Estimates of Geoacoustic Model Parameters from Inversions of Horizontal and Vertical Line Array Data

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Abstract - **This paper describes results from geoacoustic inversion of low-frequency acoustic data recorded at a receiving array divided into two sections, a sparse bottom laid horizontal array (HLA) and a vertical array (VLA) deployed in shallow water. The data are from an experiment conducted by Forsvarets forskningsinstitutt (FFI) in the Barents Sea, using broadband explosives (shot) sources. A two-layer range-independent geoacoustic model, consistent with seismic profiles from the area, described the environment. Inversion for geoacoustic model parameters was carried out using a fast implementation of the hybrid adaptive simplex simulated annealing (ASSA) inversion algorithm, with replica fields computed by the ORCA normal mode code. Low-frequency (40-128 Hz) data from six shot sources at ranges 3-9 km from the array were considered. Estimates of sediment and substrate p-wave velocities and sediment thickness were found to be consistent between independent inversions of data from the two sections of the array.**

I. INTRODUCTION

Inversion of acoustic field data for the estimation of seabed geoacoustic parameters has been a topic of considerable recent attention in underwater acoustics [1-5]. The objective is to estimate environmental parameters relevant for use in acoustic propagation models for e.g. sonar performance predictions or matched-field localization. Several applications of matched-field inversion methods have been demonstrated using vertical array (VLA) data. The VLA has often been the preferred tool of research due to its ability to sample the spatial variability of the acoustic field with respect to depth in the water, which in general carries a high sensitivity to seabed parameters. The use of a horizontal line array (HLA) deployed on the seafloor can be preferable in many practical respects, such as covertness, relative ease of deployment and stability of the array. A relevant question is whether similar sensitivity to seabed parameters can be obtained from HLA data.

To investigate the utility of a bottom deployed HLA for geoacoustic inversion, Forsvarets forskningsinstitutt (FFI) conducted an experiment in shallow water in the Barents Sea in 1999. The experiment made use of explosive sound (shot) sources and a 31-element acoustic array. The array was divided into two sections: a 21-element VLA spanning the lower two-thirds of the water column (water depth 320 m) and a 10-element HLA (length 820 m) at the seabed. This paper presents results from geoacoustic inversion of low-frequency (40-128 Hz) acoustic data from both these arrays. Data from six sources at ranges 3-9 km from the arrays were considered. Inversion for geoacoustic model parameters was carried out using a fast implementation of the hybrid adaptive simplex simulated annealing (ASSA) inversion algorithm.

The remainder of this paper is organized as follows: the matched-field inversion method is described in section 2. The acoustic experiment and supporting geophysical measurements is described in section 3. Inversion of synthetic and experimental data is described in sections 4 and 5 respectively. Section 6 summarises results.

II. INVERSION METHOD

A. Matched-field inversion

Matched-field inversion is in essence a correlation process where the acoustic field measured at an array is matched with replica fields generated by an acoustic propagation model for a given model environment. A search over candidate models is performed, and the optimal model is taken to be the one yielding the best correlation or match with measured data. Match is here measured by the incoherent broadband Bartlett objective function

$$E(m) = 1 - \frac{1}{M}\sum_{k=1}^{M}\left[\frac{\left|\sum_{j=1}^{N} p_{jk}^{+} q_{jk}(m)\right|^2}{\sum_{j=1}^{N}\left|p_{jk}\right|^2 \sum_{j=1}^{N}\left|q_{jk}(m)\right|^2}\right] \qquad (2.1)$$

with j a summation index over N hydrophones (summation in depth for a VLA and in range for a HLA), k is a summation index over M frequencies, p the measured complex pressure field, q the replica pressure field for model m, and ($^{+}$) is the complex conjugate. The objective function takes values between 0 and 1, with 0 indicating a perfect match.

B. Search algorithm

The search for an optimal seabed model is facilitated using the adaptive simplex simulated annealing (ASSA) algorithm [6]. The method combines a global random search method (simulated annealing) with a local directed search method (downhill simplex), using an adaptive adjustment of the search space for maximum efficiency. There are a few control parameters; the most crucial are the initial temperature (T_0), temperature reduction factor (β), and maximum number of temperature steps in simulated annealing. The method has been validated using synthetic data from geoacoustic inversion workshops for range-independent [7] and range-dependent environments.

C. Environment model

A two-layer range-independent fluid geoacoustic model described the environment of the experiment site. The

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model included the following features: (i) a constant-gradient p-wave velocity profile in sediment, the gradient restricted to positive values, (ii) an equivalent flat bathymetry approximation to a sloping bottom and (iii) no variation of sediment thickness with range. This permitted use of a range-independent propagation model in replica field computations. To account for a small slope in bathymetry, the following approach was adopted. The effective water depth for an equivalent flat bathymetry [8] was computed for each source range by:

$$D \equiv \left(\frac{1}{r_S} \int_0^{r_S} \frac{dr}{d^2(r)} \right)^{-1/2} \qquad (2.2)$$

with r_S the source range and $d(r)$ a measured bathymetry profile. For the resulting model water depth D, source depths and VLA element depths were then scaled [9] with the water depth ratios by:

$$z_{se} = z_s D / H_s \qquad (2.3)$$

with z_s the nominal source depth and H_s the measured water depth at the source position, and

$$z_{re} = z_r D / H_r \qquad (2.4)$$

with z_r the nominal element depth, and H_r the measured water depth at the array site. For the HLA, all elements were set to the model water depth D.

D. Forward model

The normal mode acoustic propagation model ORCA [10] for range-independent media was used to compute replica pressure fields. The model correctly accounts for attenuation and shear wave effects as well as near-field effects due to leaky (continuous) modes. The model can also be executed in a fast "real-axis" mode using a perturbation approach to approximate modal attenuations and ignoring the two other effects.

III. THE EXPERIMENT

The acoustic experiment was conducted in a relatively flat area of the Barents Sea, with water depth decreasing from 340 m at 8 km east of the array to 300 m at 8 km west of the array, and a near constant slope in between. Water depth at the array site was 320 m. The geometry of the experiment and a bathymetry profile is shown in Fig. 1.

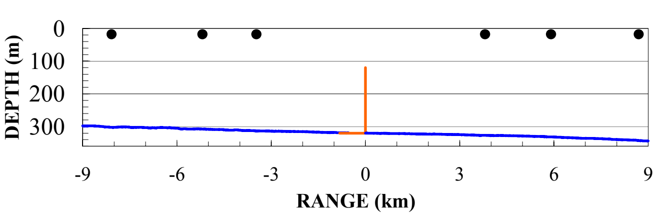


Fig. 1. Bathymetry profile. Black circles indicate ranges of analysed shots, yellow lines indicate the VLA and HLA.

The area of experiment was surveyed by geophysical equipment including a SIMRAD EA-500 echosounder, an airgun seismic system and SIMRAD TOPAS PS018 bottom penetrating sonar. The seismic profiles indicated a Quaternary sediment layer of thickness 20-35 m overlying consolidated sediment (siltstone) of several hundred metres thickness. An interpreted seismic section for the area east of the array is shown in Fig. 2. A seismic refraction velocity measurement indicated a compressional (p-wave) velocity of 2400 m/s for the siltstone layer. No in-situ measurements of Quaternary sediment properties were conducted at the site. Average p-wave velocity and density values were assigned based on measurements of cores of Quaternary sediment collected in the general region [11]. For p-wave attenuation, reference values from Hamilton were assigned.

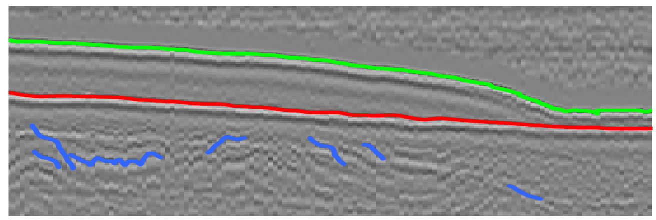


Fig. 2. Seismic section eastward from the array. Range to 12 km, VLA position at left edge of picture.

The baseline geoacoustic model developed for the area is shown in Table 1. The model uses a homogeneous fluid sediment layer over a fluid halfspace. In the water column, a sound speed profile measured at the array site prior to the collection of acoustic data (Fig. 3) was used.

TABLE 1
BASELINE GEOACOUSTIC MODEL

Layer	Thickness [m]	P-wave velocity [m/s]	P-wave attenuation [dB/λ]	Density [g/cm³]
Water	320	profile	-	1.00
Sediment	20-35	1770	0.40	2.00
Substrate		2400	0.10	2.40

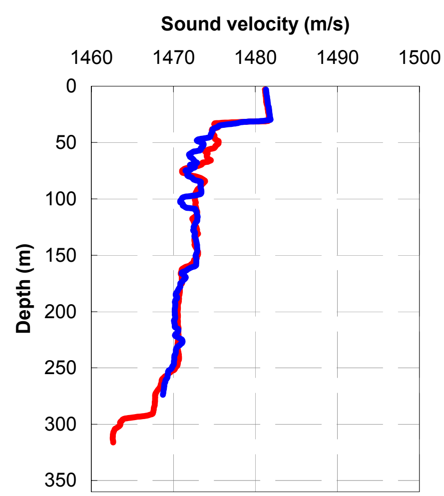


Fig. 3. Sound velocity profile in water, measured at array site before (red line) and at end of (blue line) acoustic data collection.

Shallow SUS (Mk-61 and Mk-64) explosive sources were detonated at ranges from 3 km to 9 km in two directions from the VLA. Data from six sources were selected for analysis; these are indicated in Fig. 1. Nominal source depths were 18 m, actual source depths were estimated by an analysis of measured bubble pulse periods. Nominal source ranges were determined by GPS ranging.

The receiving acoustic array was divided into two sections. The vertical section consisted of 21 hydrophones uniformly spaced 10 m apart. This 200 m long section was bottom moored, with the deepest element 4 m above the seabed. The horizontal section consisted of 10 hydrophones of geometric element spacing from 20 m to 240 m, total length 820 m, bottom-laid and oriented westward from the vertical array. The VLA position and HLA orientation was estimated from measured travel times from twelve explosive sources detonated in a circle of 2 km radius around the arrays. For estimation of array tilt, three current meters were attached near the top, middle and bottom of the vertical section.

Data were sampled at 3 kHz and transmitted via radio link to a nearby receiving ship. A 16k FFT was applied to unfiltered time segments of length 6.0 sec around each shot. Complex pressure vectors were then stored to file for further processing. Data at five selected frequencies of 40.4, 58.5, 81.4, 97.1 and 127.5 Hz are considered in this paper. These frequencies were selected at peaks of the frequency spectra and within a predetermined range of frequencies of interest, such that array design criteria for matched-field processing [12] were met both for the VLA and HLA configurations used in the present experiment.

IV. INVERSION OF SYNTHETIC DATA

Sensitivity of geoacoustic model parameters can be assessed by use of synthetic data. This is here done using noisy data (complex spatially white Gaussian noise added to the synthetic pressure field to obtain an array signal-to-noise ratio of SNR=3dB) generated for a model environment modified slightly from the baseline model of Table 1: a sediment layer of constant thickness 30 m and with a gradient in p-velocity increasing from 1600 m/s at the top to 1680 m/s at the bottom of the layer is used. Data at the five selected frequency components from 40 Hz to 128 Hz is used, for sources at ranges 8.7 km, 5.9 km and 3.2 km to a 21-element VLA and to the closest element of a 10-element HLA. The source ranges, frequencies, SNR, arrays and environment model were chosen to most closely resemble the present experiment.

The six geoacoustic model parameters included in the inversions and their search bounds are listed in Table 2. In addition, p-wave attenuations were held fixed at baseline values and a measured sound speed profile was used in water. A model water depth computed from (2.2) was used for each source. A small search interval around this depth was included to represent the possibility of a constant offset in measured bathymetry. Source range was also included as a search parameter, using a small interval around the nominal value. Source depth was fixed to a value of 18 m. Finally, a straight-line tilt was included as a search parameter for the VLA. The geometric parameters included in all inversions are listed in Table 3.

TABLE 2
MODEL GEOACOUSTIC PARAMETERS AND SEARCH BOUNDS

Parameter	Symbol	Unit	Lower Limit	Upper Limit
Sediment thickness	h	m	4.0	36
Sediment p-velocity top	c_{1T}	m/s	1500	1900
Sediment p-velocity bottom	c_{1B}	m/s	1500	1900
Substrate p-velocity	c_2	m/s	1900	3800
Sediment density	ρ_1	g/cm³	1.50	2.00
Substrate density	ρ_2	g/cm³	1.80	2.40

TABLE 3
MODEL GEOMETRIC PARAMETERS AND SEARCH BOUNDS

Parameter	Symbol	Unit	Lower Limit	Upper Limit
Water depth[a]	D	m	-6.5	+6.0
VLA tilt	T	deg	-3.0	+3.0
Source range[a]	r	m	-80	+85

[a]Relative to a nominal value

Inversions on synthetic VLA and HLA data were run using the ASSA with initial temperature T_0=0.30, reduction factor β=0.975, five perturbations at each of maximum 500 temperature steps and a maximum of 300 additional local downhill steps after "quenching" (the temperature set to zero). The ASSA inversions converged after about 250 temperature steps and an additional 100 local downhill steps, with a total number of about 6000 calls to the forward model (accepted and rejected models). This took less than two minutes on a Pentium III 1.0 GHz laptop computer. The HLA inversion converged after about the same number of iterations but in this case took ten to fifteen minutes; the ORCA model then had to be run in the complex mode to correctly account for attenuation in sediment.

One-dimensional plots of the objective function (2.1) value versus geoacoustic model parameter value are provided in Figs. 4 and 5 for the two arrays respectively. Each individual frame displays energy versus parameter value for all accepted models. From left to right are the six geoacoustic parameters of each problem (Table 2); from top to bottom are source ranges 8.7 km, 5.9 km and 3.2 km. A total of about 1500 accepted models are included in each frame. Geometric parameters (Table 3) were included in the inversions but are not considered here.

The model parameter estimates are close to but not exactly at nominal values, this is to be expected when using noisy data as here. Estimates of p-wave velocity in sediment and substrate are observed to degrade with range. For the farthest source, a stronger than nominal gradient of p-velocity in sediment is obtained, yet the average p-wave velocity is reasonably well estimated. Energies of 0.25-0.40 were obtained for the best models for each inversion; these were slightly lower than the energies at the true model.

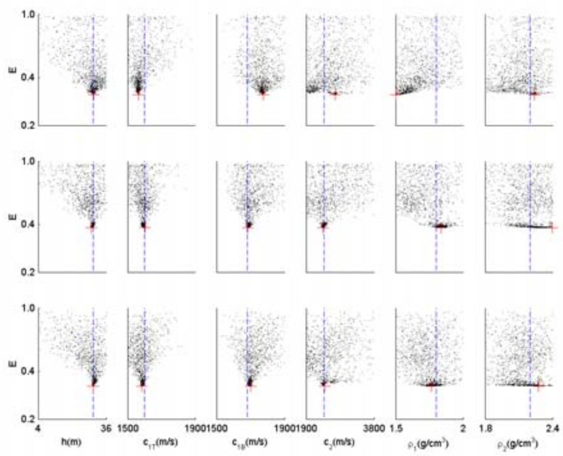


Fig. 4. Model parameter sensitivity for noisy synthetic data received at 21-element VLA. Results are from inversion of multi-frequency data (40-128 Hz) at source range 8.7 km (upper frames), 5.9 km (middle frames) and 3.2 km (lower frames). Each dot represents a model accepted by ASSA. The vertical dotted lines are the true geoacoustic model parameter values. A red cross indicates the lowest energy model.

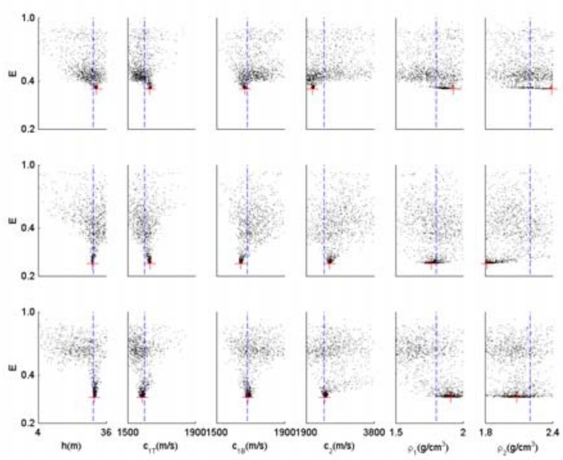


Fig. 5. Model parameter sensitivity for noisy synthetic data received at 10-element HLA. Results are from inversion of multi-frequency data (40-128 Hz) at source range 8.7 km (upper frames), 5.9 km (middle frames) and 3.2 km (lower frames). Each dot represents a model accepted by ASSA. The vertical dotted lines are the true geoacoustic model parameter values. A red cross indicates the lowest energy model.

The relative widths of the distributions near the minimum energy give an indication of model parameter sensitivity. In this case, the most sensitive parameters are the sediment p-wave velocity (top and bottom) and sediment thickness. Sensitivity to substrate p-wave velocity is less, while densities are not sensitive parameters. For HLA data, sensitivity to individual model parameters is similar to that observed for VLA. One notable difference is seen: for sediment p-wave velocity, a positive gradient is not estimated except for the short-range source, in fact estimated values of p-wave velocity are nearly equal at top and bottom of sediment for the other two sources.

The observations made pertaining to individual parameter sensitivity should not be generalized to every VLA or HLA configuration. The present examples provide an indication of what to expect from inversion of experimantal data for the array configurations, sources and processing frequencies selected.

V. INVERSION OF EXPERIMENTAL DATA

A. Inversion Details

Inversions were run using experimental data from six sources (one source at a time) at five selected frequency components within 40-128 Hz. (Inversions were also run using other combinations of frequency components, but results from these inversions are not considered in this paper.) The six geoacoustic parameters of Table 2 and three geometric parameters of Table 3 were included each inversion. Source depths were set to measured values, then scaled by (2.3). The ASSA cooling scheme determined from inversion of noisy synthetic data was used. Inversions converged after about 250 temperature steps and an additional 300 downhill steps after quenching, using a total of 6000-7000 calls to the forward model. The VLA inversions took 1-2 minutes whereas HLA inversions took from 10-15 minutes, since a correct treatment of attenuation in sediment was required for this case.

B. Parameter Estimates and Sensitivity

Results are tabulated in Tables 4 and 5 for the VLA and HLA respectively. The listed values are those of the lowest energy model from each inversion. The energy is quoted in the last column of each table. Shot labels (from W8 for the shot at 8 km west of the array to E8 at 8 km east of the array) have been introduced. Sediment and substrate densities and geometric parameters are not included in the tables.

TABLE 4
GEOACOUSTIC PARAMETER ESTIMATES BY INVERSION OF EXPERIMENTAL VLA DATA

Shot	D [m]	h [m]	c_{1T} [m/s]	c_{1B} [m/s]	c_2 [m/s]	E
W8	310.9	35.8	1543	1693	2070	0.30
W5	314.4	30.8	1512	1665	2150	0.27
W3	314.8	31.0	1515	1652	2112	0.27
E3	320.6	32.1	1502	1657	2132	0.33
E5	322.7	33.4	1517	1676	2234	0.34
E8	327.3	33.9	1520	1681	3330	0.34

TABLE 5
GEOACOUSTIC PARAMETER ESTIMATES BY INVERSION OF EXPERIMENTAL HLA DATA

Shot	D [m]	h [m]	c_{1T} [m/s]	c_{1B} [m/s]	c_2 [m/s]	E
W8	312.8	33.5	1618	1618	2367	0.30
W5	313.4	30.0	1584	1622	2196	0.47
W3	316.1	23.8	1567	1599	2564	0.50
E3	321.0	26.5	1704	1720	2247	0.53
E5	322.7	31.9	1549	1651	2182	0.38
E8	325.8	32.4	1577	1631	2653	0.37

Estimates of p-wave velocity in sediment are in all cases considerably lower than the baseline model value of 1770 m/s. A strong gradient of p-wave velocity in sediment is estimated by use of VLA data, this is weaker or absent when using HLA data. Estimates of substrate p-velocity fluctuate between values below and above the baseline value of 2400 m/s.

The energy of short-range shots (E3 and W3) received on the HLA is high. This could be related to quality of data (saturation), simplifications in the geoacoustic model (range-independence) or sensitivity to array geometry (precise array element locations). Possible contributors to mismatch have not been investigated further.

Sensitivity plots from inversion of VLA and HLA data is shown for three shots (E8, E5 and E3) in Figs. 6 and 7.

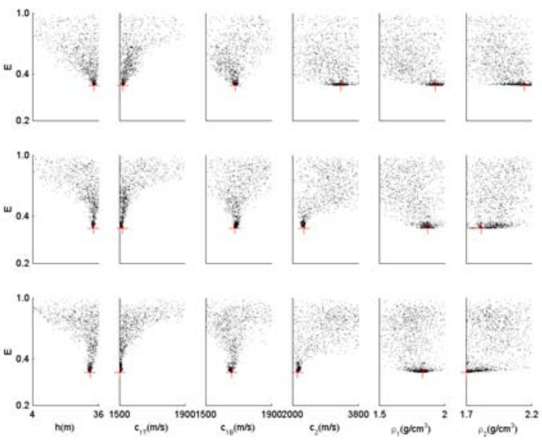


Fig. 6. Model parameter sensitivity for data received at 21-element VLA. Results are from inversion of multi-frequency data (40-128 Hz) at source range 8.7 km (upper frames), 5.9 km (middle frames) and 3.2 km (lower frames). Each dot represents a model accepted by ASSA. A red cross indicates the lowest energy model.

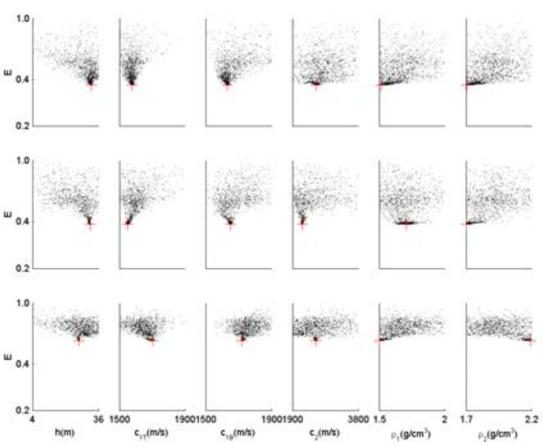


Fig. 7. Model parameter sensitivity for data received at 10-element HLA. Results are from inversion of multi-frequency data (40-128 Hz) at source range 8.7 km (upper frames), 5.9 km (middle frames) and 3.2 km (lower frames). Each dot represents a model accepted by ASSA. A red cross indicates the lowest energy model.

In all cases, parameter sensitivities are qualitatively similar to those observed for synthetic data. This indicates that meaningful assessments of parameter sensitivity can be obtained using simulated data, provided noisy data with a realistic SNR (in this case +3 dB) is used.

C. VLA and HLA comparison

The parameter estimates from the best energy models from each VLA and HLA inversion are plotted in Figs. 8 and 9 for selected geometric and geoacoustic model parameters.

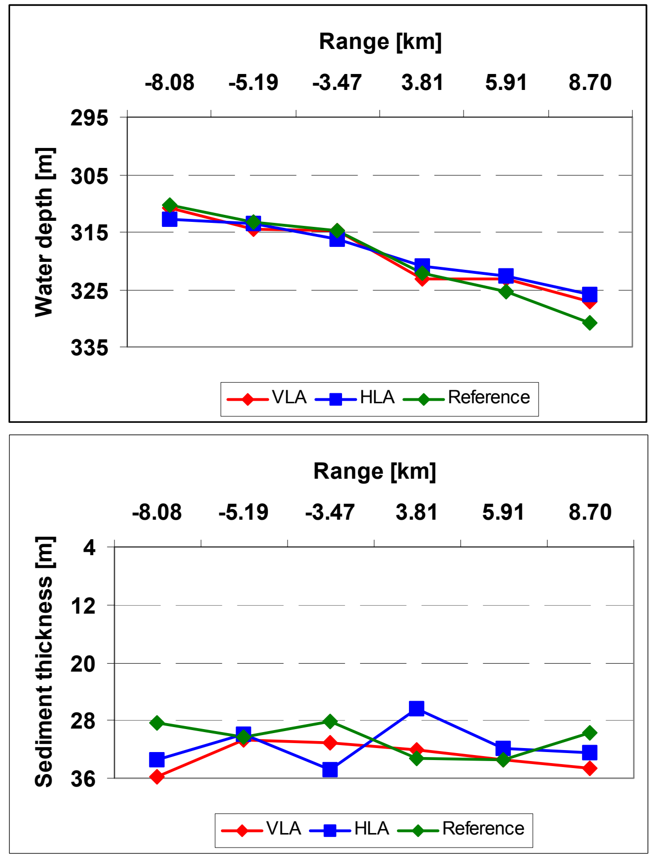
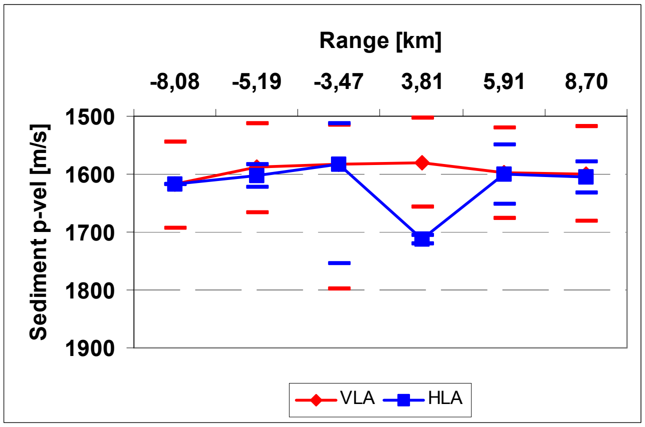


Fig. 8. Estimates of geoacoustic model parameters by matched-field inversion of shot data (40-128 Hz) recorded at a 21-element VLA (red markers) and endfire to a 10-element HLA at the seabed (blue markers). Six sources, range to the VLA (positive eastward) indicated. Reference values (green markers) are from the baseline geoacoustic model.

In Fig. 8, reference values for water depth and sediment thickness are included. The reference water depth is the effective depth computed from (2.2) using a measured bathymetry profile. There is good agreement between these reference values and the model water depths obtained from inversions. This suggests the validity of the equivalent flat bathymetry approximation for this environment. The reference values for sediment thickness are based on measured two-way seismic travel times and a p-velocity in sediment of 1600 m/s. There is good agreement between these reference values and estimates from inversions; in all cases values within 28-34 m were obtained.

In Fig. 9, estimates of p-wave velocity at top and bottom of the sediment layer as well as the mean value are included. A good agreement of mean p-wave velocity values (1580-1620 m/s) is obtained in all inversions, except for shot E3 on the HLA. Gradients in p-wave velocity are estimated differently by VLA and HLA data. This was also observed by inversions of synthetic data.



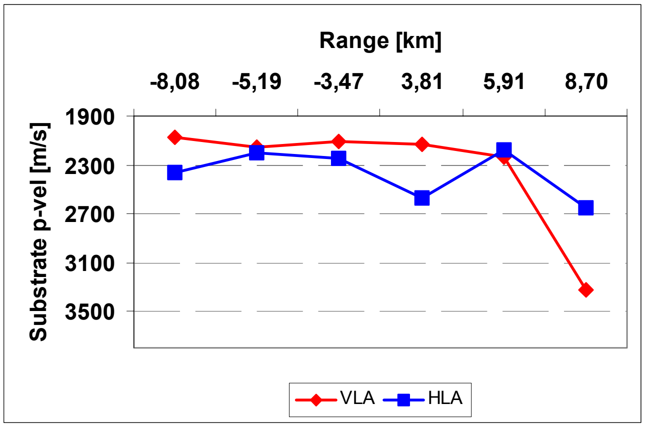


Fig. 9. Estimates of geoacoustic model parameters by matched-field inversion of shot data (40-128 Hz) recorded at a 21-element VLA (red) and endfire to a 10-element HLA at the seabed (blue). Six sources, range to the VLA (positive eastward) indicated. For p-wave velocity in sediment (upper panel) closed symbols represent the average value; estimated values at top and bottom of sediment are indicated by minus signs.

Estimates of p- wave velocity in substrate are within 2070-2370 m/s (except for shot E8 data and shot E3 on the HLA). Reference values for p-wave velocities of sediment and substrate are found in the baseline geoacoustic model (Table 1). For both parameters, the estimates obtained by inversion of acoustic data are lower than the baseline model values.

VI. SUMMARY AND DISCUSSION

Low-frequency acoustic data from shot sources endfire to a ten-element HLA at the seabed and a 21-element VLA have been inverted for geoacoustic model parameters. Data at frequencies within 40-128 Hz from six sources deployed at ranges from 3 km to 9 km in two directions from the array were used. A range-independent seabed model consisting of a fluid sediment layer over a fluid halfspace was used for the area, consistent with prior geophysical information. A fast implementation of the hybrid adaptive simplex simulated annealing (ASSA) inversion algorithm was used, with replica fields computed by the ORCA normal mode propagation model.

Results from independent inversions of VLA and HLA data were consistent between the six shots. An average p-wave velocity in sediment of 1580-1620 m/s was estimated. From VLA data, a fairly strong positive gradient of p-wave velocity in sediment was estimated; a weaker or no gradient was estimated from HLA data. Substrate p-wave velocity was estimated to 2070-2370 m/s (except when using data from one shot at longest range). These estimated values are lower than the baseline geoacoustic model values of 1770 m/s and 2400 m/s respectively. For sediment thickness, estimates varied between 28-36 m, in good agreement with values of the baseline geoacoustic model.

The consistency of geoacoustic model parameter estimates obtained from separate inversions of HLA and VLA data is the main result achieved in this paper. This suggests that in situations where data from a vertical array is not readily available, data from sources endfire to a horizontal array at the seabed can alternatively be used for geoacoustic inversion. An array of some minimum length and number of elements in relation to processing frequencies is required. Requirements to HLA design for matched-field inversion is a matter for further research.

ACKNOWLEDGMENT

The data were collected by the Norwegian Defence Research Establishment under project 776 SWASI.

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