# Multichannel Blind Deconvolution Application To Marine Seismic

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***Abstract*— In seismic deconvolution, blind approaches must be considered in situations where the reflectivity sequence, the source wavelet signal and the noise power level are unknown. In this paper, we propose a new method of blind deconvolution involving a multichannel version of the SEM algorithm. We considerer that the reflectivity sequence is the same for each hydrophone receiver while the noise level may differ at each channel. Compared to blind deconvolution of a single seismic trace, multichannel processing provides a faster convergence of the estimated parameters and reflectivity sequence.**

## I. INTRODUCTION

The aim of reflection marine seismic exploration is to obtain an image of the sea bottom lithography from reflected acoustic waves emitted by various seismic sources. The seismic source, which produces the seismic wavelet, is tracked by a ship, which also drags a multitrace streamer consisting in an array of hydrophones, which record the reflected acoustic signals (Figure1). The traveling distance between two shots is equal to the half of the distance between two sensors, which means that each sensor will receive signals coming from the same Common reflection Mid Point (CMP) of the seabed during a shot sequence. This allows to average the signals in order to increase the Signal-to-Noise Ratio (SNR). The received signals can be modeled as the convolution of an unknown source wavelet and a reflectivity sequence, which describes the most important interfaces met by the emitted wave [1]. Such blind deconvolution is considerably more difficult than simple deconvolution, where the wavelet is known a priori. The problem is highly ill posed and a priori knowledge must be introduced to achieve a convenient solution. In cases where several recordings are available, a new blind deconvolution approach is proposed in this paper, to improve the wavelet and reflectivity estimation. This method is based on the use of the Common Mid Point (CMP) information. In fact, the CMP corresponds to the position subsurface (reflectivity) observed by different (hydrophone, shot) pairs.

Here, we address the problem of parameter estimations of the a priori model via to the Maximum Likelihood approach [2]. Unfortunately direct maximization of the likelihood is not feasible, since we are in the presence of an incomplete data model. In practice, maximizing the likelihood functional of incomplete data models (here the model is completed by incorporating the Bernoulli reflectivity sequence) can be done by using the Expectation-Maximization (EM) algorithm [3]. However, for our problem, this algorithm requires high dimensional integrations that are very difficult to calculate by means of classical numerical methods. MCMC (Markov Chain Monte Carlo) methods enable to circumvent this difficulty by solving the integration and optimization problems by simulating random variables. This approach leads to stochastic versions of the EM algorithm (SEM, SAEM) [9]. Then the estimation of the Bernouilli-Gaussian model and of the wavelet is solved by the simulation of random variables via MCMC algorithms such as the Gibbs sampler [7], [8], [9]. In this paper, propose a new extension of blind deconvolution by applying the SEM algorithm to multichannel signals. Here, considerer that the reflectivity sequence is the same for each hydrophone receiver while the noise level may differ at each channel.

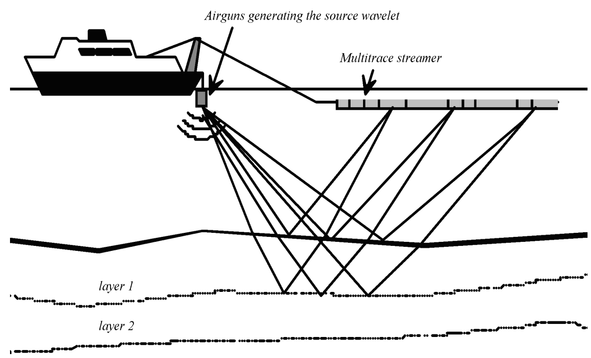


Fig. 1. Acquisition system

This multichannel processing provides the faster convergence of the estimated parameters and sequence of reflectivity compared to a channel per channel processing. In fact, for single-channel processing two steps are necessary [10]. First, parameters are estimated, then a deconvolution via an MPM (Maximum Posterior Mode ) approach yields the final estimate of the reflectivity sequence. In multi-channel processing, the MPM step appears to be useless because an accurate estimation of the reflectivity sequence is supplied by the SEM algorithm.

The paper is organized as follows: Section II describes the data model, while section III is devoted to parameters estimation. Finally, in section IV we present simulation results. A conclusion is presented in section V.

## II. Data Model

The observed multichannel signal $(\mathbf{y^1}, \ldots, \mathbf{y^d})$ is of the form

$$\begin{bmatrix} \mathbf{y^1} \\ \mathbf{y^2} \\ \vdots \\ \mathbf{y^d} \end{bmatrix} = \begin{bmatrix} \mathbf{h^1} \\ \mathbf{h^2} \\ \vdots \\ \mathbf{h^d} \end{bmatrix} \star \mathbf{r} + \begin{bmatrix} \mathbf{b^1} \\ \mathbf{b^2} \\ \vdots \\ \mathbf{b^d} \end{bmatrix}, \quad (1)$$

where

$$\mathbf{y^l} = \begin{pmatrix} y_0^l \\ y_1^l \\ \vdots \\ y_N^l \end{pmatrix}, \quad \mathbf{b^l} = \begin{pmatrix} b_0^l \\ b_1^l \\ \vdots \\ b_N^l \end{pmatrix}, \mathbf{h^l} = \begin{pmatrix} h_0^l \\ h_1^l \\ \vdots \\ h_p^l \end{pmatrix}.$$

where $l = 1, \ldots, d$ is the index of the different channels or traces. The $\mathbf{h}^l$ is the wavelet finite impulse response of length $p$ in the channel $l$. $\mathbf{w^1} = (w_n^l)_{n=0,N}$ stands for additive measurement error and noise in the channel $k$, $N$ is the trace length. The $(\mathbf{w^l})_{l=1,\ldots,d}$ are mutually independent and have $\mathcal{N}(0, \sigma_{w^l}^2)_{l=1,\ldots,d}$ Gaussian distributions with mean zero and an unknown variance $\sigma_{w^l}^2$.

The reflectivity sequence $\mathbf{r} = (r_n)_{n=0,\ldots,N}$ assumed to be white with $r_n$ distributed according to a mixture of Bernoulli and Gaussian distribution [6]. This distribution is characterized by an underlying state model $\mathbf{q} = (q_n)_{n=0,N}$, with $q_n = 1$ at high reflectivity points and $q_n = 0$ for low reflectivity points. The corresponding reflectivity $r_n$ is distributed according to a gaussian zero mean distribution depending on the value of $q_n$, with variance $\sigma_1^2$ or $\sigma_0^2$ respectively, with $\sigma_1^2 \gg \sigma_0^2$.

Equation (1) can be written

$$\mathbf{y} = \mathbf{h} \star \mathbf{r} + \mathbf{b}, \quad (2)$$

where

$$\begin{aligned} \mathbf{y} &= [\mathbf{y^1}^T, \mathbf{y^2}^T, \ldots, \mathbf{y^d}^T]^T, \\ \mathbf{h} &= [\mathbf{h^1}^T, \mathbf{h^2}^T, \ldots, \mathbf{h^d}^T]^T, \\ \mathbf{b} &= [\mathbf{b^1}^T, \mathbf{b^2}^T, \ldots, \mathbf{b^d}^T]^T. \end{aligned}$$

## III. Parameter Estimation

We address the blind deconvolution problem through the classical Maximum Likelihood criterion [3] which leads to calculate

$$\hat{\theta}_{\mathbf{MV}} = \arg\max lnp(\mathbf{y}|\theta), \quad (3)$$

where $\theta$ is the parameter vector of interest. Here,

$$\theta = (\mathbf{h^1}, \ldots, \mathbf{h^d}, \lambda, \sigma_0^2, \sigma_1^2, (\sigma_w^1)^2, \ldots, (\sigma_w^d)^2). \quad (4)$$

In fact, we are faced to an incomplete data problem [3][5], where the complete data are given by $\mathbf{z} = (z_n)_{n=0,N}$, with $z_n = (q_n, r_n)$. The joint probability density is expressed by:

$$p(\mathbf{y}, \mathbf{z}|\theta) = p(\mathbf{y}|\mathbf{z}, \theta)p(\mathbf{z}|\theta). \quad (5)$$

As $\mathbf{z} = (\mathbf{q}, \mathbf{r})$ , it can also be written:

$$p(\mathbf{y}, \mathbf{z}|\theta) = p(\mathbf{y}|\mathbf{z}, \theta)p(\mathbf{r}|\mathbf{q}, \theta)p(\mathbf{q}|\theta). \quad (6)$$

Each component of this equations can be easily expressed. As $\mathbf{q}$ is a vector of independent Bernoulli variables,

$$p(\mathbf{q}) \quad = \prod_{k=1}^{N} p(q_k) \quad = \prod_{k=1}^{N} \lambda^{q_k}(1-\lambda)^{1-q_k}. \quad (7)$$

The variable $r_k$ are independent, conditional to the variables $q_k$. Thus,

$$\begin{aligned} p(\mathbf{r}|\mathbf{q}, \theta) &= \textstyle\prod_{k=1}^{N} p(r_k|q_k, \theta) \\ &= \prod_{k=1}^{N} \left[ \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp(\frac{-r_k^2}{2\sigma_1^2}) \right]^{q_k} \\ &\times \left[ \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp(\frac{-r_k^2}{2\sigma_0^2}) \right]^{1-q_k}. \end{aligned} \quad (8)$$

Then, the complete log-likelihood $L(\mathbf{y}, \mathbf{z}|\theta)$ can be derived. Its expression is given in the appendix.

When the complete data vector $\mathbf{z}$ is known the derivation of $\hat{\theta}_{\mathbf{MV}}$ is straightforward. In practice, the complete data can be simulated using a Gibbs sampler [5]. It consists in a random drawing from the probability distribution $p(z_k|\mathbf{y}, \mathbf{z}_{-k})$, where $\mathbf{z}_{-k}$ represents the vector $\mathbf{k}$ except its $k^{th}$ entry $z_k$. In this case, generalizing [4] it can be shown, that the posterior probability becomes

$$\begin{aligned} p(z_k|\mathbf{y}, \mathbf{z_{-k}}) &= [p(z_k = (1, r_k)|\mathbf{y}, \mathbf{z_{-k}}]^{q_k} \\ &\quad \times [p(z_k = (0, r_k)|\mathbf{y}, \mathbf{z_{-k}}]^{1-q_k} \\ &= [P(q_k = 1|\mathbf{y}, \mathbf{z_{-k}}).p(r_k|\mathbf{y}, q_k = 1)]^{q_k} \\ &\quad \times [P(q_k = 0|\mathbf{y}, \mathbf{z_{-k}}).p(r_k|\mathbf{y}, q_k = 0)]^{1-q_k} \\ &= \left[ \frac{d_k}{\sqrt{2\pi V_1}} \exp \frac{-(r_k - m_1)^2}{2V_1} \right]^{q_k} \\ &\quad \times \left[ \frac{1-d_k}{\sqrt{2\pi V_0}} \exp \frac{-(r_k - m_0)^2}{2V_0} \right]^{1-q_k}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} V_u &= \left( \tfrac{1}{\sigma_u^2} + \textstyle\sum_{j=1}^{d} (\sigma_w^j)^{-2} \displaystyle\sum_{n=0}^{p} (h_n^j)^2 \right)^{-1} \\ m_u &= V_u \sum_{j=1}^{d} (\sigma_w^j)^{-2} \sum_{n=0}^{p} (U_{k,n}^j h_n^j). \end{aligned} \quad (10)$$

where $d_k = P(q_k = 1|\mathbf{y}, \mathbf{z_{-k}})$. In addition,

$$p(z_k|\mathbf{y}, \mathbf{z_{-k}}) \propto p(\mathbf{y}|\mathbf{z}).p(r_k|q_k).P(q_k), \tag{11}$$

that is, for $u = 0, 1$,

$$p(z_k = (q_k = u, r_k)|\mathbf{y}, \mathbf{z_{-k}}) = \frac{\lambda^u.(1-\lambda)^{1-u}}{(2\pi)^{\frac{d+1}{2}}\sigma_u \left(\prod_{j=1}^d(\sigma_w^j)\right)} \times \exp\left[-\frac{r_k^2}{2\sigma_u^2} - \sum_{l=1}^d \frac{\|\mathbf{y^l} - \mathbf{H^l r}\|^2}{2(\sigma_w^l)^2}\right]. \tag{12}$$

From (9) and (12), it comes that (see appendix)

$$d_k = \left[1 + \frac{1-\lambda}{\lambda}\sqrt{\frac{V_0\sigma_1^2}{V_1\sigma_0^2}} \exp\left(\frac{m_0^2}{2V_0} - \frac{m_1^2}{2V_1}\right)\right]^{-1}. \tag{13}$$

For the simulation of $\mathbf{z}$, the Gibbs sampler is implemented as follows [1][8]:

For $i > 1$ and $k = 1, \ldots, N$,

- Choice of k.
- Compute $p(q_k = 1|\mathbf{y}, \mathbf{z}_{-k})$.
- draw $u$ randomly in $[0, 1]$ and simulate $q_k$: $q(k) = 1_{[0, p(q_k=1|\mathbf{y}, \mathbf{z}_{-k})]}(u)$
- simulate $r_k$ according to a $\mathcal{N}(m_q, \sigma_q^2)$ distribution
- update $z_k^i$: $z_k^i = (q_k, r_k)$

The SEM (Stochastic Expectation maximization) [9] is then used to solve the estimation problem:

1) Initialization: choose $\theta^{(0)}$ and $\mathbf{z}^{(0)}$

2) For $i = 1, \ldots, i_{\max}$
   - SE step: simulate $\mathbf{z^{(i)}} \sim p(\mathbf{z^{(i)}}|\mathbf{y}, \theta^{\mathbf{i-1}})$ with a Gibbs sampler.
   - M step: estimation of the parameters.

     For $l = 1, \ldots, d$,

     – $\hat{\mathbf{h}}^\mathbf{l} = ((\mathbf{R^l})^\mathbf{T}\mathbf{R^l})^{-1}(\mathbf{R^l})^\mathbf{T}\mathbf{y^l}$.

     – $\hat{\sigma}_w^{l2} = N^{-1} \parallel \mathbf{y^l} - \mathbf{R^l}\hat{\mathbf{h}}^\mathbf{l} \parallel_2^2$ .

     – $\hat{\lambda} = N^{-1}\mathbf{q^T q}$.

     – $\hat{\sigma}_0^2 = \dfrac{\mathbf{r^T}(I - \mathbf{Q})\mathbf{r}}{N - \mathbf{q^T q}}$.

     – $\hat{\sigma}_1^2 = \dfrac{\mathbf{r^T Q r}}{\mathbf{q^T q}}$.

## IV. Simulations and Practical Results

In this section, we illustrate on a simulation example the proposed multichannel blind deconvolution method.

We define the signal to noise ratio for channel $l$ by:

$$SNR^{(l)} = \frac{\lambda E_h^{(l)}\sigma_1^2}{\sigma_w^{2\,(l)}}, \tag{14}$$

where $l = 1, \ldots, d$ is the index of channel $l$ and $E_h^{(l)}$ is the source wavelet energy:

$$E_h^{(l)} = \frac{\sum_{i=1}^p (h_i^{(l)})^2}{p}. \tag{15}$$

$p$ is the wavelet length.

A synthetic reflectivity sequence is generated as a Gaussian mixture (Figure 2) with the following parameters ($\sigma_1^2 = 0.7,\ \sigma_0^2 = 0.00025,\ \lambda = 0.06$). We assume that d=4 channels are available. The first, third and fourth channel are obtained by filtering Ricker's wavelet by FIR filters whose magnitude frequency responses are respectively whown in Figure 8b, Figure 8c and Figure 8d.

This corresponds to wavelet differences between successive shots due to geometrical modifications of the channels. The second channel corresponds to a Ricker's wavelets (Figure 8a). These channels are convolved with the synthetic reflectivity sequence yielding four synthetic traces.

A white noise is added at each trace, whose signal to noise ratio is adjusted respectively to $SNR^{(1)} = 7$dB, $SNR^{(2)} = 12$dB, $SNR^{(3)} = 11$dB, $SNR^{(4)} = 9$dB.

A synthetic seismic trace is shown in Figure 3. The estimation of the different parameters are shown in Figures 4 to 7.

Figure 9 shows the impulse response of true and estimated wavelets for the four channels. Figure 10 shows the estimated reflectivity sequence.

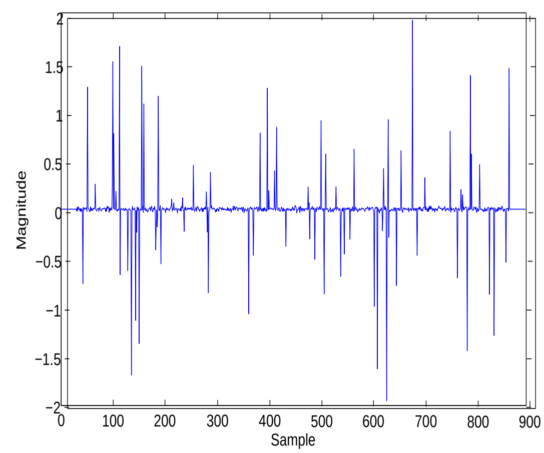


Fig. 2. Gaussian mixture reflectivity sequence.

## V. Conclusion

In this paper, we propose a new method for multichannel blind deconvolution using SEM algorithm.

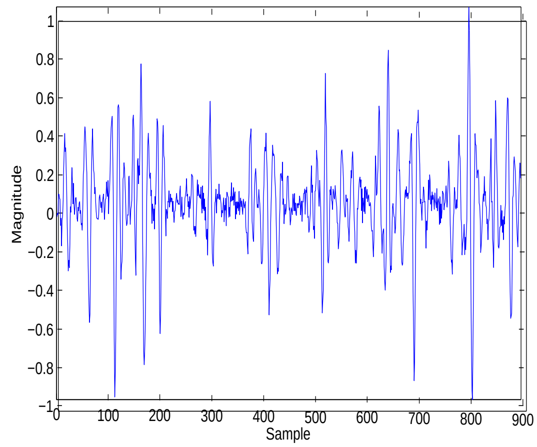


Fig. 3. Noisy seismic data (SNR=14 dB).

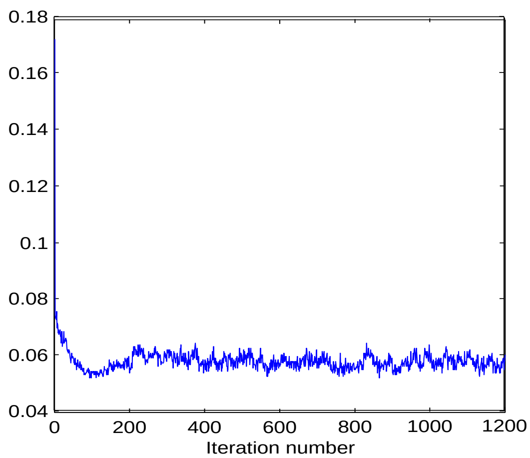


Fig. 4. Estimation of the parameter $\lambda$.

An application on simulated data shows the good behavior of the algorithm and the improvement obtained compared to single-channel deconvolution .

ACKNOWLEDGEMENT

The authors gratefully acknowledge IFREMER and Region Bretagne for supporting this work.

## APPENDIX

- Expression of $L(\mathbf{y},\mathbf{z}|\theta)$:

$$\begin{aligned} L(\mathbf{y},\mathbf{z}|\theta) = & -\sum_{l=1}^{d}\frac{(\mathbf{y^l}-\mathbf{H^l r})^T(\mathbf{y^l}-\mathbf{H^l r})}{2(\sigma_w^i)^2} \\ & -S_1(\mathbf{z})(2\sigma_1^2)^{-1} - S_0(\mathbf{z})(2\sigma_0^2)^{-1} \\ & -(N/2)\sum_{i=1}^{d} ln((\sigma_w^i)^2) \\ & +n_1(\mathbf{q})ln(\frac{\lambda}{\sigma_1}) + n_0(\mathbf{q})ln(\frac{(1-\lambda)}{\sigma_0}), \end{aligned} \quad (16)$$

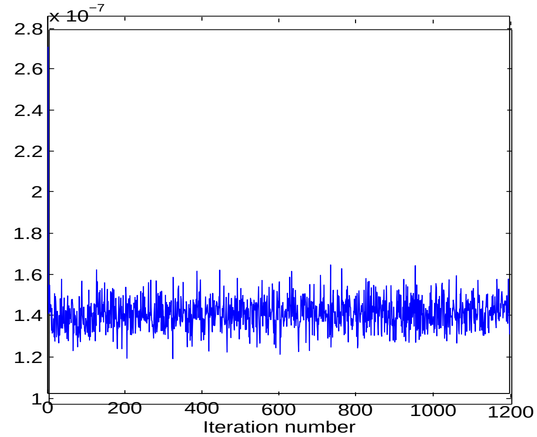


Fig. 5. Estimation of the parameter $\sigma_0^2$.

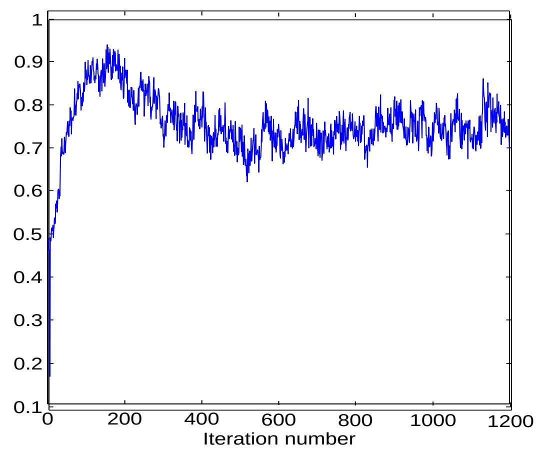


Fig. 6. Estimation of the parameter $\sigma_1^2$.

with

$$\begin{aligned} S_1(\mathbf{z}) &= \mathbf{r^T Q r}, \\ S_0(\mathbf{z}) &= \mathbf{r^T}(I-\mathbf{Q})\mathbf{r}, \\ \mathbf{Q} &= diag(\mathbf{q}), \\ n_1(\mathbf{q}) &= \mathbf{q}^T\mathbf{q}, \\ n_0(\mathbf{q}) &= N - n_1(\mathbf{q}), \\ \mathbf{H^l r} &= \mathbf{R h^l}. \end{aligned} \quad (17)$$

$\mathbf{H^l}$ and $\mathbf{R}$ are the convolution matrices associated with $\mathbf{h^l}$ and $\mathbf{r}$ respectively.

- Calculation of $d_k$(equation( 13))

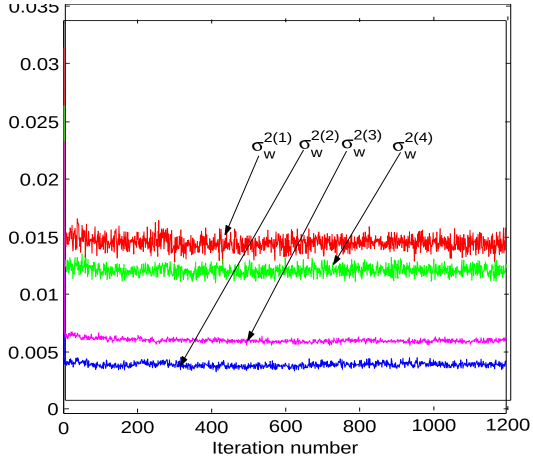


Fig. 7. Estimation of the parameter $\sigma_w^2$ for differents channels.

From equation (12),

$$p(z_k|\mathbf{y},\mathbf{z_{-k}}) = \left[\frac{\lambda}{(2\pi)^{\frac{d+1}{2}}\sigma_1\left(\prod_{j=1,d}(\sigma_w^j)\right)} \exp\left(-\frac{r_k^2}{2\sigma_1^2} - \sum_{l=1}^{d}\frac{\|\mathbf{y^l}-\mathbf{H^l r}\|^2}{2(\sigma_w^l)^2}\right)\right]^{q_k} \times\left[\frac{1-\lambda}{(2\pi)^{\frac{d+1}{2}}\sigma_0\left(\prod_{j=1,d}(\sigma_w^j)\right)} \exp\left(-\frac{r_k^2}{2\sigma_0^2} - \sum_{l=1}^{d}\frac{\|\mathbf{y^l}-\mathbf{H^l r}\|^2}{2(\sigma_w^l)^2}\right)\right]^{1-q_k}. \tag{18}$$

We set an equality between (9) and (18) to get the corresponding coefficients $m_0$, $m_1$, $V_0$, $V_1$ et $d_k$, Let's start rewriting $\|\mathbf{y^1}-\mathbf{H^1 r}\|^2$:

$$\begin{aligned} \|\mathbf{y^l}-\mathbf{H^l r}\|^2 &= \sum_{n=0}^{N}\left(y_{n+k}^l - \sum_{i=0}^{p} h_i^l r_{k+n-i}\right)^2. \qquad (19)\\ &= \sum_{n=0}^{p}\left(\mathcal{U}_{k,n}^l - h_n^l r_k\right)^2 + f^l(\mathbf{z_{-k}}), \end{aligned}$$

where

$$U_{k,n}^l = y_{n+k}^l - \sum_{i=0,i\neq n}^{p} h_i^l r_{k+n-i}. \tag{20}$$

$$f^l(\mathbf{z_{-k}}) = \sum_{n=p+1}^{N}\left(\mathcal{U}_{k,n}^l - h_n^l r_k\right)^2. \tag{21}$$

Next,

for $u = 0, 1$,

$$\begin{aligned} &p(z_k = (q_k = u, r_k)|\mathbf{y},\mathbf{z_{-k}}) \\ &\propto \frac{\lambda^u.(1-\lambda)^{1-u}}{\sigma_u} \times\exp\left(-\frac{r_k^2}{2\sigma_u^2} - \frac{1}{2}\sum_{j=1}^{d}\frac{\sum_{n=0}^{p}\left[U_{k,n}^j - h_n^j r_k\right]^2}{(\sigma_w^j)^{-2}}\right) \\ &\propto \frac{\lambda^u.(1-\lambda)^{1-u}}{\sigma_u} \times\exp\left(-\frac{r_k^2}{2}\left[\frac{1}{\sigma_u^2} + \sum_{j=1}^{d}(\sigma_w^j)^{-2}\sum_{n=0}^{p}(h_n^j)^2\right] - r_k\left[\sum_{j=1}^{d}(\sigma_w^j)^{-2}\sum_{n=0}^{p}U_{k,n}^j h_n^j\right]\right) \\ &\propto \left[\frac{\lambda^u.(1-\lambda)^{1-u}}{\sigma_u}\exp\left(\frac{m_u^2}{2V_u}\right)\right] \times\exp\left(\frac{-[r_k-m_u]^2}{2V_u}\right). \end{aligned} \tag{22}$$

From (9) and (22), we get the likelihood ratio for $z_k$ knowing $(\mathbf{y},\mathbf{z_{-k}})$:

$$\begin{aligned} \frac{p(z_k=(0,r_k)|\mathbf{y},\mathbf{z_{-k}})}{p(z_k=(1,r_k)|\mathbf{y},\mathbf{z_{-k}})} &= g(r_k)\frac{1-d_k}{d_k}\sqrt{\frac{V_1}{V_0}} \\ &= g(r_k)\frac{1-\lambda}{\lambda}\frac{\sigma_1}{\sigma_0}\exp\left[\frac{m_0^2}{2V_0} - \frac{m_1^2}{2V_1}\right]. \end{aligned} \tag{23}$$

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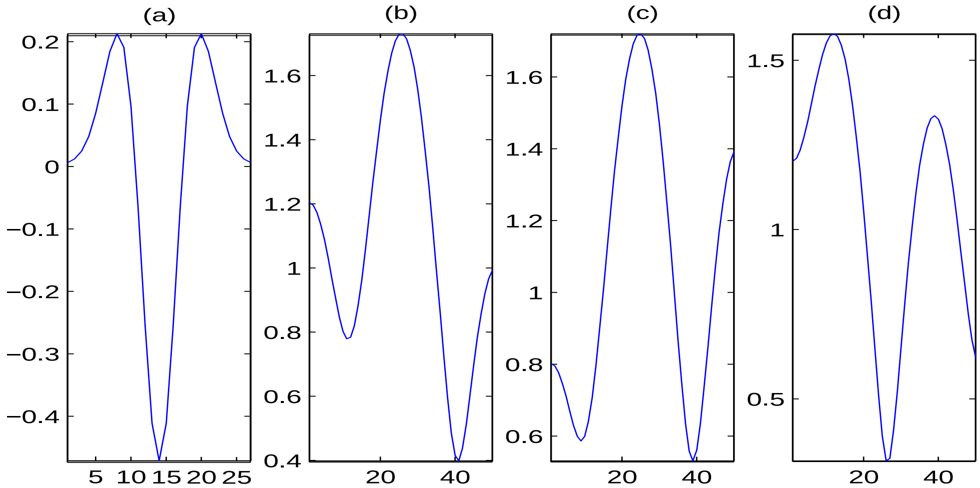


Fig. 8. Ricker's wavelet impulse response (a), magnitude frequency response applied to Ricker's wavelet: (b) for first channel, (c) for third channel, (d) for fourth channel .

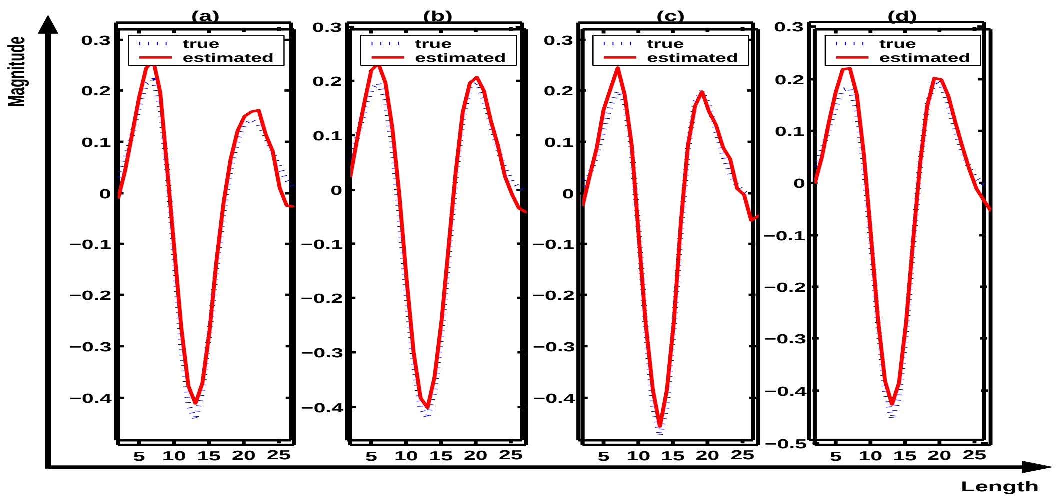


Fig. 9. True and estimated wavelets for differents channels.

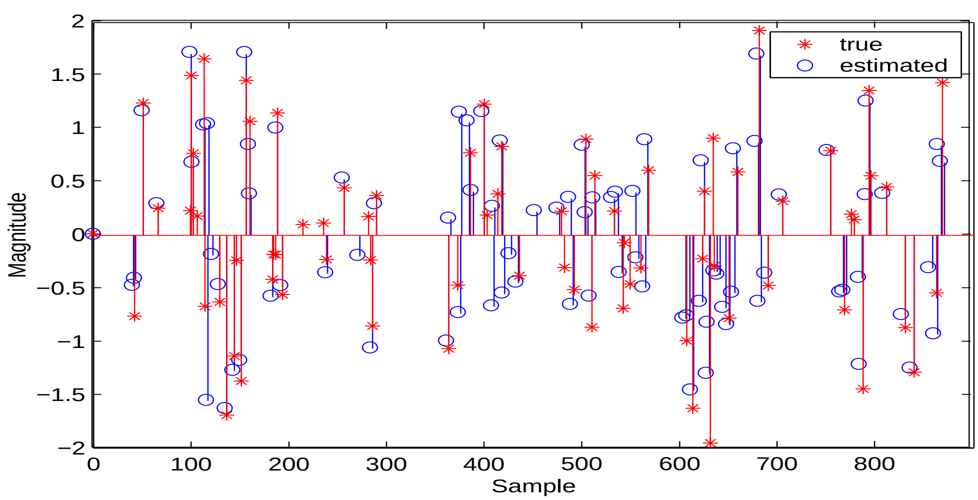


Fig. 10. True and estimated reflectivity sequence.