# Performance of a Receiver in Large Time-Frequency Spreading Channels

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**Abstract: In this paper, we investigate the communication over time-varying frequency selective channels that exhibit large delay spread and Doppler spread. The receiver consists of a decision-feedback equalizer (DFE) and a Kalman filter for channel estimation. Brief derivations for the algorithms are given. Simulation results show that there exists an irreducible error floor depending on the Doppler spread; the receiver can obtain a bit error rate (BER) of $10^{-2}$ for channel with spreading factor up to 0.2.**

## I. Introduction

Communication over time-varying frequency-selective channels is challengeable since the channel spreads the transmitted signal both in time (delay spread) and frequency (Doppler spread) directions. Delay spread results in intersymbol interference (ISI), and equalizer is demonstrated as an efficiency technique to cancel ISI in slow-fading channels [1]. On the other hand, Doppler spread augments the signal spectrum, and produces interfrequency interference (IFI). To mitigate the effect due to IFI, techniques for dynamic channel tracking are required.

Large time-frequency spreading channel may be often met in underwater acoustic (UWA) communication. But only a few papers study the channel measurements [3] and receiver [4, 5] in rapidly time-varying channel. Eggen presented a receiver that can work in practical scenarios with delay spread of about 10ms and Doppler spread up to 4Hz [4]. Kalman filter and linear equalizer are used for channel estimation and demodulation. X.Yu analyzed the impact of Rayleigh-fading on the performance of MLSE [6] where only second-order statistics of the channel are known, and found that MLSE exhibits an irreducible error floor that depends on the channel characteristics, such as Doppler spread and delay spread.

In this paper, we investigate performance of a receiver in large time-frequency spreading channel, where the spreading factor is in the range of [0.1, 1]. The receiver is made up of Kalman filter and a decision-feedback equalizer (DFE) because a) Kalman filter is the optimal MMSE estimator of the channel process that obeys a Gauss-Markov model, provided that the knowledge of the model parameters is known [7]; b) DFE has a good tradeoff between performance and computational complexity [8, 9]. The two techniques are throughout studied in the past [1, 10].

The paper is organized as follows. Channel model of randomly time-varying linear filter and system model is given in section II. Recursive Kalman algorithm for channel estimation is derived in section III. Finite length MMSE-DFE in time-varying channel is discussed in section IV. In section V simulation results are presented. Finally, we summarize our conclusions in section VI.

## II. Channel and System Model

### *A. Channel Characterization*

Characterization of randomly time-varying linear channel is well studied in [2]. Let $h(t,\tau)$ be the channel impulse response at time $t$ with respect to the delay $\tau$. If the input to the channel is $b(t)$, then the output $x(t)$ can be written as

$$x(t)=\int_{-\infty}^{\infty}h(t,\tau)b(t-\tau)d\tau\,. \tag{1}$$

This means that the input convolutes with channel impulse response in delay direction, which results in time spread. The channel is called frequency-selective fading channel if delay spread is not much smaller than the symbol duration. Frequency-selective fading results in inter-symbol interference (ISI), and equalizer is an effective technique to suppress ISI [1]. Taking the Fourier transform of $h(t,\tau)$ with respect to $t$ yields the Delay-Doppler spreading function

$$H(f,\tau)=\int_{-\infty}^{\infty}h(t,\tau)\exp(-j2\pi ft)dt\,. \tag{2}$$

Without loss of generality, $h(t,\tau)$ is assumed to be zero-mean, wide-sense stationary uncorrelated scattering (WSSUS) random process, i.e.

$$E\{H(f,\tau)H^*(f',\tau')\}=E\{|H(f,\tau)|^2\}\delta(f-f')\delta(\tau-\tau')\,, \tag{3}$$

where the power $E\{|H(f,\tau)|^2\}$ for given Doppler $f$ and delay $\tau$ is defined as the scattering function $S_h(f,\tau):=E\{|H(f,\tau)|^2\}$. With the above definitions, we can then obtain the input-output relation in spectrum domain

$$S_x(f)=\int_{-\infty}^{\infty}S_b(f)*S_h(f,\tau)d\tau\,. \tag{4}$$

This means that the input spectrum is expanded by the scattering function, i.e. a spread in frequency direction (Doppler). Doppler spread and delay spread can be thought of as a pair of duals [2], and Doppler spread results in time-selective fading [4]. Similarly, we call the interference arises from the Doppler spread as inter-frequency interference (IFI).

From the scattering function, we can obtain the delay power profile, $S_h(\tau)=\int_{-\infty}^{\infty}S_h(f,\tau)df$, and the Doppler power profile, $S_h(f)=\int_0^{\infty}S_h(f,\tau)d\tau$. Without loss of generality, we assume that $h(t,\tau)$ is causal, $\tau_{\max}$ is the multipath spread, i.e. $h(t,\tau)=0$, $\tau<0$ or $\tau>\tau_{\max}$; the Doppler spread is $2f_{D\max}$, which is defined as the range of the values over which Doppler power spectrum is nonzero. The product $2f_{D\max}\tau_{\max}$ is called the spread factor of the channel. If $2f_{D\max}\tau_{\max}<1$, the channel is said to be underspread; otherwise, it is overspread. Generally, it is extremely difficult to communicate reliable information through overspread channel [1]. In this paper, we consider a channel that disperses largely in delay and Doppler domains, i.e, $2f_{D\max}\tau_{\max}$ approaches but less than 1.

### *B. Channel Model*

Let $T$ be the symbol duration and $B_s=1/T$ be the signal bandwidth, i.e. the maximum resolvable component for the channel impulse response is separated by $T$. Thus, time-varying frequency-selective fading channel can be represented as

$$h(t,\tau)=\sum_{l=0}^{L_c-1}h_l(t)\delta(\tau-lT)\,, \tag{5}$$

where $L_c = \lfloor \tau_{\max} / T \rfloor + 1$ is the channel length, and $\lfloor x \rfloor$ represents the maximum integer less than $x$. The corresponding delay-Doppler spread function is then

$$H(f,\tau) = \sum_{l=0}^{L_c-1} \int_{-\infty}^{\infty} h_l(t) e^{-j2\pi ft} dt \delta(\tau - lT) := \sum_{l=0}^{L_c-1} H_l(f)\delta(\tau - lT), \quad (6)$$

and the output (1) can be rewritten as

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(f,\tau) b(t-\tau) e^{j2\pi ft} df d\tau \\ &= \sum_{l=0}^{L_c-1} \int_{-\infty}^{\infty} H_l(f) b(t-lT) e^{j2\pi ft} df \\ &= \lim_{M\to\infty} \sum_{l=0}^{L_c-1} \sum_{m=-M}^{M} b(t-lT) H_l(m\Delta f) e^{j2\pi m\Delta ft} \Delta f \end{aligned}, \quad (7)$$

where $\Delta f = f_{D\max} / M$. Here, $H_l(f)$ is the spectrum of the wide-sense stationary process $h_l(t)$. The physical interpretation of $H_l(f)$ is the complex gain density of the received signal arriving at delay $lT$ and Doppler $f$. Moreover, it can be regarded as the gain density due to a distinct scatter [4]. If the scatters are fixed, $H_l(f)$ should be constant in the observed time. To be simple, we use $H_l(m)$ to replace $H_l(m\Delta f)$ in the following description.

*C. System Model*

A system model in baseband is given in Fig.1 (in the last page). In this model, $h(n,l)$ is the equivalent channel impulse response for the shaping pulse filter, transmission channel, receiver filter and the sampler at the rate of $1/T$. $h(n,l)$ represents the channel impulse response at time $nT$ with respect to $lT$. $L_c$ and $2f_{D\max}$ are channel length and Doppler spread respectively. The input data sequence $\{b(n)\}$ is a sequence of independent, identically distributed (i.i.d) complex random variable $b(n)$, with zero-mean and unit-variance, $E\{b(n)b^*(m)\} = \delta(n-m)$. Signals are transmitted frame by frame. Each frame consists of two parts, training sequence with length $L_t$ and data with length $L_d$. The received signal $r(n)$ can be expressed as

$$r(n) = \sum_{l=0}^{L_c-1} h(n,l) b(n-l) + w(n). \quad (8)$$

where $w(n)$ is additive white Gaussian noise with zero-mean and variance $E\{w(n)w^*(n)\} = \sigma^2$.

The receiver in this paper consists of two parts: channel estimation and decision-feedback equalizer. Received signal is filtered by a feedforward filter with coefficients $\mathbf{f}^{[f]}(n) := [f_0^{[f]}(n) \ \cdots \ f_{L_f-1}^{[f]}(n)]^{\mathrm{T}}$, where $L_f$ is the length of the feedforward filter. Meanwhile, the estimated data sequence $\{\hat{b}(n-\delta)\}$ is filtered by a feedback filter with coefficients $\mathbf{f}^{[b]}(n) := [f_1^{[b]}(n) \ f_2^{[b]}(n) \ \cdots \ f_{L_b}^{[b]}(n)]^{\mathrm{T}}$, where $L_b$ is the length of the feedback filter. The feedforward filter is not assumed to be causal, but the feedback filter must be strictly causal[1]. The received signal at the decision point is then given by

$$y(n) = \sum_{k=0}^{L_f-1} f_k^{[f]*}(n) r(n-k) - \sum_{k=1}^{L_b} f_k^{[b]*} b(n-\delta-k) + \sum_{k=0}^{L_f-1} f_k^{[f]*}(n) w(n-k). \quad (9)$$

We assume that the DFE coefficients are calculated indirectly via channel estimates, which is always more robust with respect to channel fading than the classical direct approach [7]. In this paper, channel estimation is aided by training sequence or the tentative decisions $\{\hat{b}_{n-\delta}\}$ from the equalizer.

## III. Channel Estimation

In this section we use Kalman filter for channel estimation. The general channel input-output relation in form of delay-Doppler spread function is given by (7). Thus, the received signal (8) can also be written as

$$r(n) = \sum_{l=0}^{L_c-1} \sum_{m=-M}^{M} b(n-l) H_l(m) e^{j2\pi\Delta fmnT} \Delta f + w(n), \quad (9)$$

where $\Delta f = f_{D\max} / M$. Taking $H_l(m)$ as the complex gain due to a distinct scatter, and assuming that the speed of moving scatter is much smaller than the speed of transmission medium (e.g. sound or radio)[2], $H_l(m)$ is slowly time-varying, and we denote it as $H_{l,m}(n)$. Thus, a first-order autoregressive model (AR(1))for $H_{l,m}(n)$ is a practical, i.e.

$$H_{l,m}(n) = \alpha_{l,m} H_{l,m}(n-1) + v_{l,m}(n), \quad (10)$$

where $|\alpha_{l,m}| \le 1$ and $v_{l,m}(n)$ is white noise. Now a state space description of channel is obtained by (10) and the received signal is given by (9). Denote

$$\mathbf{h}(n) := [H_{0,-M}(n) e^{j2\pi(-M)\Delta fnT} \ \cdots \ H_{L_c-1,-M}(n) e^{j2\pi M\Delta fnT}]^{\mathrm{T}},$$

$$\mathbf{A} := diag\left(\alpha_{0,-M} e^{j2\pi(-M)\Delta fT}, \ldots, \alpha_{L_c-1,M} e^{j2\pi M\Delta fT}\right),$$

$$\mathbf{v}(n) := [v_{0,-M}(n) \quad v_{L_c-1,M}(n)]^{\mathrm{T}},$$

$$\mathbf{b}(n) := [b(n) \ \cdots \ b(n-L_c+1)]^{\mathrm{T}} \otimes \mathbf{1}_{M\times 1},$$

where $\otimes$ represents Kronecker product and $\mathbf{1}_{M\times 1}$ is a $M \times 1$ vector with all one, and the order subscript $(l,m)$ is from $(0,-M),\ldots,(0,M)$ to $(L_c-1,-M),\ldots,(L_c-1,M)$. From (9)(10), we can get

$$\mathbf{h}(n+1) = \mathbf{A}\mathbf{h}(n) + \mathbf{v}(n), \quad (11)$$

$$r(n) = \mathbf{b}^H(n)\mathbf{A}\mathbf{h}(n) + w(n). \quad (12)$$

It is clear that (11)(12) are the state space equation and measurement equation, and recursive Kalman algorithm [10] can be applied for channel estimation. Table I lists the algorithm, and $\mathbf{R}_{\mathbf{v}}$ is the autocorrelation function of the noise vector $\mathbf{v}(n)$, i.e.

$$\mathbf{R}_{\mathbf{v}} := E\{\mathbf{v}(n)\mathbf{v}^H(n)\} = diag\left((1-\alpha_{0,-M}^2)\sigma_{H(0,-M)}^2, \ldots, (1-\alpha_{L_c-1,M}^2)\sigma_{H(L_c-1,M)}^2\right),$$

where $\sigma_{H(l,m)}^2 := E\{|H_{l,m}(n)|^2\}$ is a constant.

In the receiver, there is a decision delay $\delta$ for DFE, i.e. only the past estimates $\hat{b}_{n-\delta-1}, \hat{b}_{n-\delta-2}\ldots$ are decided at time $n$. Consequently the estimator can only calculate $\hat{H}_{l,m}(n-\delta), \hat{H}_{l,m}(n-\delta-1),\ldots$ with the past decisions. However,

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[1] Denote $\widetilde{\mathbf{f}} = [1 \ \ \mathbf{f}^{[f]\mathrm{T}}]^{\mathrm{T}}$. The coefficients of the feedback filter are then $\widetilde{\mathbf{f}} - \mathbf{e}_1$, which is the same as the definition in [6, 7]. Thus, the DFE structure given in [7] is the same as Fig.1.

[2] Another explanation is that the signal spectrum of channel impulse response $h(t,\tau), t = nT, \tau = lT$ changes small in a symbol interval. A typical numerical analysis in underwater acoustic communication is given in [4].

channel information $H_{l,m}(n), H_{l,m}(n-1),...$ is required for DFE to estimate the $\hat{b}(n-\delta)$. Therefore, prediction of $H_{l,m}(n),...,H_{l,m}(n-\delta+1)$ is required at time $n$. From (10), it is easy to obtain the optimum prediction of $\hat{H}_{l,m}(n),...,\hat{H}_{l,m}(n-\delta+1)$, which is given by

$$\begin{aligned}&\hat{H}_{l,m}(n-k)=\alpha_{l,m}^{\delta-k}\hat{H}_{l,m}(n-\delta)\\&k=0,...\delta-1; l=0,...,L_c-1; m=-M,...,M.\end{aligned} \tag{13}$$

Table I. Recursive Kalman algorithm

$$\begin{aligned}
&\hat{\mathbf{h}}(n+1)=\mathbf{A}\hat{\mathbf{h}}(n)+\mathbf{k}(n)e(n)\\
&\hat{r}(n)=\mathbf{b}^{\mathrm{H}}(n)\mathbf{A}\hat{\mathbf{h}}(n)\\
&e(n)=r(n)-\hat{r}(n)\\
&\widetilde{\mathbf{P}}(n)=\mathbf{A}\mathbf{P}(n-1)\mathbf{A}^{\mathrm{H}}+\mathbf{R}_{\mathbf{v}}\\
&\sigma^2(n)=\mathbf{b}^{\mathrm{H}}(n)\widetilde{\mathbf{P}}(n)\mathbf{b}(n)+\sigma_w^2\\
&\mathbf{k}(n)=\widetilde{\mathbf{P}}(n)\mathbf{b}(n)/\sigma^2(n)\\
&\mathbf{P}(n)=\widetilde{\mathbf{P}}(n)-\sigma^2(n)\mathbf{k}(n)\mathbf{k}^{\mathrm{H}}(n)
\end{aligned}$$

Finally, since delay-Doppler spread is the Fourier transform of the channel impulse response, as given by (2), the estimated channel impulse response can be calculated by

$$\hat{h}(n,l)=\sum_{m=-M}^{M}\hat{H}_{l,m}(n)e^{j2\pi m\Delta f nT}\Delta f. \tag{14}$$

Performance of the Kalman algorithm for channel estimation is given in [5], and impacts of channel estimation errors on the performance of linear equalizer (LE) and decision-feedback equalizer are given in [5, 7], respectively.

## IV. Decision-Feedback Equalizer

Finite-length MMSE-DFE is discussed in this section. The input of the equalizer is the output of the channel estimator, i.e. $\hat{h}(n,l), \hat{h}(n-1,l),....$ at time $n$. It is seen from (14) that the channel changes with each symbol, and thus the filter coefficients should be updated with the channel information. We define the channel matrix as

$$\hat{\mathbf{H}}(n):=\begin{bmatrix}\hat{h}(n,0) & \cdots & \hat{h}(n,L_c-1) & 0 & \cdots & 0\\ 0 & \hat{h}(n-1,0) & \cdots & \hat{h}(n-1,L_c-1) & & \vdots\\ \vdots & & \ddots & & \ddots & 0\\ 0 & \cdots & 0 & \hat{h}(n-L_f+1,0) & \cdots & \hat{h}(n-L_f+1,L_c-1)\end{bmatrix}_{L_f\times(L_c+L_f-1)}, \tag{15}$$

and some vectors to derive the MMSE-DFE,

$$\begin{aligned}
&\mathbf{f}^{[f]}(n):=[f_0^{[f]}(n)\ \cdots\ f_{L_f-1}^{[f]}(n)]^{\mathrm{T}},\\
&\mathbf{f}^{[b]}(n):=[f_1^{[b]}(n)\ \ f_2^{[b]}(n)\ \cdots\ f_{L_b}^{[b]}(n)]^{\mathrm{T}},\\
&\mathbf{f}(n):=[\mathbf{f}^{[f]^{\mathrm{T}}}(n)\ \ -\mathbf{f}^{[b]^{\mathrm{T}}}(n)]^{\mathrm{T}},\\
&\mathbf{r}(n):=[r(n)\ \cdots\ r(n-L_f+1)]^{\mathrm{T}},\\
&\hat{\mathbf{b}}_{\delta}(n):=[\hat{b}(n-\delta-1)\ \cdots\ \hat{b}(n-\delta-L_b)]^{\mathrm{T}},\\
&\mathbf{x}(n):=[\mathbf{r}^{\mathrm{T}}(n)\ \ \hat{\mathbf{b}}_{\delta}^{T}(n)]^{\mathrm{T}},\\
&\vec{\mathbf{b}}(n):=[b(n)\ \cdots\ b(n-L_c-L_f+2)]^{\mathrm{T}},\\
&\mathbf{w}(n):=[w_0(n)\ \cdots\ w_{L_f-1}(n)]^{\mathrm{T}}.
\end{aligned}$$

According to (8)(9), the received signal $r(n)$ and the signal at the decision point $y(n)$ can be expressed in form of matrix, i.e.

$$\mathbf{r}(n)=\hat{\mathbf{H}}(n)\vec{\mathbf{b}}(n)+\mathbf{w}(n), \tag{16}$$

$$y(n)=\mathbf{f}^{\mathrm{H}}(n)\mathbf{x}(n)+\hat{b}(n-\delta). \tag{17}$$

The mean-square error (MSE) is defined as

$$J_{\mathrm{MSE}}(n):=E\{|y(n)-b(n-\delta)|^2\}, \tag{18}$$

and MMSE-DFE is to attain the coefficient $\mathbf{f}(n)$ that minimizes the MSE. Solving the equation $\frac{\partial J_{\mathrm{MSE}}(n)}{\partial \mathbf{f}(n)}=0$, we can obtain the Wiener solution given by

$$\mathbf{f}(n)=\mathbf{R}_{\mathbf{x}}^{-1}(n)\mathbf{R}_{\mathbf{x}b}(n), \tag{19}$$

where

$$\mathbf{R}_{\mathbf{x}b}(n):=\mathrm{E}\{\mathbf{x}(n)b^*(n-\delta)\}=\begin{bmatrix}\mathrm{E}\{\mathbf{r}(n)b^*(n-\delta)\}\\ \mathrm{E}\{\hat{\mathbf{b}}_{\delta}(n)b^*(n-\delta)\}\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{H}}(n)[:,\delta+1]\\ 0_{L_b\times 1}\end{bmatrix},$$

$$\mathbf{R}_{\mathbf{xx}}(n):=\mathrm{E}\{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)\}=\begin{bmatrix}\mathrm{E}\{\mathbf{r}(n)\mathbf{r}^{\mathrm{H}}(n)\} & \mathrm{E}\{\mathbf{r}(n)\hat{\mathbf{b}}_{\delta}^{\mathrm{H}}(n)\}\\ \mathrm{E}\{\hat{\mathbf{b}}_{\delta}(n)\mathbf{r}^{\mathrm{H}}(n)\} & \mathrm{E}\{\hat{\mathbf{b}}_{\delta}(n)\hat{\mathbf{b}}_{\delta}^{\mathrm{H}}(n)\}\end{bmatrix}$$

and

$$\mathrm{E}\{\mathbf{r}(n)\mathbf{r}^{\mathrm{H}}(n)\}=\hat{\mathbf{H}}(n)\hat{\mathbf{H}}^{\mathrm{H}}(n)+\sigma_w^2\mathbf{I}_{L_f}$$

$$\mathrm{E}\{\mathbf{r}(n)\hat{\mathbf{b}}_{\delta}^{\mathrm{H}}(n)\}=\hat{\mathbf{H}}(n)\begin{bmatrix}\mathbf{0}_{\delta+1\times L_b}\\ \mathbf{I}_{L_b}\\ \mathbf{0}_{(L_c-+L_f-L_b-\delta-2)\times L_b}\end{bmatrix}=\hat{\mathbf{H}}(n)[:,\delta+2:\delta+L_b+1]$$

$$\begin{aligned}\mathrm{E}\{\hat{\mathbf{b}}_{\delta}(n)\mathbf{r}^{\mathrm{H}}(n)\}&=\begin{bmatrix}\mathbf{0}_{L_b\times\delta+1} & \mathbf{I}_{L_b} & \mathbf{0}_{L_b\times(L_c-+L_f-L_b-\delta-2)}\end{bmatrix}\hat{\mathbf{H}}^{\mathrm{H}}(n),\\ &=\hat{\mathbf{H}}^{\mathrm{H}}(n)[\delta+2:\delta+L_b+1,:]\end{aligned}$$

$$\mathrm{E}\{\hat{\mathbf{b}}_{\delta}(n)\hat{\mathbf{b}}_{\delta}^{\mathrm{H}}(n)\}=\mathbf{I}_{L_b}$$

and $\mathbf{0}_{m\times n}$ is a $m\times n$ matrix with all elements of zero. In the derivation, we assume all the past decisions are correct, i.e. $\hat{b}(n-\delta-m)=b(n-\delta-m),\ m=1,2,...$ . For convenience, we have used the colon to signify all the elements of range as is done in Matlab, i.e. $\hat{\mathbf{H}}(n)[:,1]=[\hat{h}(n,0)\ \cdots\ \hat{h}(n,L_c-1)\ \cdots\ 0]^{\mathrm{T}}$.

Zero-Forcing DFE (ZF-DFE) and MMSE-DFE can be regarded as special cases of MMSE-DFE. Let $\sigma_w^2=0$ in (19), it becomes the coefficients of ZF-DFE. Let $L_b=0$, (19) becomes MMSE-LE.

It is seen from (19) that the Wiener solution has the same form as that in case of time-invariant channel. The only difference is that the channel matrix $\hat{\mathbf{H}}(n)$ defined by (15). It is clear that the row vectors in time-varying channel are not simply a right-shift version of the upper row vector, and information for the time-varying channel is implied in the elements $h(n,l), h(n-1,l),....$ . Both information for frequency-selective fading (described by $h(n,0), h(n,1),...$) and time-selective fading are fully included in (15).

To investigate the performance of MMSE-DFE in time-varying frequency-selective channel, we assume the perfect channel information is known, i.e. $\hat{h}(n,l)=h(n,l)$, and hence the effect due to estimation errors is excluded. Also, we set $\delta=L_f-1$ and $L_b=L_c-1$, which is the optimal setting for $\delta$ and $L_b$ in case of time-invariant channel. With the same derivation procedure as given in [9], the minimum MSE (MMSE) at time $n$ can be obtained as

$$\mathrm{MMSE}(n)=\frac{\sigma_w^2}{d_{\max}(n)}, \tag{20}$$

where $d_{\max}(n)$ is the maximum eigenvalue of the matrix

$$\mathbf{R}(n) := \mathbf{H}^{\mathrm{H}}(n)\mathbf{H}(n) + \sigma_w^2 \mathbf{I}_{L_f} . \qquad (21)$$

It is clear that performance of MMSE-DFE depends on noise power $\sigma_w^2$ as well as the maximum eigenvalue of $\mathbf{H}^{\mathrm{H}}(n)\mathbf{H}(n)$ .

## V. Simulation Results

In this section, we present numerical examples to illustrate the receiver performance due to Doppler spread and parameter setting. A channel with delay spread 100ms is simulated according to Clarke model. Four independent paths are assumed, and the expected power in each path is 0.25. Doppler spread $2f_{D\max}$ varies in range of [1, 10]Hz, i.e. spreading factor is $2f_{D\max}\tau_{\max} \in [0.1, 1]$ . To balance channel length and normalized Doppler spread ( $2f_{D\max}T$ ), we assume that data rate is 100bit/s, i.e. symbol interval is $T$=0.01s. BPSK is used for modulation. Signal structure is a training sequence for initialization (256 bits) followed by frames. Each frame consists of two parts: training sequence and data. Lengths of training sequence and the data in a frame are $L_t = 16$ , $L_d = 16$ . The default setting for DFE is $L_f = 30$ , $L_b = 10$ , $\delta = 15$ . Frequency interval of Kalman filter is $\Delta f = 0.5$Hz .

*A. Performance of equalizer with channel known.*

First, we look at performance of MMSE-DFE and MMSE-LE ( $L_b = 0$ ) due to Doppler spread. Perfect channel knowledge is known to avoid the impact of channel estimation errors on the equalizer performance. Training sequence is not required, i.e. $L_t = 0$ . Fig.2 depicts, for various values of Doppler spread, bit error rate (BER) performance vs received signal to noise ratio (SNR). It is shown that BER performance can be improved by increasing SNR. However, there is an irreducible error floor for MMSE-DFE and MMSE-LE where BER is almost a constant with respect to SNRs that are larger than a threshold. Moreover, the larger the Doppler spread, the larger the error floor. For example, error floor of MMSE-LE in channels with Doppler spread of 1, 2, and 5Hz is about $3\times10^{-3}$, $2\times10^{-2}$ and $4\times10^{-2}$ respectively. The error floor of MMSE-DFE is much lower than that of MMSE-LE. When Doppler spread is 5Hz, the error floor is smaller than $10^{-5}$.

*B. Performance of the receiver and parameters setting.*

In the following examples, performance of the receiver, i.e. equalizer with channel estimation by Kalman filter, is investigated. MMSE-DFE, MMSE-LE and ZF-DFE are compared. Fig.3 plots BER vs SNR in case of $2f_{D\max} = 2$ Hz, and Fig.4 depicts BER vs Doppler spread in case of SNR=30dB. It is seen that MMSE-DFE has the best performance. If BER of $10^{-2}$ is the required performance, MMSE-DFE can work in Doppler spread of 2Hz, i.e. spreading factor of 0.2, whereas MMSE-LE can only work in channel with spreading factor of 0.1. When SNR is high, ZF-DFE is better than MMSE-LE. However, in case of low SNR, ZF-DFE has even worse performance than MMSE-LE, since noise dominates the residual interference at the output. Also, compared with the results in Fig. 2, it is shown that receiver performance suffers largely from channel errors, which are analyzed in [7][5].

Lastly, we investigate the receiver performance due to decision delay $\delta$ . Results are shown in Fig.5. Since there are estimation and prediction errors, the optimum value of $\delta$ for MMSE-DFE is 20, but not $L_f - 1 = 29$ , which is the optimal set for MMSE-DFE with perfect knowledge [9]. Optimal decision delays of MMSE-LE and ZF-DFE are 10 and 30.

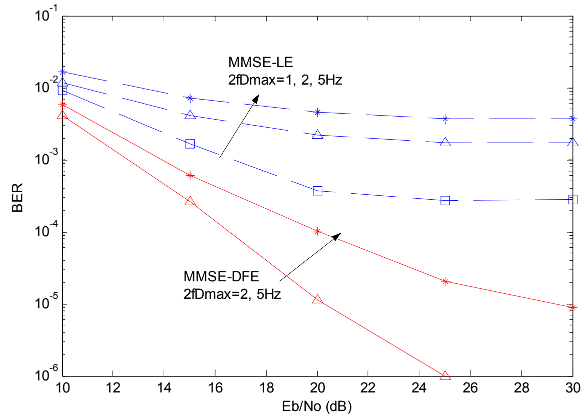


Fig.2 Performance of equalizer with perfect channel known.

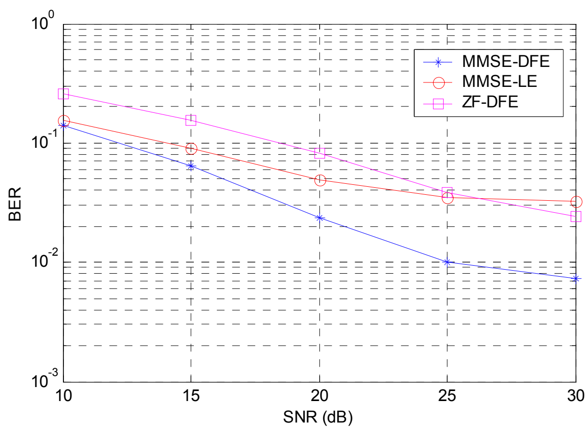


Fig.3 BER vs SNR, $2f_{D\max} = 2$ Hz.

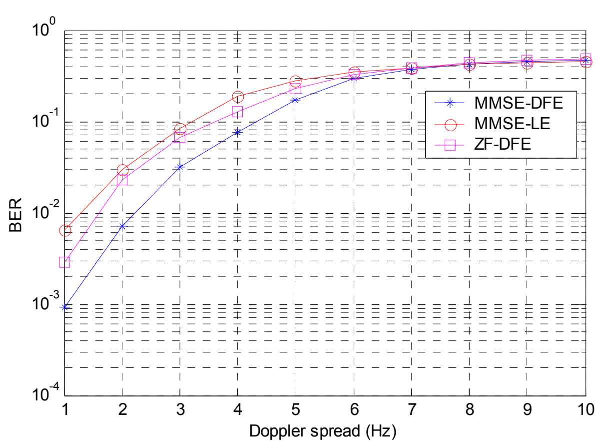


Fig.4 BER vs Doppler spread, SNR=30dB.

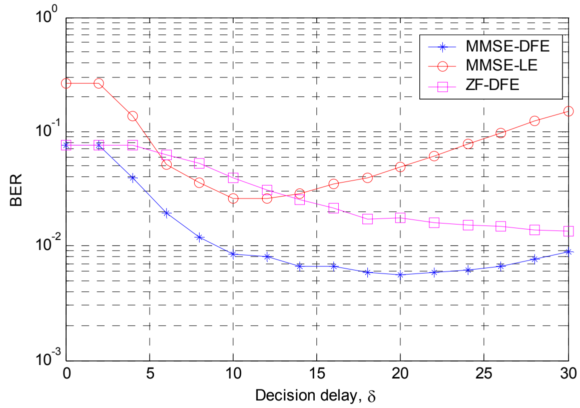


Fig.5 BER vs decision delay, $\delta$.

## VI. Conclusions

Since the time-varying frequency-selective channel spreads the transmitted signal in time and frequency directions, received signals suffer from ISI, IFI and noise. In time-invariant channel, MMSE-DFE is proved to be one of effective techniques to suppress ISI. However, our simulation results (without strict proof) show that there is an irreducible error floor for both MMSE-DFE and MMSE-LE in time-varying channels, which depends on the Doppler spread. This partly illustrates that DFE is not powerful to mitigate IFI. Also, it verifies the difficulty to communicate in overspreading channel, as indicated in [1]. An analysis study of the error floor is one of the future works. In the receiver, Kalman filter is chosen for channel estimation, since it is a good technique to track dynamic channels if channel process obeys Gauss-Markov model. Performance of the receiver due to noise, Doppler spread and decision delay is simulated. It is shown that the receiver with MMSE-DFE can work in a channel with spreading factor up to 0.2 if $10^{-2}$ is the required BER floor, whereas the receiver with MMSE-LE can only work in a channel with spreading factor up to 0.1.

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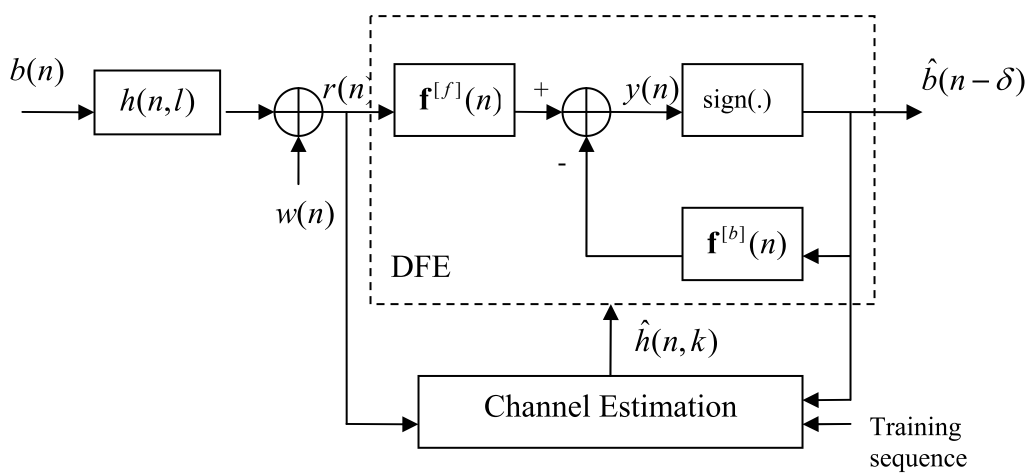


Fig.1 System model