An Integrated Vision-Based Positioning System for Video Stabilization and Accurate Local Navigation and Terrain Mapping

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***ABSTRACT*: This paper deals with the design, construction and calibration processes in the use of angle sensors to improve the accuracy of a vision-based positioning system. These sensors are used for measurements that cannot be determined accurately (enough) from motion vision techniques. Furthermore, they provide a mechanism to stabilize the video data with respect to undesirable motion effects, e.g., cyclic motions in the surf zone. To use the integrated system effectively, a high-precision calibration procedure is employed to determine the transformations among the coordinate systems of the angle sensors and the vision system. With the positioning information presented in a common coordinate frame, the ROV control system can be optimized for operations that require high-precision navigation, trajectory following, terrain mapping, etc. Results of the calibration process and the testing of the system are presented to evaluate effectiveness and performance.**

I. INTRODUCTION

Video stabilization, high-precision positioning and navigation, and vehicle control in the operation of ROVs and other submersible platforms are among factors that contribute to the need for highly accurate motion and positioning sensors. For example, video stabilization is an important task during the operations of submersible vehicles in turbulent waters. Ideally, a vehicle may be programmed to move along a smooth trajectory, while collecting video or a sequence of images. The "corrupted" data, due to inevitable deviations from the desired camera pose, can be rectified knowing the vehicle angular motions.

Development of a 6 degrees of freedom (d.o.f.) high-precision positioning systems for airborne systems has been achieved by utilizing gyros, GPS/DGPS, INS, laser range finders and other optical systems; e.g., [2, 9]. A number of underwater sensing technologies provide solutions with various trade-offs. For example, most AUVs employ an appropriate combination of gyros and angle sensors, compass, INS, velocity Doppler, long/short baseline acoustic systems, as well pencil-beam and multi-beam sonars for terrain navigation and mapping; e.g. [12]. Optics-based systems are known to provide very high precision, resolution and data rate, but they can have drifts which grow unboundedly with time, as in INS and velocity Doppler systems. Additionally, image-based methods are known to provide much higher accuracy for certain motion types than others. The primary complexity arises from the coupling between the pitch and roll rotations and horizontal translations, when the vehicle navigates over a terrain; due to the so-called rotation-translation ambiguity of visual motion[1]. Therefore, sensors that directly give absolute angle readings are desirable since they 1) can resolve the ambiguity, 2) reduce the motion estimation computational complexity, and 3) often have bounded errors that are independent of the mission duration. The ideal scenario is to overcome the shortcomings of individual sensor modalities by exploiting the advantages of individual types in building an integrated multi-modal positioning system.

In recent years, video servo has received growing interest in the application of submersibles for local short-range automatic navigation and mapping. The underlying methods attempt to estimate all or a subset of the ROV motion components directly from the video e.g., [5, 6, 10, 11, 13, 14].

The most general system, handling *all 6 degrees of freedom* can estimate the vehicle trajectory, construct a photomosaic, and use it for map-based navigation [5, 6]. However, at least two important factors are critical for satisfactory system performance: planarity of the terrain that is mapped with a down-look camera, and the viewing of any local region from multiple nearby positions to ensure large image overlaps in several frames. The latter can be accommodated by revisiting various local regions more than once, e.g., through looping trajectories. In particular, it is the use of high redundancy (global information from several viewpoints) that enables overcoming the translation-rotation ambiguity (of local measurements from two nearby views). The improved estimates requires off-line computing (post-processing), since the entire data set is necessary for the trajectory estimation and photo-mosaicing. The Stanford/MBARI ROV work deals with the same capabilities by utilizing gyros, angle sensors and video cameras. However, it is not the accurate estimation of the vehicle trajectory and (or) mosaic construction, but the ability to navigate successfully from one point on the mosaic to another, that is the prime objective. More precisely, the accuracy (quantitative scale) of the constructed mosaic is irrelevant to the extend that it provides a sufficient map for local navigation.

This paper deals with the development of an integrated 6 d.o.f. positioning system utilizing sensors readily available on most inexpensive ROVs, with emphasis on positioning accuracy for precise[1] based on instantaneous measurements. We take advantage of angle sensors for the measurements of vehicle pitch and roll motions, and a magnetic compass for

[1] to achieve no worse than sub-pixel errors in registering video frames at standard resolution.

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rough heading control. We estimate the translation and fine-scale heading variations directly from video. In order to optimize accuracy, strict procedures for sensor/camera installation and calibration must be followed; to minimize additional contribution to the total error of the integrated system. To meet the objectives, a positioning platform has been designed and constructed, which can rotate about the three axes, while supporting the ROV. This allows us to achieve any arbitrary vehicle pose, acquire images at different camera orientations for calibration, while measuring the pitch, roll and yaw angles with the compass and angle sensors. A 360 degrees rotating mechanism supports a planar checkerboard pattern that serves as a calibration object. Precise movement of this calibration target are necessary to ensure both accuracy and repeatability. Measurements in the coordinate frames of the angle sensors, magnetic compass and the video camera are used to compute the transformation among them. Once calibrated, the angle sensors and compass readings can be transformed to the camera coordinate system to obtain measurements in a common coordinate system, e.g., to rectify the raw video before processing by the ROV vision system to estimate translational and fine heading motions.

The balance of the paper describes the constructed positioning platform, and the processes of camera calibration, estimation of the camera rotational motions from images, and the calculation of the transformation to/from the coordinate frames of the sensors and the camera. Experimental results of the calibration process and the testing of the system are presented to evaluate effectiveness and performance.

II. SYSTEM CALIBRATION

This paper deals with an overall calibration of the system. It begins with the intrinsic calibration of a single camera and the extrinsic stereo calibration (since we deploy a stereovision system on our ROV), proceeding with angle sensor calibration, and finalizing with the transformation between the vision and sensor coordinate systems to rectify the raw video.

A. Intrinsic Calibration To determine the relation between a scene and its image taken by the camera, we need knowledge of certain internal camera parameters [15]. These parameters include:

- Focal length (pixel units), stored in the 2x1 vector f_c;
- Principal point coordinates, where optical axis intersects the image plane, stored in the 2x1 vector c_c;
- Image distortion coefficients (comprising both radial and tangential components), that depend on the degree of distortion of a camera lens. The same 5 parameters, commonly used is most calibration methods, are stored in the 5x1 vector k_c.

These parameters are calculated by the publicly available Matlab code, [8]. This information is also imperative for stereo calibration where we determine the translation and rotation of one camera with respect to the other.

B. Extrinsic Calibration

For stereo imaging, as in our system, the relative geometric position and orientation between the stereo cameras is critical for applications involving Euclidean reconstruction of the scene from image correspondences. This can also be determined from the same or other calibration algorithms [8, 7]. In our convention, the right camera represents the reference frame of the integrated positioning system.

The calibration of the extrinsic parameters gives:

- Translation vector $\boldsymbol{t}$, which describes the 3x1 vector from the right to the left optical center, expressed in coordinate system of the right camera;
- Rotation between the coordinate systems of the two cameras, can be represented by the 3x3 rotation matrix Ω. Alternatively, it can be expressed by the 3x1 vector $\omega = \theta\hat{\omega}$, comprising a rotation angle θ about the axis $\hat{\omega}$. The relation between ω and Ω is given by the Rodriguez formula:

$$\Omega = (1-\cos\theta)\hat{\omega}\hat{\omega}^T + \cos\theta\, \boldsymbol{I} + \sin\theta \begin{bmatrix} 0 & -\hat{\omega_3} & \hat{\omega_2} \\ \hat{\omega_3} & 0 & -\hat{\omega_1} \\ -\hat{\omega_2} & \hat{\omega_1} & 0 \end{bmatrix}$$

Knowing extrinsic parameters, we can readily transform measurements, including the representation of a (reconstructed) scene, to the left camera. In general, a point P in 3-D space with coordinate vectors $\boldsymbol{x}_L$ and $\boldsymbol{x}_R$ in the left and right camera reference frames are related by $\boldsymbol{x}_R = \Omega\, \boldsymbol{x}_L + \boldsymbol{t}$.

C. Calibration of Angle Sensors

The compass and angle sensors output absolute voltages measuring the orientation of the vehicle for each of the three pitch, roll and yaw axes. Two procedures may be adopted: 1) Determine the direct transformation between the sensor measurements (voltage) and camera rotations (radians); 2) Calibrate the sensors first to determine voltage to angle transformation, and then determine the transformation between the sensor measurements (radians) to camera rotations (radians). The sensor voltage/angle calibration needs to be done once, but the transformation from sensor readings to the camera coordinate system has to be repeated any time the sensors and (or) the cameras are removed and reinstalled.

The angle sensor calibration involves calculating the gains and offsets, and to correct for misalignment and (or) coupling between pitch p and roll r measurements. The physical characteristics of the device determine the voltage $\{p, r\}$ to angle $\{P, R\}$ transformation, which is ideally linear over the operation range. Accordingly, we utilize the model

$$\begin{bmatrix} P \\ R \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} p \\ r \end{bmatrix} + \boldsymbol{b} \qquad (1)$$

in a least-square formulation to determine the transformation parameters– 2x2 matrix $\boldsymbol{A}$ and 2x1 vector $\boldsymbol{b}$– from redundant measurements. A scalar form is applied for the compass heading readings; h (voltage) and H (radians).

D. Transformation between Sensors and Camera Frames

The sensors and the vision system (right camera) each has its own coordinate system. The rotation of the positioning system can be determined from the sensors and the vision system, however, each is based on the coordinate system of the equipment used. The objective is to find the transformation between these coordinate frames from redundant measurements in various frames. The angle measurements

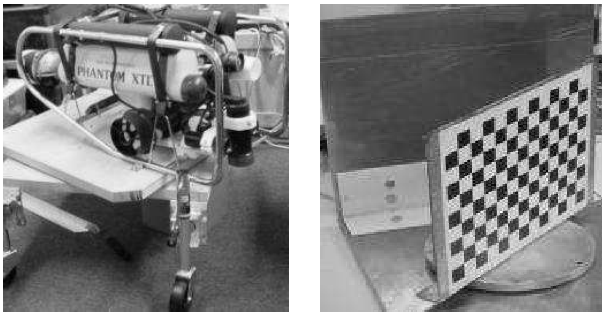

Figure 1: ROV on the positioning platform and the calibration target.

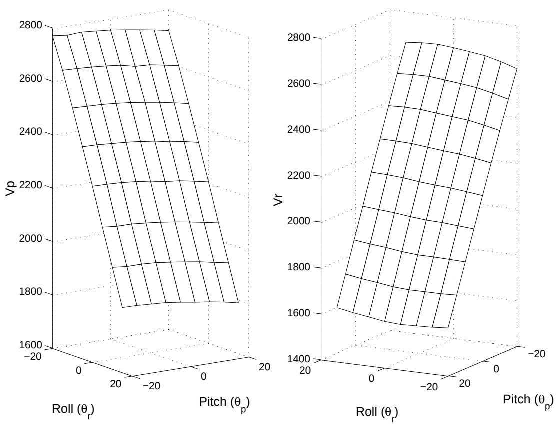

Figure 2: Pitch (left) and roll (right) sensor readings (voltage) for various rotations of the calibration turntable verify linearity of these sensors.

$\{P, R, H\}$ from the sensors to $\{P_c, R_c, H_c\}$ for the camera are related through the transformation:

$$\begin{bmatrix} P_c \\ R_c \\ H_c \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} P \\ R \\ H \end{bmatrix} + \boldsymbol{g} \tag{2}$$

Ideally, $\boldsymbol{T}$ is a rotation matrix when orthogonal (Cartesian) systems represent the coordinate frames of the camera and angle/compass devices. In practice, the pitch and roll sensors may not be mounted perfectly to have orthogonal axes. In our system, we use a dual-axes crossbow unit that measures both angles [3], and is mounted on (and aligned with the XY-axes of) the down-look stereo camera assembly. Furthermore, the compass heading sensor may typically be independent, as for the Phantom XTL. The offset vector $\boldsymbol{g}$ is necessary to account for the absolute and relative nature of the measurements of the angle/compass sensors and image-based methods, respectively. In particular, while the first two elements of $\boldsymbol{g}$ are fixed once the system is calibrated, the last element can change depending on the ROV heading at the start of the mission. Thus, it can be reset with the first readings at the beginning of video recording.

III. POSITIONING PLATFORM & CALIBRATION TARGET

The calibration processes requires sensor measurements at a minimum of 4 ROV orientations. For robust results, many more data points are to be recorded. To carry out the calibration effectively and efficiently requires a platform that would support the positioning of the ROV at arbitrary orientations quite readily, before deployment. For this purpose, a positioning platform, shown in fig. 1, was designed and constructed. The 3'x 3' platform is capable of simulating pitch, roll and yaw movements with a range of 30 degrees in each orientation. It consists of two wood plates attached to each other by means of a swivel that allows for the heading movement. The top plate supports the vehicle, while the bottom support the legs. Four manual jacks were used as legs to create the pitch and roll movements. These jacks were tested to withstand 750lb each and have a lift distance of 10 ft. The lowering of the two front/side jacks simultaneously creates the roll/pitch movements.

Fig. 1 shows the checkerboard pattern used in the calibration process, attached to a rotating platform with the ability to rotate into 36 different orientations with accuracy and repeatability. This rotating target functions independently of the positioning platform, allowing us to move it into different setting points while the ROV is in a given pose to collect data for intrinsic and extrinsic camera calibration. To calibrate the integrated sensor-camera positioning system, we instead move the ROV in different poses, while recording images of the pattern for calculating the rotational motion with the vision system.

IV. EXPERIMENTS

A. Sensor Calibration

The stereo cameras were calibrated first, to determine both the extrinsic and extrinsic parameters. Next, the angle sensors were calibrated to determine the transformation between voltage outputs and degree angles, for the pitch and roll sensors, as well as for the magnetic compass. As stated, our crossbow pitch and roll sensors are set up on a fixed mount, defining a single coordinate system. The heading sensor, installed in the closed frame of our Phantom XTL ROV, was calibrated separately. The pitch and roll sensors were positioned on a calibration turn-table, taking measurements every 5 degrees in the [-15,20] range in each direction. This is highly redundant with a total of 72 readings to estimate 6 coefficients, however, it is done only once. To verify the linearity of the sensors, the voltages for various rotations have been given as a mesh plot, for each of the pitch and roll sensors; see fig. 2. The proposed model in (1) was then applied in a least-square formulation to determine the 6 transformation parameters in $\boldsymbol{A}$ and $\boldsymbol{b}$. The same calibration process was repeated for the heading sensor.

For the calibration of the integrated positioning system, the ROV is tighten into place on the calibration platform with metal straps. The checkerboard pattern is placed underneath the right camera (down-look configuration), keeping it in the field of view over the entire data set collected for calibration. The first set of images is the reference set, where the calibration platform has no intentional pitch and roll angles (as stated before, the heading reference may be set arbitrarily). A set comprises six images of the checkered pattern, each at a different orientation. These different orientations are obtained by rotating the pattern six times, from left to right every 10 degrees, while keeping the pattern in the field of view. Simultaneously, the pitch, roll and heading measurements of the ROV pose are recorded. As a result, the set consists of six

angle sensors (deg)			vision system (deg)		
pitch	roll	yaw	pitch	roll	yaw
+0.01	-0.19	+3.79	+9.25	-0.11	206.83
+0.03	-0.47	+7.52	+9.25	-0.15	201.51
-0.04	+0.16	-2.79	+9.20	+0.02	216.67
-0.12	+0.31	-6.58	+9.19	+0.14	222.54
-0.17	+0.58	-9.38	+9.17	+0.06	226.94
+0.00	-0.84	+12.06	+9.26	-0.23	195.27
+0.43	-10.27	-1.59	+9.54	+10.11	217.85
+0.51	-11.79	-2.08	+9.55	+11.82	218.91
-0.22	+5.13	+0.23	+9.00	-5.38	211.51
-0.39	+10.53	+0.33	+8.94	-10.48	211.14
-0.50	+12.52	+0.54	+8.87	-12.32	210.82
-4.87	-0.26	+2.30	+4.47	-0.07	209.18
-9.32	-0.07	-0.35	+0.01	-0.51	213.11
-12.13	+0.69	-0.63	-2.91	-0.82	213.22
+4.35	+0.23	+0.94	+13.55	-0.27	211.80
+9.24	+0.17	+1.56	+18.53	-0.26	211.79
+5.28	-0.02	+7.26	+14.25	-0.87	202.75
+0.22	-2.61	-5.56	+9.22	+2.63	222.39
-2.86	-3.20	+3.70	+6.12	+2.46	208.36
+4.32	+2.03	+6.51	+13.19	-2.88	203.60

Table 1: Pitch, roll and yaw angles (deg) as measured by the angle sensors and compass (first 3 columns) and the vision system (last three columns).

images with the corresponding pose measurement from the sensors. The ROV is then rotated to another orientation, e.g., -5 degrees of pure pitch, and the same steps are repeated. The objective is to fix the ROV in different orientations obtaining a number of image sets with sensor angles. For our experiment, a total of 21 sets were used in order to obtain robust results. Each set is then processed relative to the reference set by the vision system to estimate the (right) camera rotation, giving us 20 data sets of rotational motions. For this, the Matlab calibration toolbox was used [8], and the results are given in table 1. It is noted that the vision system estimates actually correspond to the changes in pitch, roll and yaw angles from the original reference position, as visual cues can yield the rotational motion from one image to the next.

A least-square formulation based on (2) was applied to determine the transformation parameters. These are given by

$$\boldsymbol{T} = \begin{bmatrix} 1.01 & 0.014 & -0.0047 \\ 0.012 & -1.03 & 0.057 \\ 0.051 & 0.122 & -0.672 \end{bmatrix} \quad \boldsymbol{g} = \begin{bmatrix} -8.2 \\ -12.3 \\ 142.7 \end{bmatrix} \tag{3}$$

A few conclusions can immediately been made. The fact that special attention was paid to aligning the axes of the angle sensors with the XY axes of the camera coordinate frames is verified by the first two columns of $\boldsymbol{T}$ (roll axes are defined in opposite directions). Furthermore, any deviation from orthogonality of the pitch and roll axes can be established from the dot/cross product of these two rows (after normalization to get unit vectors). For our unit, this was estimated to be 90.2 degrees from the data. The angle errors, given in fig. 3, verifies the accuracy of model in (2) and the results of the least-square fit.

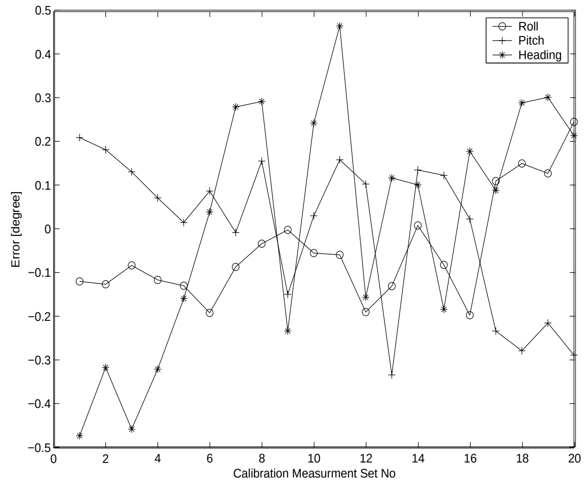

Figure 3: Angle errors for each of the 20 data sets in the computation of the transformation matrices in (2) verifies the accuracy of the fit.

This transformation parameters can be used to rectify the raw video for pitch, roll, and yaw stabilization. The new video involves solely the translational motions of the ROV and potentially some residual yaw motion, due to low-accuracy of the compass readings. These can be determined from a 4 d.o.f. motion estimation algorithm, implemented and extensively tests on our Phantom XTL (Trademark of Deep Ocean Engineering, San Leandro, CA). The primary advantage of processing the rectified video is to have resolved the translational-rotational ambiguity of visual motion, by eliminating the pitch and roll effects.

B. Testing of Calibrated System

We next demonstrate the pitch-roll stabilization through two experiments. The first involves a reference image of the calibration pattern (see top row of fig. 4), and several frames after a certain rotation of the ROV, while fixed on the positioning platform (see left column of fig. 4). The ROV pose is determined from the compass and angle sensors. Using the transformation in (2) and the calibration parameters in (3), these measurements are transformed to the coordinate system of the right camera; that is, we calculate its rotation due to ROV motion. This is then used to rectify each of the images acquired at the rotated position of the ROV, which are given in the middle row of fig. 4. One can immediately note that the ROV rotation not only induces a camera rotation, but also some 3-D translation. Comparing the reference frame (top row) with each rotation-rectified image visually, it is hard to determine 1) if the rotation effects are entirely removed (aside from some fine-scale yaw motion due to inaccuracy of the compass), 2) the amount of translation between the two frames. This translation is hard to quantify visually. To assess both quantitatively, this translation between the reference and each rectified image, is determined from our 4 d.o.f. motion estimation algorithm. Finally, the rotation-rectified images are again warped (shifted, scaled and rotated) according to these new measurements. The final results are given in the right column, with the estimated parameters (yaw rotation in degree, calling, and XY shifts in

pitch	roll	yaw
-0.9071	-14.1105	0.1685
-1.0367	-7.7973	0.3295
-0.6936	-0.4146	0.6523
-0.3647	6.8398	0.6767
-0.4755	10.1550	2.4101
-0.6583	11.7243	2.3241

Table 2: Camera rotation angles (degrees) computed from the transformation of sensor angles; second data set with 5 images involving the roll motion of the ROV.

pixels) below each image in the 2nd column. The alignment of each final image with the reference frame can be visually examined. Quantitatively, the alignment error over the checkerboard corner points (108 points in total) has a mean of less than 0.5 pixel.

The next experiment involves a sequence (6 frames) while the vehicle undergoes a pitch motion, e.g., simulating cyclic motion due to sideways wave disturbances, while viewing a rock as a 3-D object on a flat bottom. The same process is repeated with the image on row 3 as the reference image. The results are shown in table 2 and fig. 5. Some small discrepancies in the visual appearance of the rock between the reference image and all other rotation-translation rectified images can be noted due to the 3-D nature of the object shape.

V. SUMMARY

High resolution and data rate of image-based techniques offer a critical capability for accurate positioning of submersible vehicles in short-range local operations. Unfortunately, not all 6 d.o.f in ROV movements can be robustly determined from vision techniques, due to the inherent visual motion ambiguities of fine rotations and translations. To increase accuracy, certain motion components need to be measured from other sensors.

We have proposed an integrated positioning system that utilizes the available sensors on standard ROVs; namely a magnetic compass, angle sensors for pitch and roll measurements, and a video camera. The calibration of the entire system has been described, including various platforms that have been designed and constructed to carry out the calibration processes effectively and accurately. The application and performance of the 6 d.o.f vision-based positioning system has been demonstrated through experiments with real images. In particular, it has been shown that the estimates of the ROV motions allows us to register video images within sub-pixel accuracy. This is critical for a number of applications, including video stabilization, photo-mosaicing, ROV positioning and 3-D mapping from stereo vision. Ongoing work for ocean trial at the end of Summer'03 involves the implementation on our phantom XTL vision system for real-time processing.

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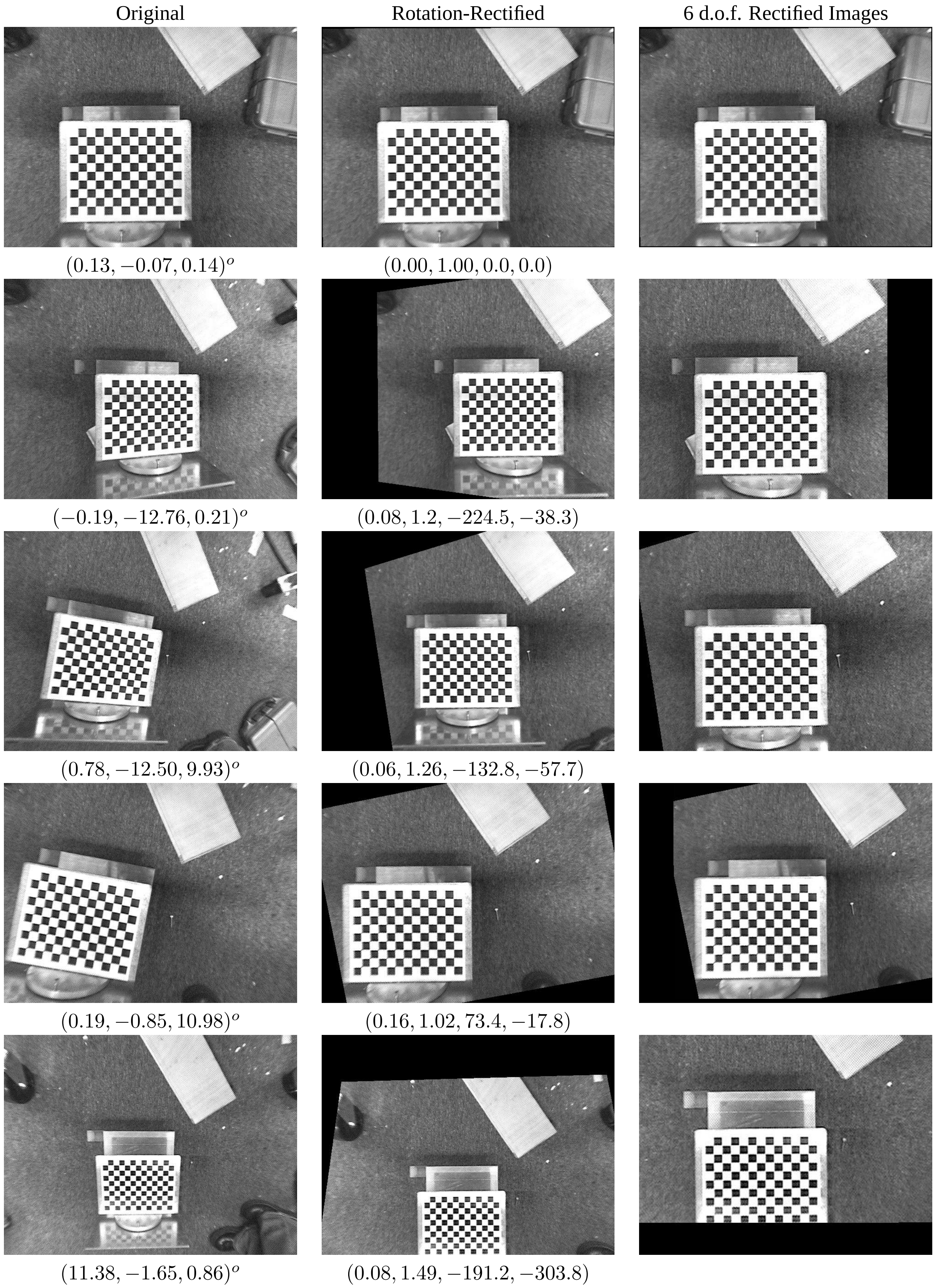

Figure 4: Original, pitch/roll/yaw rectified, and final images after fine rotation, scaling, and shift as computed from the 4 d.o.f. motion estimation algorithm. Below each image in the first column are the corresponding rotation angles. Numbers in the 2nd column are the fine rotation, scale and the xy shifts for the alignment of each image with the reference frame (given in the first row).

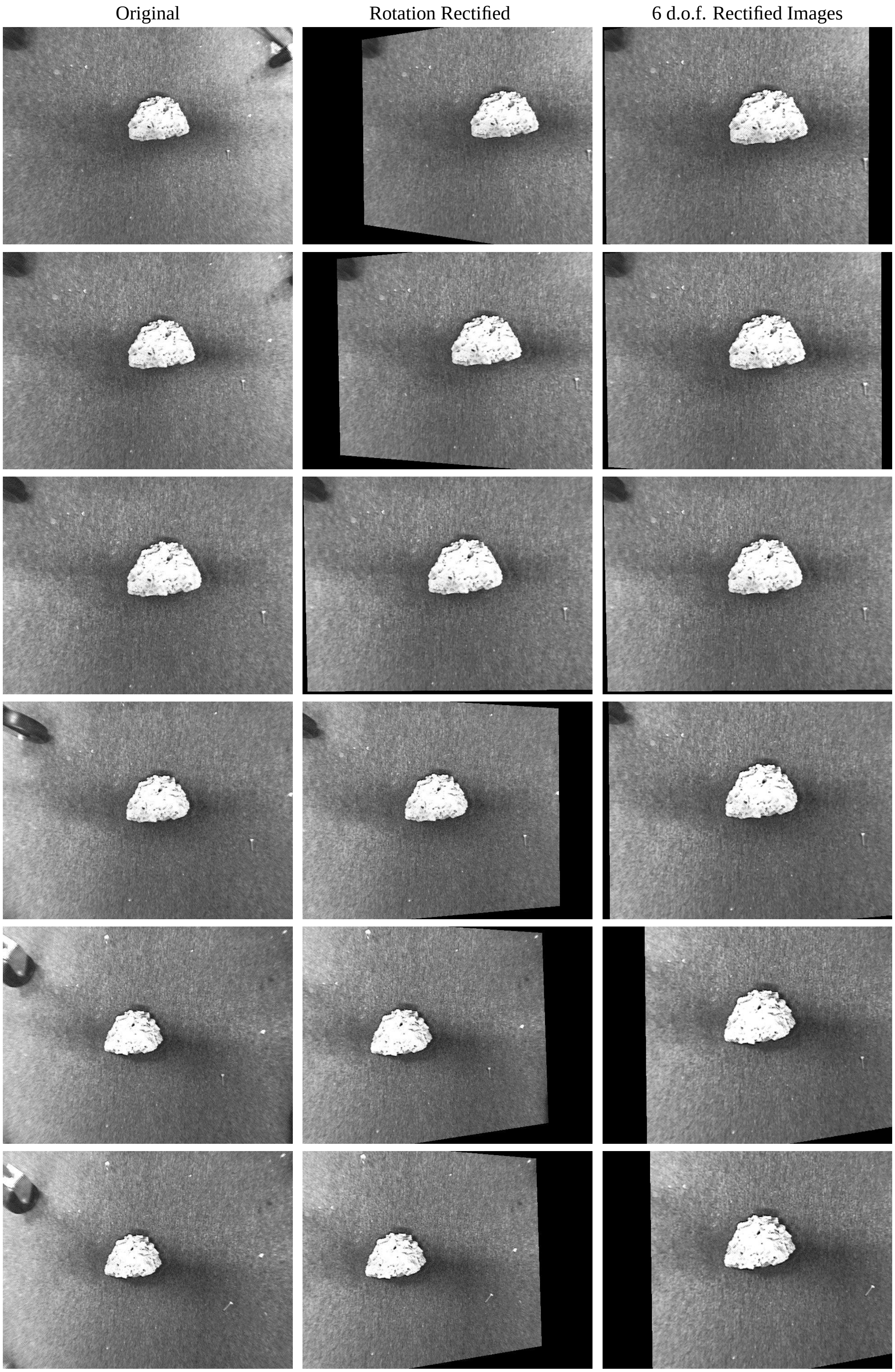

Figure 5: Original data set with ROV roll motion (left), rectified frames with computed camera rotation based on transformed readings from the compass and angle sensor (middle), and finally the shifted and scaled images based on estimates from the 4 d.o.f. translation/yaw motion algorithm (right). The image in the 3rd row is the reference frame for the 2nd rectification step.