Multivariable Optimal Control of an Autonomous Underwater Vehicle for Steering and Diving Control in Variable Speed

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Abstract - **This paper describes a multivariable optimal control for a Semi-Autonomous Underwater Vehicle (SAUV) developed in Korea Ocean Research and Development Institute (KORDI). The SAUV is a test-bed for evaluation of navigation algorithms and manipulator technologies for a mine disposal vehicle (MDV) in military use and for a light working underwater vehicle in scientific application. The vehicle was designed to control its cruising speed, heading and depth with 4 horizontal thrusters installed at the rear of the hull. The decoupled control methods are limited to be applied to the SAUV because the thrust forces are highly coupled with the surging, yawing, and pitching motion of the vehicle. The multivariable Linear Quadratic (LQ) control method is chosen to control steering and diving in variable speed motion automatically. This paper presents the results of tank-test with the controller and compares them with the simulation results to validate the performance.**

I. INTRODUCTION

A semi-autonomous underwater vehicle (SAUV) named DUSAUV (Dual Usage Semi-Autonomous Underwater Vehicle) has been developed in Korea Ocean Research and Development Institute (KORDI). The DUSAUV is a test-bed for the evaluation of navigation algorithms and manipulator technologies for a mine disposal vehicle (MDV) in military use and for a light working underwater vehicle in scientific application [1]. The external appearance of DUSAUV is depicted in the Fig. 1. In order to satisfy the requirements as a test-bed for the two technologies, the DUSAUV is designed to include proper measurement and actuator units. An inertial measurement unit (IMU), super short baseline (SSBL), Doppler velocity log (DVL), magnetic compass, depth sensor, altitude sonar and a navigation computer are equipped to implement the navigation algorithm. A four function electric driven manipulator, CCD camera and identification sonar on pan/tilt unit, and a manipulator control computer are equipped for the implementation of the underwater manipulation system. A vehicle control computer is included to carry out the basic functions of the test-bed such as supervisory task, communication, sensor interface and so on. The four horizontal thrusters are mainly used for steering and diving autopilot at the cruising mode while a lateral and two vertical thrusters are used only at the hovering mode. The video image transmission and digital data communication between the DUSAUV and surface unit are implemented using fiber optic link while the electric power is supplied from the lead-acid batteries embedded on the vehicle. The arrangement of equipment is described in Fig. 2 and the principal specification is in TABLE 1.

In this paper, the design, simulation and experiment of the control system, which is one of the basic functions for DUSAUV to carry out its role as a test-bed for the testing of the two key technologies, are described. A number of controller design method for the dynamic control of underwater vehicle is presented like as the sliding mode control [2,3], robust control [4] and so on. Most of the design methods are designed assuming that the vehicle model has rudder and stern plane to control its motion on the steering and diving plane respectively. However, because the DUSAUV controls its motion with only four horizontal thrusters at the cruising mode, the input of each thruster is coupled with all of the speed, steering and diving motion. Therefore, the combined model for the control of speed, steering and diving motion is developed, and the optimal linear quadratic controller based on the developed model is designed for the motion control with optimal thrust distribution. The optimal gain of the controller is tuned using computer simulation environment with nonlinear 6-DOF numerical vehicle model. To validate the performance of controller, the tank-test is carried in the ocean engineering basin in KRISO. The experimental results and the simulation results under the same condition are compared.

Fig. 1 Appearance of DUSAUV

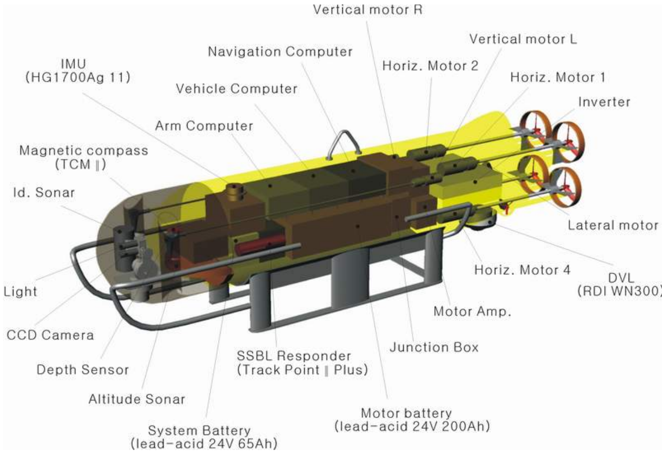

Fig. 2 The arrangement of equipment

The three-dimensional nonlinear dynamic equation of submarsible motion has been described in [5, 6, 7]. Ignoring the effect of currents and fiber optec cable, we can decribe the dynamic equation of DUSAUV as the form in [5] or [6]. Considering the body coordinates system in Fig. 3 and classifing the terms in the dynamic equation, we can get the following equation for DUSAUV.

$$M\dot{v} = F_{CC} + F_{DL} + F_{rest} + F_{thrust}\ , \tag{2.1}$$

where $\dot{v} = \{u, v, w, p, q, r\}^T$ is linear and angular velocity vector with respect to the body coordinates frame, M is inertial term including the added mass, F_{CC} is coriolis and centrifugal force term, F_{rest} is restoring force, and F_{thrust} is thrust force from the four horizontal thrusters. Note that there is no F_{plane} term because the DUSAUV has no plane for motion control. Each element terms of (2.1) are listed in APENDIX A. The hydrodynamic coefficients in the model are developed by VPMM (Vertical Planner Motion Mechanism) test

TABLE 1
Principal Characteristics of the DUSAUV

Principal Dimension	3.188(L) x 0.745(W) x 0.988(H) m
Weight in air	627kg
Speed/operating time	Max. 7 knots /0.5 hour Cruising 3.5 knots / 6 hours
Operating depth	400m
Thrusters	4 Horizontal – 600W each 2 Vertical – 200 W each 1 Lateral – 150 W
Navigation & Measurement	IMU – HG1700AG11 (Honeywell) SSBL – Track Point II plus (ORE) Compass – TCM II (Precision Nav.) Speed Log – WN 300kHz (RDI) Depth Sensor – PS10000 (Schaevitz) Id. Sonar – SeaKing (Tritech) Altitude Sonar – PA500 (Tritech) CCD Camera – OceanSpy (Tritech) Light – Mini-SeaLite 100W (Deepsea)
Communication	Fiber Optic Communication
Manipulator	Four function electric

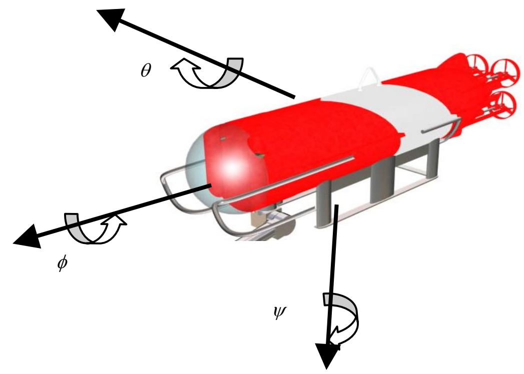

Fig. 3 Body coordinates system of DUSAUV

which is carried out in the towing tank (L200m ×W16m×D7m) in KRISO with 1:1 scale model of DUSAUV. The developed coefficients are nondimensionalized with length of vehcile and listed in APPENDIX B. The roll damping coefficient which can not be devlopted by the facility, is analogized from that of similar vehicles.

The v in (2.1) can be written with respect to the earth-fixed coordinates frame with the transformation matrices J 's as follows.

$$\dot{\eta}_1 = J_1(\eta_2)v_1 \Leftrightarrow v_1 = J_1^{-1}(\eta_2)\dot{\eta}_1 \tag{2.2}$$

$$\dot{\eta}_2 = J_2(\eta_2)v_2 \Leftrightarrow v_2 = J_2^{-1}(\eta_2)\dot{\eta}_2\ , \tag{2.3}$$

where $v_1 = \{u, v, w\}^T$, $v_2 = \{p, q, r\}^T$ are linear and angular velocity with respect to the body coordinates frame respectively, $\eta_1 = \{X, Y, Z\}^T$, $\eta_2 = \{\phi, \theta, \psi\}^T$ are position and attitude vector with respect to the earth-fixed frame respectively, and J_1, J_2 are linear and angular velocity transformation matrices refered to APPENDIX A.

III. CONTROLLER DESIGN

The simplified equations for surging, steering and diving motion can be derived from (2.1), (2.2) and (2.3) by ignoring the cross-flow terms and higher order terms of thrust model. Considering only the first order term of thruster model (refer to APPENDIX B), we get (3.1)-(3.6) from (2.1), (2.2) and (2.3) as simplified model for the surging, pitching, and yawing motion of DUSAUV.

$$(m - \frac{1}{2}\rho l^3 X_{\dot{u}})\dot{u} = \frac{1}{2}\rho l^2 b_X (u_{T1} + u_{T2} + u_{T3} + u_{T4}) \tag{3.1}$$

$$(I_y - \frac{1}{2}\rho l^3 M_{\dot{q}})\dot{q} = \frac{1}{2}\rho l^4 u M_q q + \frac{1}{2}\rho l^3 b_M (u_{T1} + u_{T2} + u_{T3} + u_{T4}) \tag{3.2}$$

$$(I_z - \frac{1}{2}\rho l^3 N_{\dot{r}})\dot{r} = \frac{1}{2}\rho l^4 u N_r r + \frac{1}{2}\rho l^3 b_N (u_{T1} + u_{T2} + u_{T3} + u_{T4}) \tag{3.3}$$

$$\dot{Z} = -u \sin\theta \tag{3.4}$$

$$\dot{\theta} = q \tag{3.5}$$

$$\dot{\psi} = r\ , \tag{3.6}$$

where ρ is density of water, l is the length of vehicle and u_{Ti} is

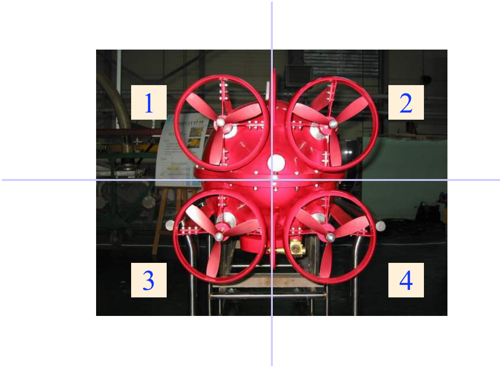

Fig. 4 Assignment of horizontal thruster number

defined as (3.7) using designed vehicle speed u_0 and designed propeller rps n_0. The number of subscript corresponds to the thruster number assigned in Fig. 4

$$u_{Ti} = \frac{u_0 n_i}{n_0 u} - 1, \quad i = 1, 2, 3, 4 \tag{3.7}$$

Linearizing (3.4) at the operating point u_0 and representing (3.1)-(3.6) as a state space form with the states $x = \{u\ q\ r\ Z\ \theta\ \psi\}^T$, we get following state space equation for the DUSAUV.

$$\hat{M}\dot{x} = A_1 x + B_1 u_T \tag{3.8}$$

$$\dot{x} = Ax + Bu_T \tag{3.9}$$

$$y = Cx, \tag{3.10}$$

where

$$A = \hat{M}^{-1} A_1, \quad B = \hat{M}^{-1} B_1 \tag{3.11}$$

$$\hat{M} = \begin{bmatrix} m - \frac{1}{2}\rho l^3 x_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & I_y - \frac{1}{2}\rho l^3 M_{\dot{q}} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_z - \frac{1}{2}\rho l^3 N_{\dot{r}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.12}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\rho l^4 u M_{\dot{q}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\rho l^4 u N_{\dot{r}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.13}$$

$$B_1 = \frac{1}{2}\rho l^2 \begin{bmatrix} b_X & b_X & b_X & b_X \\ -lb_M & -lb_M & lb_M & lb_M \\ lb_N & -lb_N & lb_N & -lb_N \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad u_T = \begin{Bmatrix} u_{T1} \\ u_{T2} \\ u_{T3} \\ u_{T4} \end{Bmatrix} \tag{3.14}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.15}$$

Equation (3.9) is linear state space form to construct the LQ controller combining the speed, steering, diving motion of DUSAUV. The optimal control law utilizing feedback from $x(t)$ and feedforward from $y_d(t)$ can be obtained as (3.17) by the minimization of the following quadratic performance index [7].

$$J = \frac{1}{2}\int_0^T \left(\tilde{y}^T \tilde{Q} \tilde{y} + u^T R u\right) d\tau = \frac{1}{2}\int_0^T \left(\tilde{x}^T \tilde{Q} \tilde{x} + u^T R u\right) d\tau, \tag{3.16}$$

where $\tilde{y} = y - y_d = C(x - x_d) = C\tilde{x}$, $\tilde{Q} = C^T Q C$, Q and $\tilde{Q}$ are semi-positive definite matrix, and R is positive definite matrix.

$$u_{opt} = G_1 x + G_2 y_d \tag{3.17}$$

$$G_1 = -R^{-1} B^T P \tag{3.18}$$

$$G_2 = -R^{-1} B^T \left(A + BG_1\right)^{-T} C^T Q, \tag{3.19}$$

where P is the solution of the following Riccati equation.

$$PA + A^T P - PBR^{-1}BR^{-1}B^T P + \tilde{Q} = 0 \tag{3.20}$$

Using the optimal control law $u_{opt} = \{u_{T1}^{opt}, u_{T2}^{opt}, u_{T3}^{opt}, u_{T4}^{opt}\}^T$ in (3.17), we get the rps command of the four thrusters from (3.7) as

$$n_i^{opt} = \frac{n_0}{u_0} u(u_{Ti}^{opt} + 1), \quad i = 1, 2, 3, 4 \tag{3.21}$$

IV. EXPERIMENT AND DISCUSSIONS

The numerical computer simulation and experiments are performed to validate the performance of control system. The numerical model for simulation is constructed using the 6-DOF non linear equation of motion (2.1) on Matlab simulink environment. The available test area of the ocean engineering basin in KRISO is restricted to 35m×50m×3.5m. Therefore, The weight matrix Q and R in the performance index of LQ controller are tuned for the responses to be adequate for the test area as

$$Q = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 40 \end{bmatrix}, \quad R = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \tag{4.1}$$

And the optimal gain matrices are obtained as

$$G_1 = \begin{bmatrix} -4.4721 & 26.6231 & -8.0006 & -8.6603 & 32.5062 & -10.0000 \\ -4.4721 & 26.6231 & 8.0006 & -8.6603 & 32.5062 & 10.0000 \\ -4.4721 & -26.6231 & -8.0006 & 8.6603 & -32.5062 & -10.0000 \\ -4.4721 & -26.6231 & 8.0006 & 8.6603 & -32.5062 & 10.0000 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 4.4721 & 8.6603 & 10.0000 \\ 4.4721 & 8.6603 & -10.0000 \\ 4.4721 & -8.6603 & 10.0000 \\ 4.4721 & -8.6603 & -10.0000 \end{bmatrix} \tag{4.2}$$

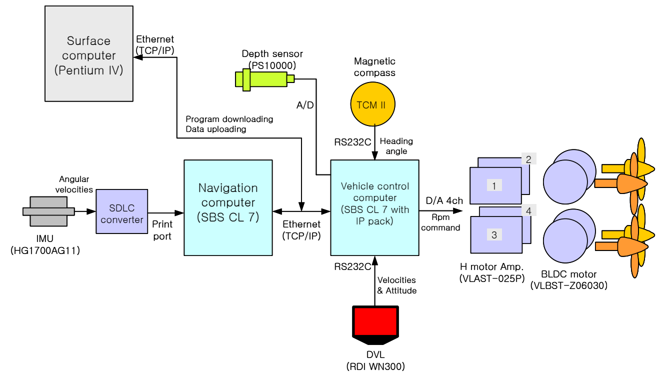

Fig. 5 Hardware configuration for experiment

Fig. 6 Tank-test of DUSAUV

All of the states are measured to feed them back to the controller in the experiments. A Doppler velocity log (RDI WH 300Hz), IMU (HG1700AG11) and a magnetic compass (TCM II) are used to measure the pitch angel and linear velocities, angular velocities, depth, and heading angle respectively. Hardware configuration for the experiment is shown in Fig. 5. The sampling time of controller is 0.1 second. The initial values of states and outputs are listed in TABLE 2. From the initial reference of speed, depth and heading given as $y_d^0 = \{u_d, Z_d, \psi_d\}^T = \{1, 0.5, 60\}^T$, the heading reference was changed to 150 degrees after 20 seconds, that is $y_d^{20} = \{u_d, Z_d, \psi_d\}^T = \{1, 0.5, 150\}^T$.

The experimental results of LQ control are depicted in Fig. 7-11 with simulation results under the same condition. The solid lines in figures show the results of tank-test and the dashed lines are simulation results. The experimental outputs show good agreements with those of simulation in Fig. 7. The difference between the velocity responses of simulation and experimental results observed at the initial acceleration section, is caused by the unmodeled uncertainties such as the cable drag and thurst leakage resulted from the exposure of propeller on the surface when it starts to pitch down as shown in Fig. 6. When the heading reference is changed to 150 degrees, it was found that the heading output follows the new reference within 15 seconds. The speed output is decreased up to 0.5 m/sec while the depth output is following the reference, which also has good agreement with the simultion result. It is supposed that the drift angle from the change of heading contributed to the decreased forwad speed, u in Fig. 7, and increased leftward drift speed, v in Fig. 8.

The responses of angular rate and attitude in Fig. 9 and 10 also show the similar trend between the simulation and experimental results. The offset angle between simulation and experimental results found in the roll response, phi in Fig. 10, is caused by the difference of the trimming condition between the simulation model and the real system. The real system has inclination about 8 degrees in roll angle.

TABLE 2
Initial condition of tank-test

Z [m]	ϕ [degree]	θ [degree]	ψ [degree]	p [rad/sec]
-0.3674	8.4	5.0	75.4	-0.0071
q [rad/sec]	r [rad/sec]	u [m/sec]	v [m/sec]	w [m/sec]
-0.0049	0.001	0.055	-0.006	-0.015

Fig. 11 shows the optimal control inputs of four thrusters. Note that the thruster input at the initial stage is limited by the forward speed of vehicle because the rps commands of thrusters are obtained by multiplying the speed of vehicle shown as (3.21). The port side

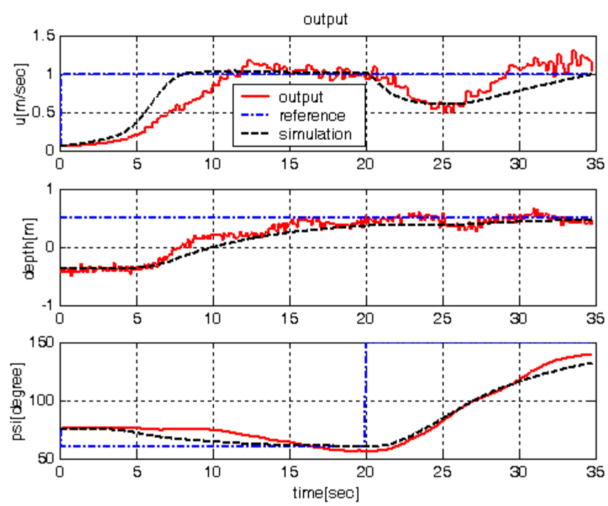

Fig. 7 Control output response

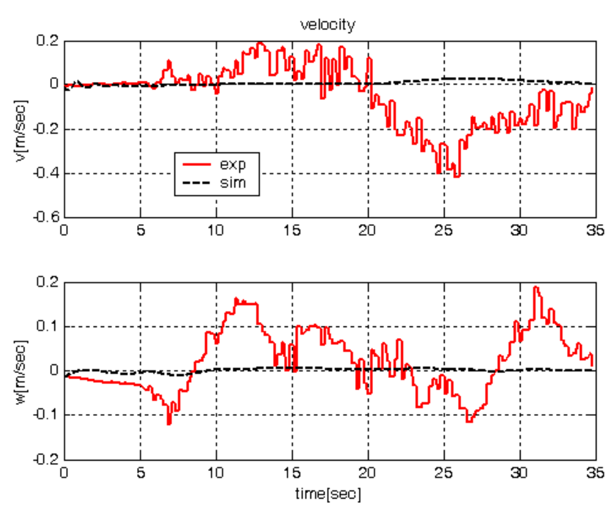

Fig. 8 Linear velocity response

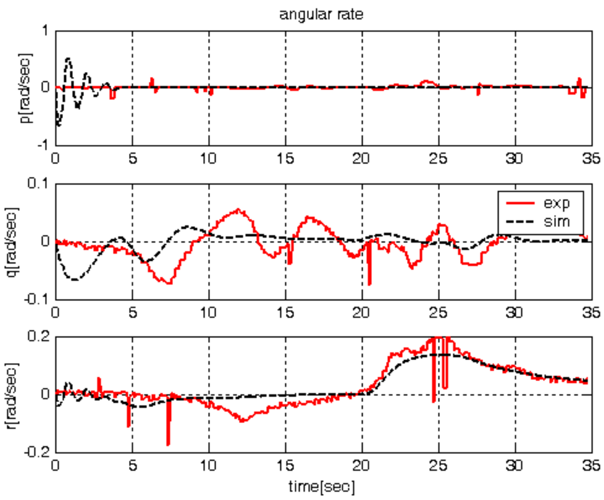

Fig. 9 Angular rate response

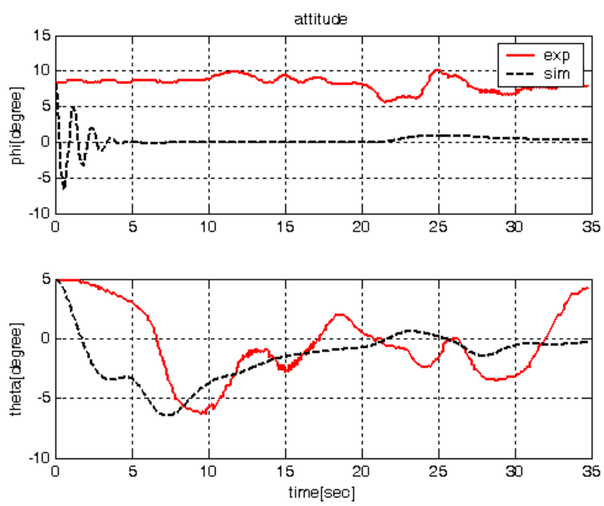

Fig. 10 Roll and pitch angle response

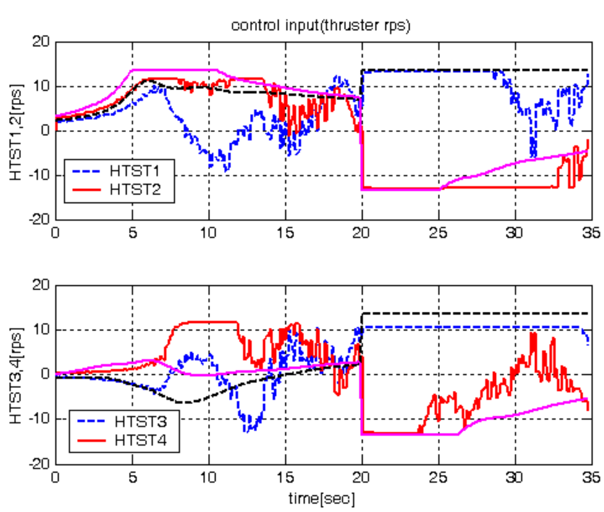

Fig. 11 Thruster control input

thrusters (HTST1, 3) are saturated to upper limits while the starboard side thrusters (HTST2, 4) are saturated to lower limits in order to get maximum rightward angular rate of heading at the time of 20 seconds. According to the decreased heading error the thruster inputs are also decreased.

V. CONCLUSIONS

The design, simulation and experiment of a multivariable optimal controller for a SAUV named DUSAUV are presented. The thruster input of DUSAUV is coupled with the speed, steering and diving motion of the vehicle because the SAUV was deigned to control its motion only with four horizontal thrusters without any fins at cruising mode. Therefore, the LQ design method was utilized to control the steering and diving motion in variable speed. The control simulation was performed with the numerical model with the hydrodynamic coefficients developed by VPMM test in towing tank. The experiment was carried out in ocean engineering basin of KRISO. It was found that the designed controller shows good performance to control the speed, depth and heading at the same time from the both of simulation and experimental results. Moreover, the simulation results with the developed numerical model are in accordance with the experimental results. It was remained as a future work to perform field trials.

ACKNOWLEDGMENT

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APPENDIX A

$m = 652.8$, $\Delta = 652.8$, $l = 3$

$I_x = 29.16$, $I_y = I_z = 360.05$, $I_{xy} = I_{yz} = I_{zx} = 0$

$(x_G, y_G, z_G) = (0,0,0)$, $(x_B, y_B, z_B) = (0,0,-0.02)$

$u_0 = 2$, $n_0 = 5.2$

- Inertial matrix

$$M = \begin{bmatrix} m-\frac{\rho}{2}l^3X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m-\frac{\rho}{2}l^3Y_{\dot{v}} & 0 & -mz_G-\frac{\rho}{2}l^4Y_{\dot{p}} & 0 & mx_G-\frac{\rho}{2}l^4Y_{\dot{r}} \\ 0 & 0 & m-\frac{\rho}{2}l^3Z_{\dot{w}} & my_G & -mx_G-\frac{\rho}{2}l^5Z_{\dot{q}} & 0 \\ 0 & -mz_G-\frac{\rho}{2}l^4K_{\dot{v}} & my_G & I_x-\frac{\rho}{2}l^5K_{\dot{p}} & -I_{xy} & -I_{zx}+\frac{\rho}{2}l^5K_{\dot{r}} \\ mz_G & 0 & -mx_G-\frac{\rho}{2}l^4M_{\dot{w}} & -I_{xy} & I_y-\frac{\rho}{2}l^5M_{\dot{q}} & -I_{yz} \\ -my_G & mx_G-\frac{\rho}{2}l^4Y_{\dot{r}} & 0 & -I_{zx}-\frac{\rho}{2}l^5K_{\dot{r}} & -I_{yz} & I_z-\frac{\rho}{2}l^5N_{\dot{r}} \end{bmatrix}$$

- Coriolis and centrifugal force

$$F_{CC} = \lfloor X_{CC} \;\; Y_{CC} \;\; Z_{CC} \;\; K_{CC} \;\; M_{CC} \;\; N_{CC} \rfloor^T$$

$$X_{CC} = mvr - mqw - my_Gpq - mz_Gpr + mx_Gq^2 + mx_Gr^2$$

$$Y_{CC} = -mur + mwp - mx_Gqp - mz_Gqr + my_Gq^2 + my_Gr^2$$

$$Z_{CC} = muq - mvp - mx_Grp - my_Grq + mz_Gp^2 + mz_Gq^2$$

$$K_{CC} = my_Guq + mz_Gur - my_Gvp - mz_Gwp - (I_z - I_y)qr + I_{yz}q^2 - I_{yz}r^2 + I_{xz}pq - I_{xy}pr$$

$$M_{CC} = mz_Gvr + mx_Gvp - mx_Guq - mz_Gwq - (I_x - I_z)rp - I_{zx}p^2 - I_{yz}pq + I_{xy}qr + I_{zx}r^2$$

$$N_{CC} = -mx_Gur - my_Gvr + my_Gwq + mx_Gwp - (I_y - I_x)pq + I_{xy}p^2 - I_{xy}q^2 + I_{yz}rp - I_{zx}rq$$

- Damping and lift force

$$F_{DL} = \lfloor X_{DL} \;\; Y_{DL} \;\; Z_{DL} \;\; K_{DL} \;\; M_{DL} \;\; N_{DL} \rfloor^T$$

$$X_{DL} = \frac{\rho}{2}l^2(X_{vv}v^2 + X_{ww}w^2) + \frac{\rho}{2}l^4(X_{qq}q^2 + X_{rr}r^2)$$

$$Y_{DL} = \frac{\rho}{2}l^2(Y_v v + Y_{v|v|}v|v|) + \frac{\rho}{2}l^3Y_r r + \frac{\rho}{2}l^4Y_{r|r|}r|r|$$

$$Z_{DL} = \frac{\rho}{2}l^2(Z_{vv}v^2 + Z_w w + Z_{w|w|}w|w|) + \frac{\rho}{2}l^3Z_q q + \frac{\rho}{2}l^4Z_{q|q|}q|q|$$

$$K_{DL} = \frac{\rho}{2}l^3K_v v + \frac{\rho}{2}l^4K_r r + \frac{\rho}{2}l^5K_{r|r|}r|r|$$

$$M_{DL} = \frac{\rho}{2}l^3(M_{vv}v^2 + M_w w + M_{w|w|}w|w|) + \frac{\rho}{2}l^4M_q q + \frac{\rho}{2}l^5M_{q|q|}q|q|$$

$$N_{DL} = \frac{\rho}{2}l^3(N_v v + N_{v|v|}v|v|) + \frac{\rho}{2}l^4N_r r + \frac{\rho}{2}l^5N_{r|r|}r|r|$$

- Restoring force

$$F_{rest} = \begin{bmatrix} -(W-B)\sin\theta \\ (W-B)\cos\theta\sin\phi \\ (W-B)\cos\theta\cos\phi \\ (y_GW - y_BB)\cos\theta\cos\phi - (z_GW - z_BB)\cos\theta\sin\phi \\ -(x_GW - x_BB)\cos\theta\cos\phi - (z_GW - z_BB)\sin\theta \\ -(x_GW - x_BB)\cos\theta\sin\phi + (y_GW - y_BB)\sin\theta \end{bmatrix}$$

- Thrust force

$$F_{thrust} = \lfloor X_{thrust} \;\; Y_{thurst} \;\; Z_{thrust} \;\; K_{thrust} \;\; M_{thrust} \;\; N_{thrust} \rfloor^T$$

$$X_{thrust} = X_{T1} + X_{T2} + X_{T3} + X_{T4}$$

$$Y_{thrust} = Z_{thrust} = K_{thrust} = 0$$

$$M_{thrust} = -M_{T1} - M_{T2} + M_{T3} + M_{T4}$$

$$N_{thrust} = N_{T1} - N_{T2} + N_{T3} - N_{T4}$$

$$X_{Ti} = \frac{\rho}{2}l^2\{a_X + b_X(\eta_i - 1) + c_X(\eta_i - 1)^2 + d_X(\eta_i - 1)^3\}$$

$$M_{Ti} = \frac{\rho}{2}l^3\{a_M + b_M(\eta_i - 1) + c_M(\eta_i - 1)^2 + d_M(\eta_i - 1)^3\}$$

$$N_{Ti} = \frac{\rho}{2}l^3\{a_N + b_N(\eta_i - 1) + c_N(\eta_i - 1)^2 + d_N(\eta_i - 1)^3\}$$

$$\eta_i = \frac{n_i u_0}{n_0 u},$$

where i = 1, 2, 3, 4, and n_0, u_0 are designed rps of propeller and designed forward speed of vehicle

- Coordinates transformation matrix

$$J_1(\eta_2) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta \\ \sin\psi\sin\theta & \cos\psi\cos\theta + \sin\phi\sin\theta\sin\psi & -\cos\psi\cos\phi + \sin\theta\sin\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

$$J_2(\eta_2) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix}$$

APPENDIX B

m =0.04835599

$I_x = 0.00024$ I_y =0.00296337 I_z =0.00296337

$I_{xy} = I_{yz} = I_{zx} = 0$

u_0 =2 n_0 =5.2

- Hydrodynamic coefficients

$X_{\dot{u}}$ =-0.0024

X_{vv} =-0.00285640 X_{ww} =-0.00454427

X_{qq} =0.04466548 X_{rr} =0.04837608

$Y_{\dot{v}}$ =-0.05257961 Y_v =-0.14264210 $Y_{v|v|}$ =0.03185948

$Y_{\dot{r}}$ =0.00750169 Y_r =0.04083242 $Y_{r|r|}$ =0.035929

Z_{vv} =-0.03450369

$Z_{\dot{w}}$ =-0.05582685 Z_w =-0.08999440 $Z_{w|w|}$ =-0.08490253

$Z_{\dot{q}}$ =-0.00477980 Z_q =-0.02568048 $Z_{q|q|}$ =-0.072053

$K_{\dot{v}}$ =0.00154146 K_v =0.00045582

K_p =-0.01

$K_{\dot{r}}$ =0.00037772 K_r =0.00045582 $K_{r|r|}$ =0.00234668

M_{vv} =0.01153473

$M_{\dot{w}}$ =-0.006320 M_w =0.00880454 $M_{w|w|}$ =0.00346087

$M_{\dot{q}}$ =-0.00308712 M_q =-0.01597730 $M_{q|q|}$ =-0.050803

$N_{\dot{v}}$ =0.00523318 N_v =-0.01129999 $N_{v|v|}$ =0.06226890

$N_{\dot{r}}$ =-0.00351482 N_r =-0.02304781 $N_{r|r|}$ =-0.034163

- Thrust coefficients

a_X =0 b_X =0.01802 c_X =0.0012 d_X =0.0002405

a_M =0 b_M =-0.00035 c_M =-0.000103 d_M =0.00041

a_N =0 b_N =0.000306 c_N =0.000084 d_N =-0.000047