A Comparison of AUV Navigation Performance: A System Approach

Edgar An
Advanced Marine Systems Laboratory
Ocean Engineering Department
SeaTech Campus
Florida Atlantic University
Dania Beach, Florida
ean@oe.fau.edu

I. ABSTRACT

This paper addresses the technical challenges in underwater navigation for autonomous underwater vehicles (AUVs) in terms of selecting an appropriate combination of navigation system and software processing. There are many existing systems that can be used onboard an AUV with different costs and capabilities. It is not immediately clear to ocean scientists which system is appropriate for a given set of navigation requirements. To address this challenge, this paper proposes a novel metric called navigation efficiency which is intended to provide a framework with which the navigation performance can be related directly to a given sensor suite and survey requirements. Three basic survey patterns are considered although more complicated survey patterns can be created using these basic patterns, and the navigation efficiency modified accordingly. To address the limitation in software processing for vehicle motion in the presence of wave disturbances, this paper provides new insights into how the existing error state Kalman filtering techniques can be used to attenuate the effect of wave disturbances. To substantiate the insights, an error state filter is applied to estimating the bias in gyro measurements although the basic idea can be extended to estimating other motion variables. The main contribution of this paper is twofolded. Firstly, it provides ocean scientists an easily understood framework with which to size and specify the needed navigation systems. Secondly, it provides navigation engineers a new appreciation of the use of error state filters to characterize complicated vehicle motion.

II. INTRODUCTION

Autonomous Underwater Vehicles (AUVs) are unmanned, untethered and self-propelled platforms. They provide marine researchers with a simple, long-range and low-cost solution with which to gather oceanographic data [4][5]. Recent trends in AUV technology are moving towards reducing the vehicle size and improving its deployability in order to reduce the operational costs. Common applications for AUVs include oceanographic sampling, bathymetry profiling, underwater system inspection, and military mine counter-measure (MCM) operations [12].

In many of these missions, it is critical that the vehicle position be known to a given precision and in real-time such that the seafloor can be mapped accurately, and bottom mines re-acquired with a high degree of confidence despite that the vehicle motion is subject to large wave forces in a littoral zone. Underwater navigation for AUVs is thus a very challenging research topic, and is a major subject of interest to both the AUV community and their end users.

Since AUVs are multi-function platforms, their navigation requirements depend highly on a specific mission and the sensor suite onboard. It is entirely feasible that, for a given suite of sensors, the vehicle can navigate adequately for one mission, but fail to meet the minimum requirements for another mission. Since the sensor suite onboard a vehicle can be modified and upgraded, the underlying questions are:

1. What sensor suite is required for a given mission?
2. Can the inadequacy of the sensor suite be alleviated by means of software processing?

These have been the main concern for many ocean scientists who are not familiar with AUV navigation. It is thus of much interest and significance to derive a mechanism with which the navigation performance can be compared on the basis of navigation requirements and sensor suites, and to determine appropriate software processing to enhance the navigation capability. The objective of this paper is to address these two inter-related aspects, and to propose a novel mechanism for comparing navigation performance and to provide new insights into the software processing. This paper is organized as follows. The next section reviews briefly the background in navigation in terms of existing navigation systems and software processing. Section IV introduces a new concept of navigation efficiency which forms a basis for navigation evaluation, and Section V addresses the impact of software processing on navigation performance. Section VI provides summary remarks and future work planned.

III. BACKGROUND

This section reviews very briefly some of the existing navigation systems and software processing that can be used on small AUVs. Note that the treatment provided here is by no means exhaustive. Additional resources are available in [1][6][7][8][9].

III.1 Existing Navigation Systems

Given the rapid development in computer technology, the navigation sensors continue to reduce in size and improve in performance. Important components for AUV navigation are composed of magnetic compass for vehicle's heading, Dopper Velocity Log (DVL) for vehicle's velocity in a body-fixed frame, global positioning system (GPS) for vehicle's position on surface, inertial measurement unit (IMU) for vehicle's acceleration and angular velocity, and acoustic positioning system (APS) for vehicle's position relative to the acoustic baseline. It is obvious that these sensor components provide different aspects of the vehicle's position and motion, and a careful mix of these components will provide synergistic improvement in navigation capability.

Platforms	Size (mm)	Accuracy (w/DVL)
PHINS INS IXSEA	160x160x160	< 5m / hr
MARPOS INS Maridan	638x370 (dia)	< 5m / hr
CORNER INS Bluefin	N/A	0.1% [15]
IMU+DVL+compass	73 x 93 (dia)	N/A
Compass + DVL	180x186 (dia)	2-3%

Table 1 Existing Navigation Systems

Table 1 presents a snapshot of some of the navigation systems that are available, and their brief capabilities in size and accuracy [16][17][18][19][1]. In the table, the first three systems are completely integrated in the sense that they can output position information directly, in addition to attitude and motion information. For examples, the PHINS system utilizes a high-end IMU based on fiber optic gyro technology whereas the MARPOS and CORNERSTONE systems utilize different high-end IMUs based on ring-laser technology. The last two options require the end users to carry out strenuous effort on sensor integration and fusion processing. The first three options by far are the most powerful systems albeit the most expensive (by at least an order of magnitude) whereas the last two options are by far the most popular methods despite that

[1] A complete description of the accuracy capability of these systems is beyond the scope of this paper. Details and additional materials should consult the references.

they are much less accurate. It is thus important to determine, given a specific mission objective, which navigation system is needed to satisfy the minimum navigation requirement with minimum costs incurred.

III.2 Existing Navigation Processing

Dead reckoning (DR) aided with Doppler velocity measurement has been, and remains, the most common method for underwater navigation. In the DR mode, the vehicle relies on a set of navigation instruments to estimate its position. These instruments include a 3-axis accelerometer and gyroscope, a flux-gate compass and a Doppler-velocity-log (DVL) sonar. The system is commonly referred to as a Doppler-aided inertial navigation system (INS) as most of these instruments rely on the inertial properties of gravity and magnetic field. The DR solution does not measure directly the vehicle's position, but instead estimates it by integrating the velocity and bearing measurements with respect to time. Any measurement errors in velocity and heading will thus result in a growing position error, up to a threshold beyond which the navigation performance becomes unacceptable. It is widely accepted that, with bottom lock measurements from DVL, the most dominant error source can be attributed to that of vehicle's heading. A detailed coverage of the DR processing can be found in [7]. To address the position error growth in the DR solution, a widely known technique called Kalman filtering can be applied.

The development of time domain Kalman filtering for parameter estimation, subject to a set of known dynamics constraints, can be dated back to the sixties, and the concept of error state filtering is not new [2][9]. The basic principle of Kalman filtering rests on minimizing the mean squared error of the parameter estimates provided that an accurate description of process uncertainty and measurement variability are available. The basic review of the full state and error state filtering and their implementation are beyond the scope of this paper, and readers should consult [2][9] for a complete coverage.

There are two methods for obtaining direct position measurements. In the first method, the vehicle surfaces, and obtains GPS position fixes on a needed basis, although at the expense of mission efficiency. An alternative method is to make use of an array of Long-Base-Line (LBL) acoustic beacons such that the vehicle position can be continuously estimated by means of triangulation [7][14]. The LBL solution requires the beacon array to be either mounted on the ocean bottom or moored on the surface (inverted LBL), and thereby restricts the AUV coverage area to be within the beacon grid. In addition, the performance of this externally aided positioning depends largely on the sound speed profile in the water column. For multiple AUV operations, independent sets of beacons must be installed, and this rapidly increases the logistical complexity of a mission.

IV. NAVIGATION EFFICIENCY

This section formulates a novel metric called navigation efficiency, η, which can be used to compare the navigation performance with respect to a given sensor suite and survey requirements. Since the vehicle's vertical positioning in the water column can be directly and accurately measured using pressure or sonar sensors, η is applied only to the horizontal navigation profile.

Three basic survey patterns are considered. They are: line, box and lawn mower patterns. It can be seen later that more complicated patterns can be created using these basic geometric patterns, and η modified. In all these surveys, it is assumed that the initial vehicle position is available (e.g. GPS fixes), and the error growth in position is solely related to speed and heading errors during the course of a survey. Equation (1) & (2) describe a first-order perturbation of a 2D kinematic transformation about the yaw axis. The perturbation is evaluated with respect to the vehicle's heading (ψ), forward and starboard speeds (u, v).

$$\Delta x' = \Delta u \cos\psi - \Delta v \sin\psi - (u \sin\psi + v \cos\psi)\, \Delta\psi \quad (1)$$

$$\Delta y' = \Delta u \sin\psi + \Delta v \cos\psi + (u \cos\psi - v \sin\psi)\, \Delta\psi \quad (2)$$

where ’ denotes a time derivative operator, Δ the perturbation of a motion variable due to its sensor error. That is, Δu and Δv are the forward and side speed biases respectively, and Δψ is the heading bias [7].

IV.1 Line Survey

Consider an AUV travels at a constant velocity and heading. Assuming the speed bias is constant, and heading bias is constant for some chosen vehicle orientation, Δx’ and Δy’ are also constant, see (1) and (2). Thus,

$$\Delta x = \Delta x'\, T \quad (3)$$
$$\Delta y = \Delta y'\, T \quad (4)$$

Define $E = \sqrt{(\Delta x^2 + \Delta y^2)}$ as the total error in position traveled during a line survey. After a few steps of derivation, we have

$$E = \sqrt{[\Delta u^2 + \Delta v^2 + \Delta\psi^2 (u^2 + v^2) + 2\, \Delta\psi\, (u\, \Delta v - v\, \Delta u)]} = D \sqrt{[(\Delta u / S)^2 + (\Delta v / S)^2 + \Delta\psi^2]} \quad (5)$$

where S is the magnitude of the vehicle speed, and is defined as $S = \sqrt{(u^2 + v^2)}$. D is the total distance traveled, and is defined as S·T. One can see that for a line survey, the navigation error is intuitively dominated by the speed and heading biases. Note that the term, (u Δv – v Δu), is set to zero based on the assumption that the speed bias is pre-dominantly due to scale factor effect, which is the same for both u and v. Let us define navigation efficiency, η, as

$$\eta = (D - E) / D = 1 - (E / D) = 1 - \sqrt{[(\Delta u / S)^2 + (\Delta v / S)^2 + \Delta\psi^2]} \quad (6)$$

Note that η is independent of D, and is a better evaluation index for navigation performance. Figure 1 illustrates the relationship of η with respect to biases in speed and heading. There are 10 contours altogether, starting with the innermost contour at η = 0.99, and ending with the outermost contour at η = 0.90.

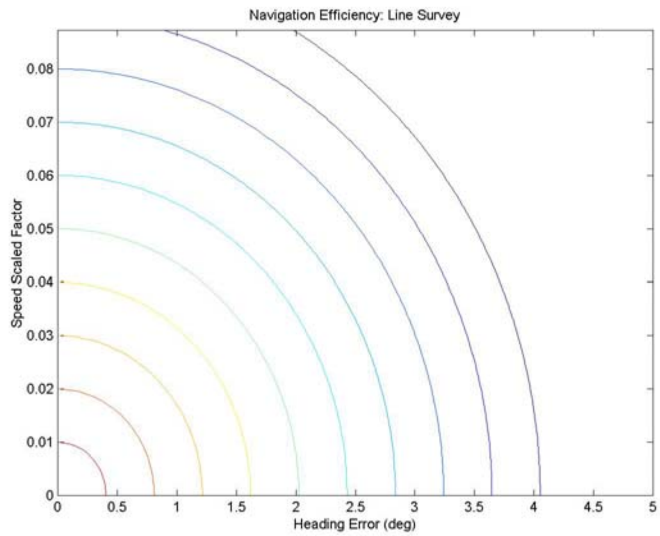

Figure 1 Navigation efficiency wrt biases in speed and heading. η goes from 0.99 to 0.90.

IV.2 Box Survey

Without loss of generality, consider an AUV performs a box survey aligned with north, east, south and west legs, and neglect the dynamics during turning. Define T_1 and T_2 as the time taken to execute an east-west leg and a north-south leg respectively.

$$\begin{aligned} \Delta x &= \Delta x_n + \Delta x_e + \Delta x_s + \Delta x_w \\ &= T_2 (\Delta u - v\, \Delta\psi_n) + T_1 (-\Delta v - u\, \Delta\psi_e) + T_2 (-\Delta u + v\, \Delta\psi_s) + T_1 (\Delta v + u\, \Delta\psi_w) \\ &= -T_2\, v\, \Delta\psi_n - T_1\, u\, \Delta\psi_e + T_2\, v\, \Delta\psi_s + T_1\, u\, \Delta\psi_w \end{aligned} \quad (7)$$

Similarly,

$$\begin{aligned} \Delta y &= \Delta y_n + \Delta y_e + \Delta y_s + \Delta y_w \\ &= T_2 (\Delta v + u\, \Delta\psi_n) + T_1 (\Delta u - v\, \Delta\psi_e) + T_2 (-\Delta v - u\, \Delta\psi_s) + T_1 (-\Delta u + v\, \Delta\psi_w) \\ &= T_2\, u\, \Delta\psi_n - T_1\, v\, \Delta\psi_e - T_2\, u\, \Delta\psi_s + T_1\, v\, \Delta\psi_w \end{aligned} \quad (8)$$

After a few steps of manipulation, we have

$$E = \sqrt{(\Delta x^2 + \Delta y^2)} \quad (9)$$
$$= S\sqrt{[T_2^2\, \Delta\psi_n^2 + T_1^2\, \Delta\psi_e^2 + T_2^2\, \Delta\psi_s^2 + T_1^2\, \Delta\psi_w^2 - 2\, T_2^2\, \Delta\psi_n\, \Delta\psi_s - 2\, T_1^2\, \Delta\psi_e\, \Delta\psi_w]}$$

Consider the worst case in which all heading biases are upper bounded by $\Delta\psi_{max}$. Also, assume the product of heading biases is negative in order to maximize E. We then have

$$E_{max} = 2\, S\, \Delta\psi_{max} \sqrt{[T_2^2 + T_1^2]} \quad (10)$$

For a square box survey, $T_1 = T_2$, and $D = 4\, S\, T_1$

$$E_{max} = \sqrt{(½)}\, D\, \Delta\psi_{max} \quad (11)$$

The navigation efficiency in this case is

$$\eta = 1 - (E / D) \geq 1 - \sqrt{(½)}\, \Delta\psi_{max} \quad (12)$$

IV.3 Lawn Mower Survey

Consider a simple lawn mower pattern shown in Figure 2. The pattern can be constructed using surveys of 2 boxes of length T_1 ($T_2 = 0$) and a box of length T_2. ($T_1 = 0$).

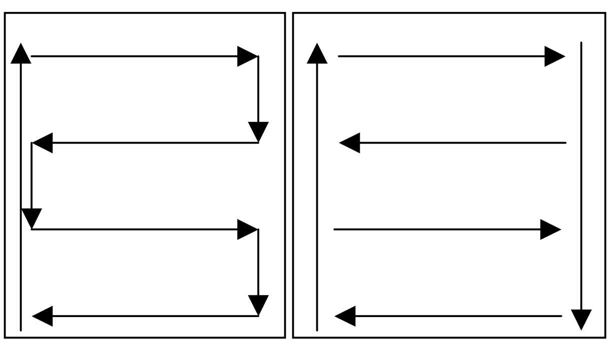

Figure 2 A simple lawn mower survey

With this interpretation, we have

$$E_{max} = 2\, (2\, S\, \Delta\psi_{max}\, T_1) + 2\, S\, \Delta\psi_{max}\, T_2 \quad (13)$$
$$= S\, \Delta\psi_{max}\, (4\, T_1 + 2\, T_2)$$
$$= \Delta\psi_{max}\, D$$

It can be easily shown that this result holds true for a more complicated lawn mower pattern using *n* survey boxes ($T_2 = 0$) and 1 survey box ($T_1 = 0$). η can now be expressed as

$$\eta = 1 - (E / D) \quad (14)$$
$$\geq 1 - \Delta\psi_{max}$$

Thus, the navigation efficiency of a lawn mower survey is generally not as high as that of a box survey. Note that the navigation efficiency can be computed for any other survey patterns straightforwardly by using box and line surveys as building blocks. It is worth mentioning that for some missions, E is subject to a maximum radius of uncertainty. Examples include large survey area missions for oceanographic sampling or mine counter-measures. It is intuitive that the total mission distance can be maximized if E can be minimized. Ways to reduce E include reducing the maximum heading error, or inserting position fixes during a mission.

IV.4 Availability of Position Fixes

The determination of navigation efficiency above assumes that there are no position fixes during a survey. In cases the vehicle surfaces for fixes, its navigation efficiency can be improved by reducing E. However, the overhead in time for performing the surfacing is also the time the vehicle is not contributing to the mission objective. It will be unfair to compare the navigation efficiencies of different sensor suites if such overhead is not incorporated into the definition. Thus, the navigation efficiency is modified as

$$\eta = \frac{(D - E)}{D} \frac{(T - To)}{T} \quad (15)$$

Where T is the total mission period, and To corresponds to the overhead in time required to execute a given mission. This might include time periods for obtaining GPS fixes (both surfacing and time-to-first-fix delay), an initial time period for performing sensor calibration or alignment, or a time perod for installing and calibrating an acoustic baseline positioning system. For the latter two cases, the navigation efficiency can be significantly deteriorated for a single mission, but can be improved if multiple missions can be performed without requiring any additional re-calibration.

IV.5 Remarks on Navigation Efficiency

It should be reminded that the navigation efficiency does not account for the dynamics of both the vehicle motion and sensor characteristics, and assumes that the biases to be constant [2]. This is not an unreasonable assumption since the majority of AUV missions utilize magnetic compasses and DVL for underwater navigation. Except for a line survey, the effect of speed bias is minimal due to the symmetry of box and lawn mower survey patterns.

To illustrate the defined navigation metric, consider a large area survey for oceanographic sampling or MCM operations. In these operations, E_{max} is typically given as a navigation requirement. Choose $E_{max} = 20$ meters, and $\psi_{max} = 3°$. If a box survey is performed, the maximum side is limited to 540 meters. Whereas a lawn mower survey results in a side length of about 382 meters only. It would appear that a box pattern should be used instead of a lawn mower pattern. However, a constraint on spatial survey resolution tends to favor the latter. Thus, a compromise in sampling and navigation requirements might be needed [2].

It was mentioned earlier that one way to reduce E or improve η is to reduce the maximum heading error by means of filtering, which is the topic for the next section.

V. KALMAN FILTERING

An error state filter makes use of the complementary characteristics of the sensor suites available so that the dynamic response of the motion variables can be maintained [2]. For marine applications, an AUV is programmed to maneuver with a varying degree of speed, depth and heading profiles. Minimally, the maneuvering dynamics must be modeled and incorporated into the Kalman filter otherwise biases in the motion estimates will result. In addition, an AUV moving near the ocean surface or in a shallow water column is subject to a varying degree of wave forces, resulting in undesirable and unpredictable vehicle motion. Thus, the Kalman filter must account for the intended maneuvering (partially known via modeling) and undesired maneuvering (due to disturbances). It is the aspect of the vehicle motion and its estimation on which the rest of the paper is focused.

A Kalman filter can be applied to estimate the six degree of motion of an AUV. Without the knowledge of the disturbance forces, it is impossible to distinguish between vehicle motion and sensor error. Typically, the process uncertainty associated with such disturbances is ignored, resulting in estimation performance which is much less predictable. However, in an error state Kalman filter, each of the error states is defined as the difference between a derived quantity and a measured quantity. As a result, the disturbance effect can be signficantly attenuated. The following presents a simplified vehicle motion modeling and simulation together with preliminary estimation results.

V.1 Vehicle Motion Modeling

In this simulation, the vehicle was chosen to maneuver in 2D although the concept can be extended to 3D motion. The state variables of interest were heading and the heading rate of the vehicle, both of which were observable and had unknown biases. The objective of the simulation is to estimate the unknown bias in the rate measurements. The vehicle was assumed to travel at a heading of 150 degrees for a period of 2000 seconds. Its vehicle motion due to wave disturbances was modeled as a combination of sinusoids as in (16).

[2] The heading bias is constant at a particular vehicle orientation.

$$\psi(t) = 0.13 \sin(2 \pi f_0 t) + 0.23 \sin(2 \pi f_0 t) + 0.41 \sin(2 \pi f_0 t) \quad (16)$$

where f_0 is the fundamental frequency, and was set to 0.1 Hz.

V.2 Sensor Modeling

The gyro measurements were modeled as a Gaussian white noise sequence (σ = 0.03°/sec) added with a constant bias of 1°/hr, whereas the compass measurements were modeled as another independent Gaussian white noise sequence (σ = 0.3°) added with a constant bias of 1 degrees. The update rates for the gyro and compass measurements were chosen to be 100 and 10 Hz respectively. Note that the above sensor characteristics are similar to those of a magnetic compass and a medium grade inertial measurement unit (IMU) available in the off-the-shelf market.

V.3 Gyro Bias Estimation

This section considers estimating the gyro bias using a full state Kalman filter and an error state Kalman filter.

V.3.1 Full State Kalman Filter

In the full state filter, there are three states: heading (ψ), heading rate (r) and gyro bias (b_g). Consider that the wave induced vehicle motion is unavailable, and the vehicle travels at a constant heading. The state equations can be arranged as

$$\psi' = r \quad (17)$$
$$r' = 0 \quad (18)$$
$$b_g' = 0 \quad (19)$$

The measurement equations are given as

$$\psi_m = \psi + v_\psi \quad (20)$$
$$r_m = r + v_g \quad (21)$$

where v represents the measurement errors, the subscripts m stands for measurement respectively. It should be reminded that ψ and r represent the true motion, both the intended straight line motion and undesired wave induced motion.

A standard linear Kalman filter was chosen for the implementation because the mechanization equations are linear with respect to the state variables. The process uncertainty matrix (**Q**) was set to a null matrix of 3x3, and the measurement noise matrix (**R**) was chosen according to the noise variances of the individual sensors [3]. The top plot in Figure 3 shows a time history of the bias estimate. One can see that the estimate is slowly converging as well as fluctuates significantly. The bottom plot in Figure 3 shows another time history of the estimate in which the matrix **Q** was adjusted so that Q(1,1) = R(1,1) and Q(2,2) = R(2,2). One can see that the noise level in the estimate is considerably less although it still exceeds 1 °/ hr. These results suggest that, without properly accounting for the vehicle motion due to wave disturbances, the gyro bias estimate can be significantly sensitive to the choice of **Q**.

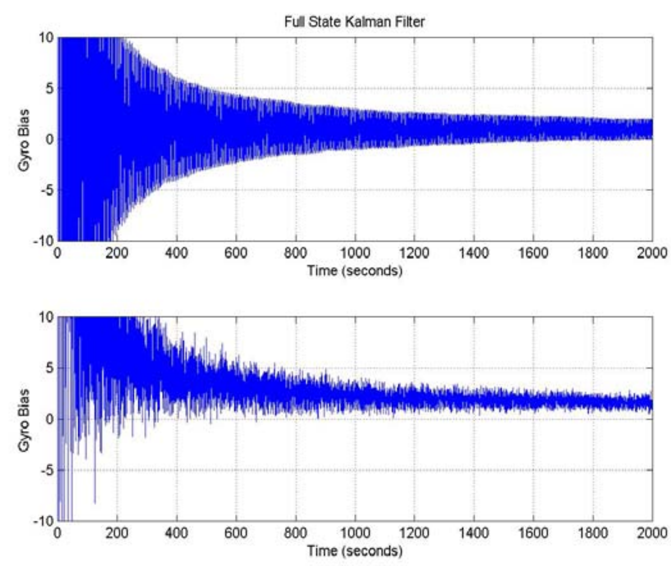

Figure 3 Estimation performance using a full state Kalman filter

V.3.2 Error State Kalman Filter

[3] The details about these matrices are beyond the scope of this paper, and can be found in [3][6].

In the error state filter, there are two states: heading error (ψ_e) and gyro bias (b_g). ψ_e is defined as the difference between a computed heading and a measured heading. The computed heading was calculated by integrating the gyro measurements over time, initialized by the first sample in the compass measurements. That is,

$$\begin{aligned}\psi_e(t) &= \psi_c(t) - \psi_m(t) \\ &= \int r_m(t)\,dt - \psi_m(t) \\ &= \int (r(t) + b_g + \sigma_g(t))\,dt - (\psi + b_m + \sigma_m(t)) \\ &= b_g\,t + \int \sigma_g(t)\,dt - b_m - \sigma_m(t)\end{aligned} \tag{22}$$

$$\psi_e'(t) = b_g + \sigma_g(t) - d/dt(\sigma_m(t)) \tag{23}$$

where the subscript c stands for a computed quantity, σ_g and σ_m are the fluctuations in the gyro and compass measurements respectively. Ignoring the last term in (23), we have the following state equations

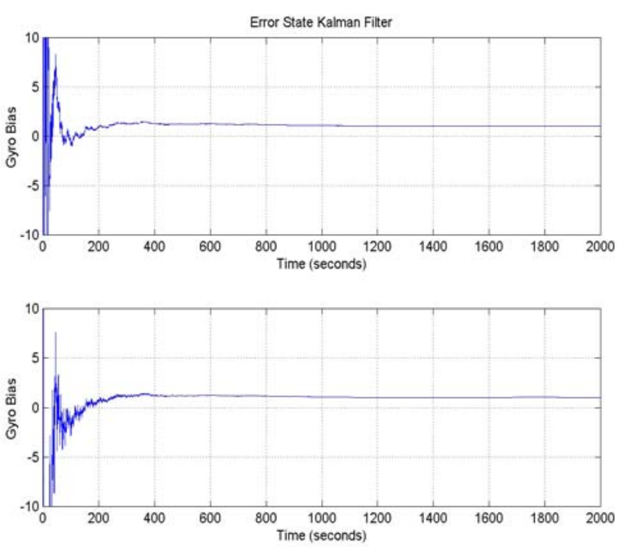

Figure 4 Estimation performance using an error state Kalman filter

$$\psi_e' = b_g + \sigma_g \tag{24}$$
$$b_g' = 0 \tag{25}$$

In this case, the only error measurement is defined as

$$\psi_{e,m} = \psi_e + v \tag{26}$$

The top plot in Figure 4 shows a time history of the bias estimate given that **Q** = **0** and **R** was chosen based on the noise variance of the compass measurements, see (22). The bottom plot in Figure 4 shows another time history of the estimate given that **Q** was chosen based on the noise variance of the gyro measurements, see (24).

By comparing the above results, one can see that the estimation performance can be significantly improved using an error state filter, and the performance is relatively insensitive to the choice of **Q**. This is due to that the vehicle motion is largely rejected due to the differencing in measurements.

VI. CONCLUSION

In this paper, the challenges in underwater navigation has been reviewed in terms of selecting an appropriate combination of navigation system and software processing. A novel metric called navigation efficiency has been proposed mainly to provide a framework with which the navigation performance can be evaluated for a given sensor suite and survey requirements. Three basic survey patterns were considered although more complicated survey patterns can be created using these basic patterns, and the navigation efficiency modified accordingly. To address the limitation in software processing for vehicle motion, new insights were provided into how an error state Kalman filter can be used to attenuate the effect of wave disturbances. The error state filter was applied to estimating an unknown bias in the simulated gyro measurements although the basic idea can be extended to estimating other motion variables. The main contribution of this paper are twofolded. Firstly, it provides ocean scientists an easily understood framework with which to size and specify the needed navigation systems. Secondly, it provides navigation engineers a new appreciation of using error state filters for characterizing complicated vehicle motion. Future work includes further extending the error state filtering method to other motion estimation, and evaluating the filtering performance using real at-sea data.

VII. ACKNOWLEDGMENTS

This research was supported by the Office of Naval Research under the grant number N00014-96-1-5029.

VIII. REFERENCES

[1] P.E. An, T. Healey, On Line Compensation of Heading Sensor Bias for Low Cost AUVs, AUV 98 Navigation Workshop, pp.35-42, Draper Lab, Massachusetts, August, 1998.

[2] J. Bellingham, S. Willcox, Optimizing AUV Oceanographic Surveys, AUV 96 Conference, pp.391-398, Monterey, CA, 1996.

[3] Introduction to Random Signals and Applied Kalman Filtering, R. Brown, P. Hwang, Wiley Press, 1997.

[4] T. Curtin, J. Bellingham, J. Catipovic, D. Webb, "Autonomous Oceanographic Sampling Network, Oceanography, Vol. 6, No. 3, pp.86-94, 1993.

[5] M. Dhanak, E. An, K. Holappa, An AUV Survey in the Littoral Zone: Small-scale Subsurface Variability Accompanying Synoptic Observations of Surface Currents, IEEE Transactions of Oceanic Engineering, Vol.26, No.4, pp.752-768, October 2001.

[6] Applied Optimal Estimation, A. Gelb, MIT Press, 1986.

[7] G. Grenon, E. An, S. Smith, A. Healey, Enhancement of the Inertial Navigation System for the MORPHEUS Autonomous Underwater Vehicles, IEEE Transactions of Oceanic Engineering, Vol.26, No.4, pp.548-560, October 2001.

[8] M. S. Grewal and A. P. Andrews, Kalman Filtering: Theory and Practice, Prentice Hall, 1993.

[9] Einar Gustafson, Edgar An, Alexander Leonessa, Samuel Smith, A Post-Processing Kalman Smoother for Underwater Vehicle Navigation, UUST 2001, Trends New Hampshire, NH,September, 2001.

[10] J. Huddle, in Inertial Systems Technology for High Accuracy AUV Navigation, AUV Navigation Workshop, pp.63-73, 1998, Boston, Mass.

[11] R. E. Kalman, A New Approach to Linear Filtering and Prediction Problems, Journal of Basic Engineering, March 1960, pp. 35 - 46.

[12] Smith, S.M., An, P.E., Holappa, K., Whitney, J., Burns, A., Nelson, K., Heatzig, E., Kempfe, O., Henderson, E., Kronen, D., Pantelakis, T., Font, G., Dunn, R., Dunn, S.E. Morpheus: ultra modular AUV for coastal survey and reconnaissance. *IEEE Journal of Oceanic Engineering*, 2001.

[13] S.M. Smith, K. Ganesan, P.E. An, S.E. Dunn, Strategies for Simultaneous Multiple AUV Operation and Control, International Journal of Systems Science, Vol.29, No.10, pp.1045-1063, 1998.

[14] T.R. Stockton et. al., Acoustic Position Measurement: An Overview, Offshore Technology Conference, Dallas, Texas, 1975.

[15] Personal communication with Bluefin.

[16] www.ixsea.com/english/products/phins

[17] www.thortech.org/thortech/submarine-specs.html

[18] www.honeywell.com

[19] www.rdinstruments.com