FEASIBILITY OF PASSIVE OCEANIC ACOUSTIC TOMOGRAPHY : A CRAMER RAO BOUNDS APPROACH

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Abstract : This paper completes the proof of the feasibility of passive Ocean Acoustic Tomography (OAT) by using the Cramer Rao Bounds (CRB) Tool. CRB are calculated on a shallow water environment modeled by an homogeneous layer over a semi infinite sediment layer. The results are that considering relative delays instead of absolute delays degrades the estimation of ocean parameters. Taking into account the spatial structure (angles of arrival) enables to improve performances of passive OAT and have similar performances as soon as accuracy of physical parameter estimations are conserved than active OAT for absolute delays only.

INTRODUCTION

Ocean Acoustic Tomography (OAT) was developed by Munk [1,2] to explore oceanic environment on a large scale and so estimating features like pressure, temperature, salinity or local current velocity directly from the acoustic pressure measurement and the resolution of an inverse problem like time inversion [1] or Matched Field Processing [3].

The ocean's structure is currently obtained by applying an active process emitting a load, repeated acoustic signal. Its features disagree with a warfare context involving discretion or respect of mammals' species life conditions. In this way, Passive Ocean Acoustic Tomography (POAT) is necessary to be developed. Some experiments tend to apply POAT using opportune acoustic sources naturally present in the ocean such as ship radiated noise, surface noise or animal noise [8, 4].

The aim of this paper is to complete the proof of the feasibility of Passive Ocean Acoustic Tomography using the Cramer Rao Bounds (CRB) tool which define a lower bound of the variance of any unbiased estimates of the ocean parameters. In this CRB study, times of arrival and angles of arrival are seen as measurement and acoustic channels properties are the parameters of interest to estimate from the measurement. The differences between active and passive tomography are that delays are absolute measurement in active tomography while they are relative in passive tomography. Angles of arrival are additional measurement describing the spatial structure of the ocean.

I. SIGNAL MODEL – OAT CONFIGURATION

We assume in this study an ocean modeled by an homogeneous horizontal layer for water at the top of an semi infinite sediment layer. All parameters of the environment are summarized on the Fig. 1.

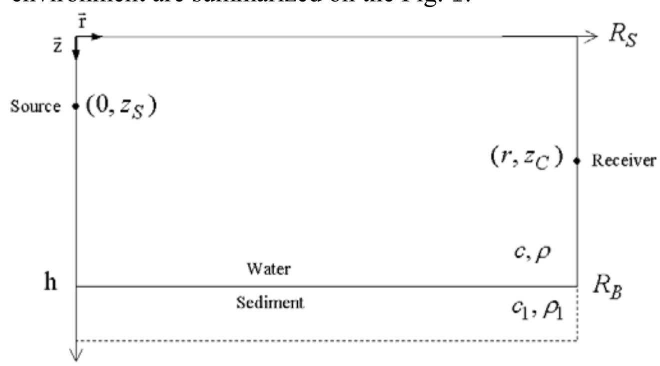

Fig 1: Ocean model

OAT based on ocean's structure inversion is composed of two steps : Firstly time and spatial structure of the ocean is estimated from the acoustic ocean pressure, then ocean parameters are estimated from the time spatial structure.

Ocean parameters could be sort in two subsets :

- the geometrical parameters
  - z_S = source depth,
  - z_C = receiver depth,
  - r = source – receiver horizontal range,
  - h = water depth.
- the physical parameters
  - c = water sound speed,
  - ρ = water density,
  - c_1 = sediment sound speed,
  - ρ_1 = sediment density,
  - R_S = surface reflection coefficient,
  - R_B = bottom reflection coefficient.

We consider modeling of the acoustic pressure by the ray theory. In this context, the receive signal is a sum of the attenuated, delayed replicas of the original signal :

$$p(r,z,t)=\sum_{k=0}^{\infty} a_k\, s(t-\tau_k)+n_k \qquad (1.1)$$

where for the k[th] ray :

a_k = attenuation function of R_S, R_B and R distance of propagation,

τ_k = time delay estimation,

n_k = white gaussian noise of zero mean and σ^2 variance.

For an harmonic source in the frequency domain, the acoustic pressure is expressed by :

$$P(r,z,f)=\frac{S_0}{4\pi}e^{i\varpi t}\sum_{k=0}^{\infty} a_k e^{ikc\tau_k}+N_k \qquad (1.2)$$

Assuming constant sound speed, rays could be estimated by the method of images [5]. In this context, 4 kinds of rays are possible :

- direct path $\Re_{n1}$,
- bottom reflected path $\Re_{n2}$,
- surface reflected path $\Re_{n3}$,
- bottom – surface reflected path $\Re_{n4}$.

Acoustic distances associated to these kinds of rays are function of the geometrical parameters. So :

$$\begin{aligned}
R_{n1} &= \sqrt{r^2+(z_C-z_S-2nh)^2}\\
R_{n2} &= \sqrt{r^2+(z_C+z_S-2(n+1)h)^2}\\
R_{n3} &= \sqrt{r^2+(z_C+z_S+2nh)^2}\\
R_{n4} &= \sqrt{r^2+(z_C-z_S+2(n+1)h)^2}
\end{aligned} \qquad (1.3)$$

where n is the index of the n[th] quadruplet of rays.

From this expression, time delays and angles of arrival are expressed :

$$\tau_{nk}=\frac{R_{nk}}{c} \qquad (1.4)$$

$$\beta_{nk}=\operatorname{atan}\left(\frac{\sqrt{R_{nk}^2-r^2}}{r}\right) \qquad (1.5)$$

Eventually, the expression of the acoustic pressure is :

$$p(r,z,f)\sim\frac{Sw}{4\pi}e^{i\varpi t}$$

$$\sum_{n=0}^{\infty}(-R_B)^n\left[\frac{e^{ikR_{n1}}}{R_{n1}}+R_B\frac{e^{ikR_{n2}}}{R_{n2}}-\frac{e^{ikR_{n3}}}{R_{n3}}-R_B\frac{e^{ikR_{n4}}}{R_{n4}}\right]$$

II. CRAMER RAO BOUNDS FORMULATION

We would calculate the Cramer Rao Bounds in the case of OAT to complete the proof of the feasibility of Passive Ocean Acoustic Tomography. The CRB are the lower bound of the variance of any unbiased estimates vector θ [6,7]. In the estimation theory, the CRB are the inverse of the Fisher Information Matrix I defined by :

$$CRB(\theta)=I^{-1}(\theta)$$

$$I_{ij}=-E\left\{\frac{\partial^2\ln(p(X,\theta))}{\partial\theta_i\partial\theta_j}\right\}=E\left\{\frac{\partial\ln(p(X,\theta))}{\partial\theta_i}\frac{\partial\ln(p(X,\theta))}{\partial\theta_j}\right\}$$

where $p(X,\theta)$ is the probability density function.

In the case of a deterministic signal in a white gaussian noise N of matrix covariance C, $X=S(\theta)+N$, the Fisher Information Matrix is expressed :

$$[I(\theta)]_{ij}=\left[\frac{\partial S(\theta)}{\partial\theta_i}\right]^T C^{-1}(\theta)\left[\frac{\partial S(\theta)}{\partial\theta_j}\right] \qquad (2.1)$$

For different situations, we consider time delays and angles of arrival as available information. From these measurements, the CRB are derived for the vector parameter θ representative of the geometrical parameters $[z_S, z_C, h, r]$. Covariance matrix on delays and angles of arrival are expressed on [Appendix A] while analytical expressions are summarized in [Appendix B].

The main objective is to study the condition number of the inverse problem and to predict performance estimation of passive OAT.

III. SIMULATED RESULTS

Different simulations have been considered to compare active and passive OAT in function of available information.

The performance calculated are the performance of the statistics of the geometrical parameters estimation knowing the time and spatial structures.

A. OAT from time delays measurements

Assuming physical parameters are known and the time structure is available, we first compare the performance of active and passive OAT (geometrical parameters estimation) based on delays information only.

We compare the active OAT and passive OAT in a shallow water channel defined by :

r = 200 m, h = 200 m, z_C = 100 m, z_S progresses from 0 to 200 m.

The measurement model for active process is defined by :

$$m(i)=\tau(i)+n(i) \qquad (3.1)$$

while the model for passive process is :

$$m(i)=\tau(i)-\tau_{direct\ path}+n(i) \qquad (3.2)$$

While acoustic pressure is composed of an infinite number of paths, simulations consider a finite number equal to p_{max} = 15 in a first time (practical number of rays really measured).

Noise $n(i)$ is a white gaussian noise of zero mean and σ^2 variance. The standard deviation is assumed to be 10 ms at 1 nautical mile [Appendix A].

From the CRB calculation, we determine the 2-norm condition number, the cross correlation and covariance over the geometrical parameters. Small magnitudes of the condition number and cross correlation terms warrant a good inversion while parameter's covariance indicates directly the expected performance of parameters' estimation. Results are summarized on Fig. 2 to the condition number and the sum cross-correlation (mean of cross-correlation terms) and Fig. 3 to geometrical parameter covariance.

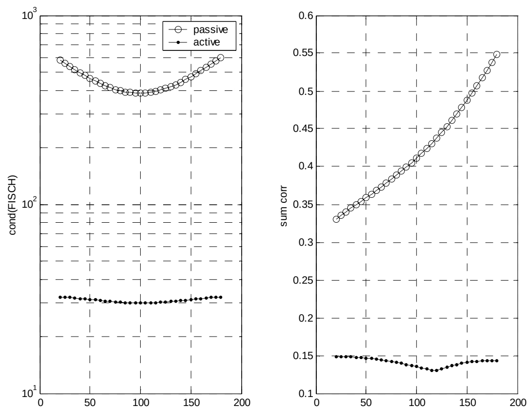

Fig 2 : Condition number and sum cross-correlation

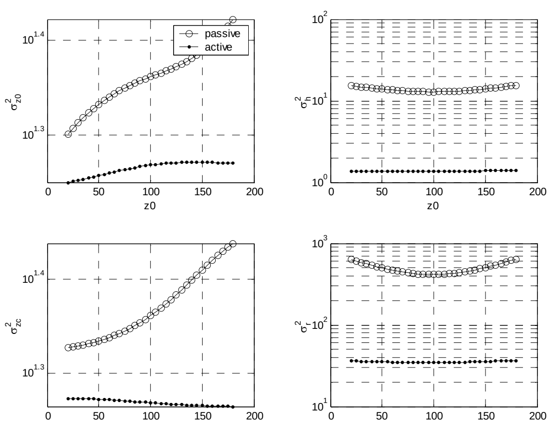

Fig 3 : Covariances of geometrical parameters

From figures 2 and 3, point line refers to active OAT and circle line refers to passive OAT.

The condition number have degraded by 10 from the transition between active and passive OAT. The variances of the parameters increase in the same time until a factor 10 (for r and h, less for z_C and z_S).

To improve the performance of passive OAT, we propose to add more information than time delay structure. In this aim, we fuse the spatial structure information to the time structure to better estimate the geometrical parameters.

B. OAT from time delays and angles of arrival

Assuming physical parameters are known and the time-spatial structure is available, we compare the performance of active OAT based on absolute delays and passive OAT based on relative delays and angles of arrival information.

The measurement model for active process is defined by :

$$\begin{aligned} m_1(i) &= \tau(i) + n_\tau(i) \\ m_2(i) &= \beta(i) + n_\beta(i) \end{aligned} \tag{3.3}$$

while the model for passive process is :

$$\begin{aligned} m_1(i) &= \tau(i) - \tau_{direct\ path} + n_\tau(i) \\ m_2(i) &= \beta(i) + n_\beta(i) \end{aligned} \tag{3.4}$$

We keep the same configuration of the ocean channel and stack delays and angle of arrival with the depth source at 10m (Typically ship source noise depth).

We consider the same number of rays and same standard deviation on delay measurement [Appendix A]. As information depends on the number of paths, we define a normalization process to fuse delays and angles.

$$\begin{aligned} I_{delays} &= I_{delays} / trace(I_{delays}) \\ I_{angles} &= I_{angles} / trace(I_{angles}) \end{aligned} \tag{3.5}$$

Normalization's objective is to distribute the same energy over all paths without dependence on the number of paths.

To compare the time or angle influence on the process of estimating parameters, we then calculate a weighted Fisher Information Matrix by :

$$I(\alpha) = \alpha.I_{delays} + (1-\alpha).I_{angles} \tag{3.6}$$

The parameter α is a weighted parameter interpreting confidence given to delay information of angle information.

We calculate the CRB function of the parameter α and present results on Fig. 4 and 5.

The figures 4 and 5 show results of experiments for active and passive OAT. Point line represents active OAT for time delays, circle line indicates passive OAT for time delays, plus line refers to active OAT for delays and angle while x mark line refers to passive OAT for delays and angles.

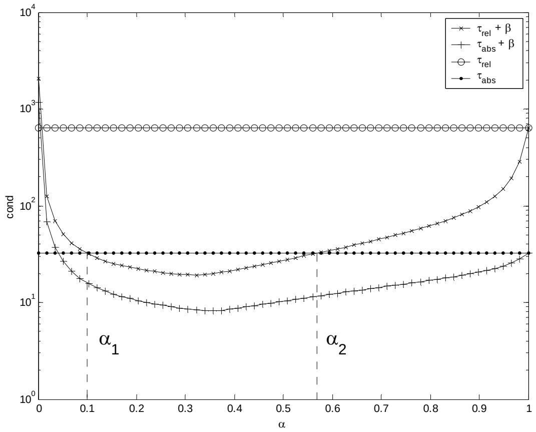

Fig 4 : Condition number

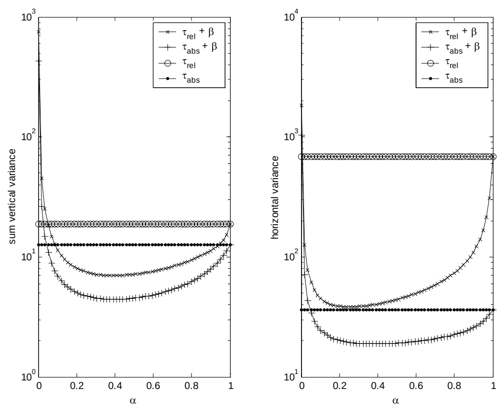

Fig 5 : Vertical and horizontal parameters variances

For any $\alpha \in [\alpha_1, \alpha_2]$, the condition number associated with relative delays and angles of arrival is lower than one associated with absolute delays only. In this condition, passive OAT based on relative delays and angles of arrival will give results with the same performance than active OAT from absolute delays. Added information from the angles of arrival improves the geometrical parameters inversion.

The effective improvement of performance is drawn in the left subplot of Fig. 5 that represent the mean of vertical covariance. For passive OAT, the addition of angles of arrival increases the performance of a 2.7 factor. The modification of active OAT from absolute delays to a new passive OAT from delays and angles of arrival increases the performance of a 1.8 factor.

C. Others experiments

Other experiments are applied with the same concept.
We apply the comparison of active and passive OAT to the global estimation of the geometrical and physical parameters. More information is added by taking into account the ray amplitude that are directly function of surface and bottom reflection coefficients. Conclusions are similar with the previous study. Addition amplitude and angles of arrival to relative delays improves the global performance of passive OAT.

In a similar way, CRB are derived from a constant sound speed channel modeled by the normal modes propagation. In a shallow water environment, the modal filtering is detected and so the available information is limited by the geometrical parameters. More complex calculus are developed by [6] under the assumption of a random emitted signal.

CONCLUSION

The Cramer Rao Bounds allow to determine the condition number and variances of the ocean's parameters of 2 different simulations. First of all, the comparison of active and passive OAT from time delays measurements affirms the superiority of active process. Developing passive OAT needs to take into account of the spatial structure to improve the performance of ocean's parameters estimation. In certain case, the passive OAT performance for delays and angles of arrival may be equivalent to active OAT performance for delays only.

Work's perspectives are the development of time delays and angles of arrival algorithms in a passive context that insure same performance than in active OAT process. These algorithms will be applied in particular to impulse noise source with time-frequency approach [8] and spatial time-frequency distributions [9].

APPENDIX

A. Rays perturbation

We suggest to classify the rays perturbations from two origins. On one hand, perturbations induced by the environment and on the other hand perturbation caused by data processing. The aim of OAT is to estimate the environmental variations, so we assume that we could design an algorithm such that data processing perturbations are small against environment perturbations.

Define the perturbation as a white gaussian noise of zero mean and variance σ^2, we could summarize next formulas :

Data processing variance :
Assuming a signal s(t) of a bandwidth B and a signal to noise ratio SNR,

$$\sigma_\tau = \frac{1}{B\sqrt{SNR}}$$

$\sigma_\beta = \dfrac{\lambda}{\pi(n-1)p\cos\theta\sqrt{SNR}}$ for a linear array of n sensors and p is the distance between two consecutive sensors.

Environmental variance :
From [10,11] the perturbation of time and spatial structure are caused by inhomogeneous area of sound speed connected to temperature variations. For internal waves, standard deviation is approximately of 2°C, so we could measure a 10m/s standard deviation for sound speed.

$$\frac{\delta c}{c} = 3.19\ 10^{-3}\delta T$$

We suppose the sound speed is $c(z) = c_0 + \delta c$. In this context, the linearization of time delays cause :

$$\tau(c) = \tau(c_0) + \frac{\partial\tau}{\partial c_0}\delta c \cdot$$

The standard deviation on time delay for an environment of a disturb sound speed c(z) is :

$$\delta\tau = \frac{\partial\tau}{\partial c_0}\delta c$$

From the modeled time delays, we calculate the standard deviation on time delay for a sound speed perturbation of 10m/s. The standard deviation on delays is assumed to be 10 ms at 1 nautical mile in all experiments.

In an equivalent manner, $\delta\beta = \dfrac{\partial\beta}{\partial c_0}\delta c \cdot$

Note : For a constant sound speed, the angles of arrival are independent of a sound speed variation. Then we consider the measurement of angles by a vertical linear array of 10 sensors spaced by 1 m and take into account the data processing variance.

B. Derivatives of delays and angles

Vector X contains the available information obtained by the measurement of the time and spatial structure. Active OAT and Passive OAT are differentiated by absolute and relative delays. We assume that absolute delays are the delays (1.4, 3.1) and relative delays are the difference between absolute delays and the direct path delay (3.2).

$\tau_{OAT} = \tau - \alpha\tau_{01}$, $\alpha = 0$ for active OAT and $\alpha = 1$ for passive OAT

Delay for path of the first kind $\Re_{n1}$

$X_{n1}(\theta) = S_{n1}(\theta) + b_{n1}$ for $n \in \{0, n\max\}$

$$X_{n1}(\theta) = \frac{\sqrt{r^2 + (z_C - z_S - 2nh)^2}}{c} - \alpha\frac{\sqrt{r^2 + (z_C - z_S)^2}}{c} + b_{n1}$$

The derivatives are :

$$\frac{\partial S_{n1}(\theta)}{\partial z_S} = -\frac{(z_C - z_S - 2nh)}{c}\frac{1}{R_{n1}} + \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n1}(\theta)}{\partial z_C} = \frac{(z_C - z_S - 2nh)}{c}\frac{1}{R_{n1}} - \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n1}(\theta)}{\partial h} = -2n\frac{(z_C - z_S - 2nh)}{c}\frac{1}{R_{n1}}$$

$$\frac{\partial S_{n1}(\theta)}{\partial r} = \frac{r}{c}\frac{1}{R_{n1}} - \alpha\frac{r}{c}\frac{1}{R_{01}}$$

Delay for path of the second kind $\Re_{n2}$

$X_{n2}(\theta) = S_{n2}(\theta) + b_{n2}$ for $n \in \{0, n\max\}$

$$X_{n2}(\theta) = \frac{\sqrt{r^2 + (z_C + z_S - 2(n+1)h)^2}}{c} - \alpha\frac{\sqrt{r^2 + (z_C - z_S)^2}}{c} + b_{n2}$$

The derivatives are :

$$\frac{\partial S_{n2}(\theta)}{\partial z_S} = \frac{(z_C + z_S - 2(n+1)h)}{c}\frac{1}{R_{n2}} + \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n2}(\theta)}{\partial z_C} = \frac{(z_C + z_S - 2(n+1)h)}{c}\frac{1}{R_{n2}} - \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n2}(\theta)}{\partial h} = -2(n+1)\frac{(z_C + z_S - 2(n+1)h)}{c}\frac{1}{R_{n2}}$$

$$\frac{\partial S_{n2}(\theta)}{\partial r} = \frac{r}{c}\frac{1}{R_{n2}} - \alpha\frac{r}{c}\frac{1}{R_{01}}$$

Delay for path of the third kind $\Re_{n3}$

$X_{n3}(\theta) = S_{n3}(\theta) + b_{n3}$ for $n \in \{0, n\max\}$

$$X_{n3}(\theta) = \frac{\sqrt{r^2 + (z_C + z_S + 2nh)^2}}{c} - \alpha\frac{\sqrt{r^2 + (z_C - z_S)^2}}{c} + b_{n3}$$

The derivatives are :

$$\frac{\partial S_{n3}(\theta)}{\partial z_S} = \frac{(z_C + z_S + 2nh)}{c}\frac{1}{R_{n3}} + \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n3}(\theta)}{\partial z_C} = \frac{(z_C + z_S + 2nh)}{c}\frac{1}{R_{n3}} - \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n3}(\theta)}{\partial h} = 2n\frac{(z_C + z_S + 2nh)}{c}\frac{1}{R_{n3}}$$

$$\frac{\partial S_{n3}(\theta)}{\partial r} = \frac{r}{c}\frac{1}{R_{n3}} - \alpha\frac{r}{c}\frac{1}{R_{01}}$$

Delay for path of the fourth kind $\Re_{n4}$

$X_{n4}(\theta) = S_{n4}(\theta) + b_{n4}$ for $n \in \{0, n\max\}$

$$X_{n4}(\theta) = \frac{\sqrt{r^2 + (z_C - z_S + 2(n+1)h)^2}}{c} - \alpha\frac{\sqrt{r^2 + (z_C - z_S)^2}}{c} + b_{n4}$$

The derivatives are :

$$\frac{\partial S_{n4}(\theta)}{\partial z_S} = -\frac{(z_C - z_S + 2(n+1)h)}{c}\frac{1}{R_{n4}} + \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n4}(\theta)}{\partial z_C} = \frac{(z_C - z_S + 2(n+1)h)}{c}\frac{1}{R_{n4}} - \alpha\frac{(z_C - z_S)}{c}\frac{1}{R_{01}}$$

$$\frac{\partial S_{n4}(\theta)}{\partial h} = 2(n+1)\frac{(z_C - z_S + 2(n+1)h)}{c}\frac{1}{R_{n4}}$$

$$\frac{\partial S_{n4}(\theta)}{\partial r} = \frac{r}{c}\frac{1}{R_{n4}} - \alpha\frac{r}{c}\frac{1}{R_{01}}$$

Angle for path of the first kind

$$X_{\beta n1}(\theta) = S_{\beta n1}(\theta) + b_{\beta n1} \text{ for } n \in \{0, n\max\}$$

$$X_{\beta n1}(\theta) = \text{atan}\left(\frac{z_C - z_S - 2nh}{r}\right) + b_{\beta n1}$$

The derivatives are :

$$\frac{\partial S_{\beta n1}(\theta)}{\partial z_S} = -\frac{1}{r}\frac{1}{1+\left(\frac{z_C - z_S - 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n1}(\theta)}{\partial z_C} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C - z_S - 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n1}(\theta)}{\partial h} = -\frac{2n}{r}\frac{1}{1+\left(\frac{z_C - z_S - 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n1}(\theta)}{\partial r} = -\frac{(z_C - z_S - 2nh)}{r^2}\frac{1}{1+\left(\frac{z_C - z_S - 2nh}{r}\right)^2}$$

Angle for path of the second kind

$$X_{\beta n2}(\theta) = S_{\beta n2}(\theta) + b_{\beta n2} \text{ for } n \in \{0, n\max\}$$

$$X_{\beta n2}(\theta) = \text{atan}\left(\frac{z_C + z_S - 2(n+1)h}{r}\right) + b_{\beta n2}$$

The derivatives are :

$$\frac{\partial S_{\beta n2}(\theta)}{\partial z_S} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C + z_S - 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n2}(\theta)}{\partial z_C} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C + z_S - 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n2}(\theta)}{\partial h} = -\frac{2(n+1)}{r}\frac{1}{1+\left(\frac{z_C + z_S - 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n2}(\theta)}{\partial r} = -\frac{(z_C + z_S - 2(n+1)h)}{r^2}\frac{1}{1+\left(\frac{z_C + z_S - 2(n+1)h}{r}\right)^2}$$

Angle for path of the third kind

$$X_{\beta n3}(\theta) = S_{\beta n3}(\theta) + b_{\beta n3} \text{ for } n \in \{0, n\max\}$$

$$X_{\beta n3}(\theta) = \text{atan}\left(\frac{z_C + z_S + 2nh}{r}\right) + b_{\beta n3}$$

The derivatives are :

$$\frac{\partial S_{\beta n3}(\theta)}{\partial z_S} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C + z_S + 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n3}(\theta)}{\partial z_C} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C + z_S + 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n3}(\theta)}{\partial h} = \frac{2n}{r}\frac{1}{1+\left(\frac{z_C + z_S + 2nh}{r}\right)^2}$$

$$\frac{\partial S_{\beta n3}(\theta)}{\partial r} = -\frac{(z_C + z_S + 2nh)}{r^2}\frac{1}{1+\left(\frac{z_C + z_S + 2nh}{r}\right)^2}$$

Angle for path of the fourth kind

$$X_{\beta n4}(\theta) = S_{\beta n4}(\theta) + b_{\beta n4} \text{ for } n \in \{0, n\max\}$$

$$X_{\beta n4}(\theta) = \text{atan}\left(\frac{z_C - z_S + 2(n+1)h}{r}\right) + b_{\beta n4}$$

The derivatives are :

$$\frac{\partial S_{\beta n4}(\theta)}{\partial z_S} = -\frac{1}{r}\frac{1}{1+\left(\frac{z_C - z_S + 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n4}(\theta)}{\partial z_C} = \frac{1}{r}\frac{1}{1+\left(\frac{z_C - z_S + 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n4}(\theta)}{\partial h} = \frac{2(n+1)}{r}\frac{1}{1+\left(\frac{z_C - z_S + 2(n+1)h}{r}\right)^2}$$

$$\frac{\partial S_{\beta n4}(\theta)}{\partial r} = -\frac{(z_C - z_S + 2(n+1)h)}{r^2}\frac{1}{1+\left(\frac{z_C - z_S + 2(n+1)h}{r}\right)^2}$$

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