control surface with t/c of $1/16$, the proportion of the total resistance by form is of 8 %.

11. The elliptical forms, as much in profiles as in revolution solids, have the characteristic to generate the smaller total resistance in a great number of analyzed geometric forms (PNA).

12. An orifice or groove generates 5 times more resistance than a protuberance of similar definitions.

Having as it bases the form of the submarine Arcangelo 40, in 1997 three prototypes of the same form but different dimensions were made, with the purpose of comparing them and of determining if it were necessary to modify the form of the hull.

The prototypes were drawn in real size, and later with aid of drawing software that facilitated the visualization of the geometry of the hull and the calculation of the wet surface.

The results obtained when analyzing these three prototypes were:

Prototype I:

Design speed: 5 Knots (2.572)

$L = 3.5$ m.

$D_{max.} = 0.58$ m.

$S = 4.46$ m²

Naked hull frictional resistance.

$R_f = C_{fp} V^2 s(1/2)$

$R_f = (3.064 \times 10^{-3}) (1/2) (998.2 \text{ Kg./m}^3) (2.572 \text{ m/s})^2 (4.46 \text{ m}^2)$

$R_f = 45.126$

Prototype II

Constants

Design speed: 5 knots.

$L = 3.5$ m.

$D_{max.} = 0.55$ m.

$S = 3.99$ m²

Naked hull frictional resistance:

$R_f = C_{fp} V^2 s(1/2)$

$R_f = (2.97 \times 10^{-3}) (998.2 \text{ Kg/m}^3) (2.572 \text{ m/s})^2 (3.99 \text{ m}^2) (1/2)$

$R_f = 39.1451 \text{ N}$

Prototype III

Constants:

Design speed: 5 knots.

$L = 3$ m.

$D_{max.} = 0.58$

$S = 3.81$ m²

Reynolds number

$R = VL / \mu$

$R = (2.572 \text{ m/s}) (3 \text{ m}) / (1.077 \times 10^{-6} \text{ m}^2/\text{s})$

$R = 7164964.4073$

Friction coefficient.

$C_f = 0.072 (R)^{-1.5}$

$C_f = 0.072 (7164964.4073)^{-1.5}$

$C_f = 3.064 \times 10^{-3}$

Naked hull frictional resistance .

$R_f = C_{fp} V^2 s(1/2)$

$R_f = 3.064 \times 10^{-3} (1/2) (998.2 \text{ kg/m}^3) (2.572 \text{ m/s})^2 (3.81 \text{ m}^2)$

$R_f = 38.54 \text{ N}$

Arcangelo 40

Constants

Design speed: 5 knots

$L = 3$ m.

$D_{max.} = 0.62$ m.

$S = 3.89$ m²

Reynolds number:

$R = VL / \mu$

$R = (2.572 \text{ m/s}) (3 \text{ m}) (1.077 \times 10^{-6} \text{ m}^2/\text{s})$

$R = 7164964.4073$

Friction coefficient:

$C_f = 0.072 (R)^{-1.5}$

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Naked hull frictional resistance:

$R_f = C_{fp} V^2 s(1/2)$

$R_f = (3.064 \times 10^{-3}) (1/2) (998.2 \text{ kg/m}^3) (2.572 \text{ m/s})^2 (3.89 \text{ m}^2)$

$$R_f = 39.358$$

Comparing the data, it is obvious that Prototype III is the one that offers better hydrodynamic characteristics, nevertheless, these dimensions are little favorable for the good performance of the pilot, it is for that reason that the decision was to use the characteristics of Arcangelo 40.

HULL DESIGN

According to the information contained in the boat design sail book, in the chapter of Aero and Hydrodynamics, the total resistance of a revolution submerged body varies in relation to its L/D ratio and it is possible to be observed that the optimal relation is around 4.5.

The next step was to measure the body shapes of the possible pilots to see the space that was need to lodge them, the results were.

Minimum Diameter: 0.60 m.

Length: 3m.

From these parameters and the values of L/D, we analyzed 30 prototypes, once obtained the parameters of these, was make a comparison and we determined, that the prototype that would be used, had to be the following one:

$$L = 3.00 \text{ m}$$

$$D = 0.62 \text{ m}$$

$$L/D = 5$$

$$A_T = 5.056 \text{ m}_2$$

$$C_F = 0.0030512$$

$$R_E = 7.3168 \times 10^6$$

$$R_F \text{ at } 5\text{Kn} = 5.201 \text{ kg}$$

$$R_{\text{APPENDICES}} \text{ at } 5 \text{ Kn} = 6.7521 \text{ lbs.}$$

$$R_T \text{ at } 5 \text{ Kn} = 18.22 \text{ lbs}$$

$$CG = 1.75 \text{ m stern}$$

$$C_p = 0.5025$$

$$J_o = 179.94 \text{ kg m}^2$$

$$C_S = 0.6766$$

$$GM_{\text{IMMERSION}} = 0.0614 \text{ m}$$

CONSTRUCTION

For the model construction, a system that allows the manufacture of castings from a male mold (one off), which purpose is to obtain a finished practical system with good.

One of the advantages that we have, is that, since the submarine design is a revolution solid,

This fact allows the construction of a single mold, obtaining two symmetrical pieces, which finally would be united.

Materials:

- Three leaves of American pine plywood (1220 x 2440 x 12 mm).
- 4 leaves of American pine (6 x 1 x 10), aligned.
- 50 alfajillas (5 xs 15 xs 3000 mm) of first, aligned.
- ½ Kg. of 2" nails.
- ½ Kg. of 1 ½" nails.
- ½ Kg. of 1" nails.
- ½ litter of glue.
- 20 Kg. Of pre-accelarated polyester resin.
- ¼ litter of catalyst.
- 15m² of fiber glass.
- 4 pieces of carbide discs.
- 10 clips type nail.

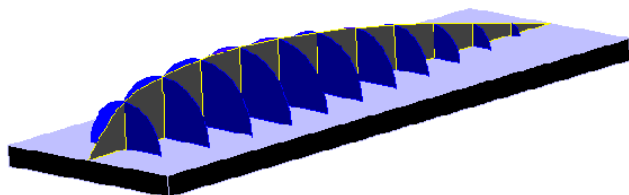
For the bed construction the following materials were select:

- 3 leaves of plywood (3"x1"x80cm).
- 2 leaves of plywood (6"x1"x3cm).
- 1 piece of Wood of 80 cm x 3m x 12mm.

FABRICATION:

- Alignment of the base
- After selecting the material, we had to construct a frame (bed), which supported the mold and made level well.
- With the support of the layout record, the contour of the hull was draw in the central line on the piece of plywood of 42cm x 3m x 12mm, which was cut and later it leagues together perfectly.
- The semicircles corresponding to the stations were draw up, on the leaf of plywood (42cm x 3m x 0.012cm) and grooves become of 2/3 of perpendicular radio to the base line.

- The central line is due to fix perpendicular to the bed and on center line, nevertheless previously this center line in the stations is due to groove, with a depth of 1/3 from the curved, perpendicular section to the straight art (as seen in the following figure.)



LINING ELABORATION AND MOUNT

This step deserves special well-taken care of its manufacture and finish, because it defines the final forms of the prototype.

The first step is the election of the alfajillas, preferring the straightest, without knots and with uniform cut.

Next, the symmetrical positioning of it on the patterns, alternate one to starboard and another in the center line to continue with others every 45° to port and starboard to continue placing them in the halving ones of the angles generated by the separation of the alfajillas On guard.

Being the design of the prototype of circular section and with an uniform profile; the alfajillas (straight, uniform elements) tend to follow a convergent line towards bow and specially to stern, which originates, that in certain points the alfajillas converge preventing the uniform and complete lining of the cover.

In order to correct this problem, we have used a wood naval construction method that is known as “morro”.

“MORRO” FABRICATION

The thickness of the alfajilla prevents that it takes the natural curve of design in the bow section that we will call “morro” and that is defined by a revolution ellipsoid, reason why it is necessary to choose another material that allows these forms and that would be resistant as well to the strip.

Plaster was our best option, armed with wire and in order to give it volume, news “paper” and cardboard.

The proportions of plaster-water mixture will be according to the consistency that is desired, being of preference that it

allows molding, in addition if a 10% of “delahuel” (a hardener) is added better results are obtained.

In order to obtain the construction of the natural curves, a layout with the dimensions indicated in the record is made, and it defines the part of the “morro”

This layout is fixed to the bed, just in the bow resentment, and a hinge allows that it rotates on the axis of the center line, thus generating, part of the revolution solid (ellipsoid) that by design it defines the bow, completing it.

The drying of plaster has to be done in a controlled environment, to avoid the appearance of cracks, is recommendable to dampen careful and periodically plaster, because of the exothermic reaction of the mixture.

“MORRO” FINISH

The finish of the “morro” is made with paste for wood, which will fill the small fissures between the alfajillas, also sandpaper is used to reduce the protuberances and imperfections.

The sandpapering on wood is made with heavy and medium wood sandpaper no. 80 and no. 120 respectively adhered to little pieces of wood to apply a uniform sandpapering avoiding the formation of blows.

COVERINGS

In order to facilitate the strip of the mold; a plastic film that is resistant to the catalyzed polyester resin is placed, covering totally the model, in such a form that when becoming the laminate is in the middle of these, and by its insulating properties, it will have to facilitate the separation.

This type of plastic, comes in presentation of 10 x 30 meters rolls, it is a very fine film, it has a great adhesion, this is of great advantage when doing your overlap in the covered one, but also it is inconvenient in inexperienced hands, because it causes tangles and by consequence, loses of material.

HULL CONSTRUCTION

For the construction of the hull is used the composed material laminate method, formed by four fiber glass layers, 3 of them are of the “colchoneta” type and 1 of the “petatillo”, saturated with r polyester resin and placed with the following order to obtain an optimal structural strength.

1. - One "colchoneta" layer. (With multidirectional stress action)
2. - One "colchoneta" layer. (With multidirectional stress action)
3. - One "petatillo" layer. (With bidirectional stress action)
4. - One "colchoneta" layer. (With bidirectional stress action)

The "colchoneta" is a type of fiber glass with an approximately thickness from 3 to 6 cms. That is arranged in all directions forming a multidirectional fabric that is commercially in roll form. also one is in the same single presentation that its form is arranged in perpendicular form, in such a way that it forms a closed weave that provides discharge resistance in both felt of the fiber mainly. This combination along with the resin throws to us as result a hull of high resistance as well as of easy handling for its construction.

Because of the form is perfectly symmetrical, a model was make that is exactly the half of the hull, making two halves of this later to unite them.

The accomplishment of the hull was carried out cutting the fiber glass by parts since the form cannot be covered with the hull totally with a single piece being the overlap in strategic places and with the total participation of the members of the team since the "gelado" process of the resin to catalyze it is of a relatively short time, in such a way that a member of the team made the resin mixture and catalyst which must be in very exact proportions, another member was in charge to wash the brushes to avoid that these became hardened, and the rest of the team was in charge to apply the fiber, that is as well as it is made manually, which consists to soak the brush with resin and to prick the fiber glass with which it happens of his characteristic white color to a transparent appearance.

The layers were placed one by one and finally pieces of paper were placed to absorb the excess of resin giving with this a better finished.

The process of drying of the hull needed 12 hours and it was continued the finished process there of the surface so that it was the smoothest possible, since the inner part of the hull was not smooth had come to sandpaper itself to disappear these harshness. First the paper was sandpapered which an as absorbent of the resin and thus it was continued until a surface with a minimum of irregularities helped with paste was obtained filler, that also use in the model previously, this paste is applied in fine layers that are sandpapered after curing. it is pertinent to stand out while more imperfections has the outer surface it will obtain minor hydrodynamic efficiency and with this the little aesthetic one that will reflect

the submarine, it is for that reason that spent great time to im to obtain the finished of the submarine.

Also two bulkheads were made and placed in the part of stern; these bulkheads contain rowlocks that are those in charge to maintain the propulsive axis. These were made in disc form and were made totally of fiber glass, doing a pressure laminate of three layers, after the drying process mounted in one of halves of the hull, so that finally the perforations were made to it in the central part where the rowlocks and other would be placed that serve as lightening.

The union of two halves of the hull was made by little pieces of wood that contained a screw in the center to make agree the two parts perfectly and thus the union with two fiber glass layers was laminated when the curing allowed to retire the wood pieces to laminate the parts in which these were, once the union strip was dried totally the sandpapering has come.

The following phase is the cuts that are due to make the accesses and a good visualization from interior to outside of the submarine, process that was made with a cutting and a pneumatic for curves very pronounced machine, it is possible to emphasize that this process is very delicate since it must carry out of a very precise way and without errors.

FLOATER

One of the basic characteristics of the submarine is to maintain a neutral buoyancy (so that this store not to sink). It is by that a floater was placed and a weight that calculated to maintain this buoyancy and to maintain a good stability or in the longitudinal and cross-sectional sense. The floater was placed in the internal superior part of the submarine and the weight in the internal inferior part so that it balances with the floater. The floater was designed by means of calculations of stability taking into account all the weights of the submarine giving like result a volume that was to adapt to the form of the submarine and the ergonomics of the pilot to be able to evolve totally in the work of propulsion, this volume was molded in polyurethane and we have laminated it with three fiber glass layers fixed to the hull so that the water did not filter to it.

ACRYLICS

The positioning of acrylics was one of the greatest satisfactions as far as the aspect of the construction since it was not required of a single rivet for his sustenation to the hull.

The submarine counts with two pairs of acrylics, it has one pair in the frontal superior part and another pair in the inferior frontal part of the hull, being those of the inferior part greater than those of the superior part since the visibility of the pilot tends to be used with more frequency towards the inferior part that to the superior one, the corners of acrylics were

well-off to diminish the concentration of efforts and to give a better appearance to the submarine.

MAIN HATCHWAY

This is the access by the pilot enters and leaves the submarine, a series of ironworks, being these two pairs of hinges in order when this was opened was not loose of the hull and fell, as well as a handle with hinges and another one in the interior adapted to it that serves so that this is opened and automatic locking by means of a pin with means. This is located in the central part of starboard of the hull.

SECURITY BUOY

One of the security requirements that are demanded in the competition is to count with a security buoy which will have to be released when the pilot is in problems. It is important to remark that this buoy does not have a complicated mechanism that put in risk the life of the pilot.

The buoy is placed in the stern port part of the submarine, this one is a small cover-hatchway ready in its interior a polyurethane floater covered with fiber glass so that it does not absorb the water and to obtain one better bouyancy of the same one.

Control surfaces

Arcangelo has 6 control surfaces. 4 of them are horizontal and the other 2 are vertical. The two front horizontal control surfaces are mobile and so are the verticals.

All these pieces are NACA profiles of the 65A010 type, based on ellipses.

To govern properly, the motions of the submarine must be controlled, vertically and horizontally with the least drag produced by the action of the rudders.

Stability

Arcangelo has a watertight tank that helps floating properly when surfacing. This tank also helps in controlling the center of gravity in light ship condition, in ballast condition or to attain the neutral floating.

Propulsion mechanism

The races organizers have established a lot of requirements or conditions for the propulsion mechanisms. The objective is to control and insure that all the teams participate under the same conditions in order to have integrity at the competition.

1. The power for the propulsion must come exclusively from the human body.
2. Only one pilot can produce propulsion energy.
3. Devices to storage energy are not allowed, like inertia wheels or springs.
4. The energy transmission system is free to choose (hydraulic, mechanic, etc.).

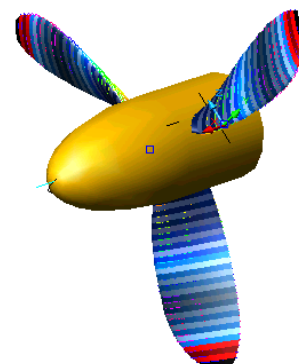
Based on this information a system was designed impulsed by pedals, in a way that the legs impulse and generate energy for the system. That energy will depend on the pilot ability and physical conditions. The strength is in a range of 3/10 to 4/10 hp.

This energy is transmitted through a gearbox to the propeller.



Gear box

The propulsion system of the submarine Arcangelo III is being finished, and is a controllable-pitch propeller of three vanes, the calculations of this system were in charge of the student of Naval engineering Julio Adrián Grajales Duray.



Power transmission-system

The system that transmits the energy for propulsion Consists in a gearbox that provides 150 revolutions per minute at a constant pedaling.

Shaft-propeller

The shaft-propeller transmits rotating power coming out from the gearbox and into the propeller. The Nylamid shaft material has excellent mechanical properties.

Safety and life support systems

The security of our submarine has been the point of greater preoccupation, since we looked for, avoiding any type of accident. It is for that reason that we are improving the mechanism that releases to the emergency buoy and the hatch, in addition we are equipping the submarine of an oxygen tank that allows the quick ascent of the pilot, in case of having some fault with the breathing system.

The good training of the support divers, is another one of the points that are of great importance for the security of the pilot, is for that reason that all of us that integrate the team, have been put under qualification, and we have taken the courses of open water and Advanced open Water diver courses of PADI.

IV. TESTING

Since the first participation of the Arcangelo submarines, in the International races of FURE, the fundamental part of all the construction and design processes have been the one to carry out the greater possible number of tests.

With these tests it is possible to detect the failures that we are having, as well as to verify the veracity of the calculations that have been done previously.

These tests have taken place with the Arcangelo submarines in different sites among them we can mention:

- Pool of the facilities of “Leyes de Reforma”. a place pertaining to the IVEDE.
- Pool of the facilities of the Heroic Naval school, in Antón Lizardo, Veracruz.
- Some tests in the sea took place, in an area pertaining to the yacht club of Veracruz.

The tests of the submarine Arcangelo III, will be carried out in this month, on the previously mentioned facilities.

V. Training

The training of the pilot and the support divers have taken place in the spinning and pool facilities of the “IVEDE”, running at the beach and diving in the Veracruz reef system.

Annex A

Calculation of Drag.

To obtain this, it is necessary to know the geometric characteristics of the hull, fundamentally its volume and its wet surface.

Calculation of the wet surface.

For these calculations we used the hull layout record and calculation computer programs to obtain the outer area that will be in contact with the water by the integration method using the 1/3 Simpsons rule:

$$W.S. = 1/3 \sum f(W.S.) dx$$

Annex B

This moment was obtained from the following form:

$$N = J_o \times x$$

(according to Saunders, SNAME)

Where:

N: is the necessary moment to move the submarine

J_o: polar moment of inertia of the submarine.

x : angular acceleration

Once found the J_o value by geometric calculations, the value of the moment necessary to move the submarine is obtained using the following equation:

$$N = J_o \times \alpha$$

Thus since $N = F \times d$, the force necessary to move the stern fin is:

$$F = N / d_2$$

and for the bow fins we have:

$$F = N / d_1$$

d₂: is the distance of the center of gravity of the submarine to Stern.

d₁: is the distance of the center of gravity of the submarine to bow .

Once we have found the force necessary to move the fins we have to find the area of each one of them by the use of the Newton formula:

$$F = q \times T_o \times C$$

Where:

F: is the resulting force.

q: is the dynamic pressure which is equal to:

$$q = \frac{1}{2} \rho V_o^2$$

ρ: is the density of fresh water.

V_o²: is the speed of the submarine square elevated.

C: is the lift coefficient (C_L) or the coefficient of drag (C_D)

Replacing the value q and clearing the area A in the formula of Newton, we have:

$$A = (F \times g) / (q \times C_L)$$

Now that we know the area of the control surfaces, we must define its geometry.

The form that we have chosen is of elliptical cuts and profiles NACA then studies that have been made in wind tunnels (PNA section 14), reveal that they tend less to the formation of end vertices and ventilation phenomena that produce eddies and increase the drag.

We know that in vehicles like ours where the EHP is limited, diminishing this type of resistance will help to facilitate the increase of the speed. According to PNA section 14, the aspect reason (L²/A) influences in the efficiency of the rudder, because to the greater aspect ratio, minor angle of attack for a same lift coefficient; what takes to us to choose an aspect ratio of 5, which allowing an angle of attack of 10° offers a lift coefficient of 0.72.

Having this way for the stern rudders:

$$\text{Aspect ratio} = L^2 / A$$

Then :

$$5 = L^2 / 472.6 \text{ cm}^2$$

Therefore:

$$L = 48.71 \text{ cm}$$