Analysis of signals spectrum in the shallow sea

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Abstract - **In the shallow waters any acoustic field is the super-position of running and standing waves. The report under consideration offered some results of stringent solution of Helmholz's equation taking into account actual parameters of bottom ground, marine water and surface. This algorithm admits to calculate dispersion of pressure, oscillation velocity and stream of energy in every point of a closed space including the space near surface. The results of mathematical exploration of array's field that arose from 20 elements in closed volume, distribution of the group velocity, energy transfer inside marine bay are displayed. These results of digital research are compared with the experimental testing performed at the Peter the Great Bay (Japan Sea).**

Key words: Helmholz's equation, oscillation velocity, array's field, bottom acoustic parameters, sea bed parameters, running waves, standing wave.

I. Introduction

The elastic waves are propagating in shallow sea in compliance with rules of linear acoustics.

Therefore the physical and mathematical models expounding laws of waves propagation could result in Helmholtz's equation solution. The "wave numbers " should be altered in the system of Helmholtz equations depending on the form of signal. In present work we shall consider narrow band signals, where it is possible to apply one Helmholtz equation in which the wave number is adequate to carrier frequency. The length of emitted pulse can be various. For such simple mathematical model the character of propagation of sonic or acoustic field is determined by acoustic parameters of marine medium of bottom crust and the sea surface. The covering surface of bottom and interface greatly influences the resultant acoustic field.

The significant influence of antenna orientation concerning seabed was remarked in acquisition of signal parameters.

The experimental explorations conducted in the 70s [1] exhibited a row of anomalous phenomena that acoustic signals underwent when having been beamed into the ocean or sea bottom. The anomaly of such experimental events lied in the magnitude of wave permeated into the bottom, namely, the magnitude appreciably was higher than the level of theoretical calculation.

Beyond this, the reducing magnitude of wave group velocity was observed in seabed sediment compared to the calculated one.

It is proved in the present report that these anomalous results are explicable within the framework of accurate theory developed by the authors [2].

Besides explanation of anomalous phenomena in bottom crust the stringent accurate theory assumes to study in details the amplitudes and phases of pressure and oscillation velocity inside closed sea basin volume. Sometimes this closed volume would be named as "marine lens" or simply "lens".

However, obtained system (description) of acoustic field: pressure, pressure gradient oscillatory velocity of medium particles, group velocity of propagation etc., allows to investigate thoroughly and predict new phenomena.

For example, the intromission level of acoustic field penetrated into the seabed or ocean crust under different angles of incidence of spherical waves might be quantified. It was noted that under angles of wave incidence being more than critical, about 30% - 40%, from the incident stream of power (wave intensity) percolated into the marine bottom. The new theoretical effects of group velocity dependence on coordinates of seabed halfspace, the waveguide nature of wave propagation in seabed halfspace and some other new effects associated with wave transmission were discovered at first that should be interesting for further exploration.

These new effects could be used in practical sphere of undersea acoustics.

II. Theory

The acoustic, sonic fields and field of elastic wave in the waveguides and closed marine volumes are being determined by acoustic parameters of the marine water basin geometrical shape of frontiers, density and elasticity of the waveguide surrounding stuff (air, bottom). Research of the field of sonic pressure displays approximate state of real conditions inside of waveguide or closed volume. Actually description of acoustic field is possible to implement using simultaneously measurements of pressure, oscillation velocity and phase of elastic wave.

Simultaneous measurement of pressure in one point of space, as well instrumentation of oscillation velocity and phase between them would be noted as " vector – phase measurements". In real marine conditions it is very much complicated to measure precisely the actual acoustic parameters of marine water, bottom, frontiers of media acoustic separation in a waveguide and in any closed space.

Therefore, an error of measurement in marine medium equals to more than 20% on pressure and 40% on power capacity. Under such big value of error to compare the experimental and theoretical results could be reliable only by their properties.

Theoretical computation of acoustic field should be fulfilled using mathematical models which are in closest propinquity to the actual conditions.

To our supposition the marine medium might be imagined as endless flat waveguide only theoretically, because actually it is not a waveguide, but closed volume restricted by bottom, shoreline, frontier between air and water.

Therefore in presented work the attempt to suppose marine medium as closed volume being an ellipsoid of rotation is considered. Meanwhile the theory expounded in [1,2] is used to admit calculation of harmonic source field that is situated in the closed volume under consideration.

Basing on this theory the propagation of arbitrary form periodical signal in marine medium is researched. Let ponder on harmonic source that works at some frequency “f”.

The physical model of task is combination of point sources with distribution density “q” that are located in the area restricted by surface “s” generating some potential “Φ” distribution on surface S. The sources are placed in the closed volume confined by surface S (Fig.1).

The mathematical model of task originated in Helmholtz’s equation (1), which solution would be in accordance with congruous conditions along the frontier S (1a):

$$\Delta\Phi + k^{2}\cdot\Phi = -4\cdot\pi\cdot q_n(\overline{r_{0n}}), \quad (1)$$

$$\left.\begin{aligned} \rho_1\cdot\Phi_1\big|_S &= \rho_2\cdot\Phi_2\big|_S \\ \frac{\partial\Phi_1}{\partial n}\Big|_S &= \frac{\partial\Phi_2}{\partial n}\Big|_S \end{aligned}\right\}, \quad (1a)$$

where: $q_n(\overline{r_{0n}})$ – density of sources distribution,

Φ – potential of oscillation velocity,

$\boldsymbol{n}$ – normal to S boundary of separation,

κ – wave number.

The task of analysis for considered closed volume could be formulated as finding of the field generated by researched combination of the sources in observation point. In this case the field of potential is identified by the next equation:

$$\Phi(\bar{r}) = \sum_n^N q_n(\bar{r}_{0n})\cdot G^{(+)}(\bar{r}_{0n},\bar{r}), \quad (2)$$

where: $G^{(+)}(\overline{r_{0n}},\bar{r})$– Green's function, i. e. a field of point sources located at the considered in medium place that was indicated by radius – vector r_0 (sign «+» points to application of divergent waves in Green's function),

$\bar{r}$ – indicates point of observation,

N – 17 elements.

Green’s function of point source placed inside of closed volume in accordance with [1] should be computed in this way:

$$\left.\begin{aligned} G_1(MM_{on}) &= \frac{e^{i\cdot k_1\cdot R_{on}}}{R_{on}} + K_{refl\_n}\cdot\frac{e^{-i\cdot k_1\cdot R_{on}}}{R_{on}} \quad - \quad inside \ \ volume \ \ W \\ G_2(MM_{on}) &= \frac{e^{i\cdot k_1\cdot R_{prn}}}{R_{prn}}\cdot K_{perc\_n} \quad - \quad outside \ \ volume \ \ W \end{aligned}\right\}, \quad (3)$$

where: K_{refl} – function depending only on angle coordinates (it means a coefficient of reflection of waves from separation frontier)

K_{perc} – function depending on angle coordinates either (it means a coefficient of percolation of waves into the second halfspace)

R_0 – distance between points of beaming and reception

R_{pr} – distance between some imaginable source and point of reception (detailed calculation of coordinates of imaginable source location is expounded in [2].

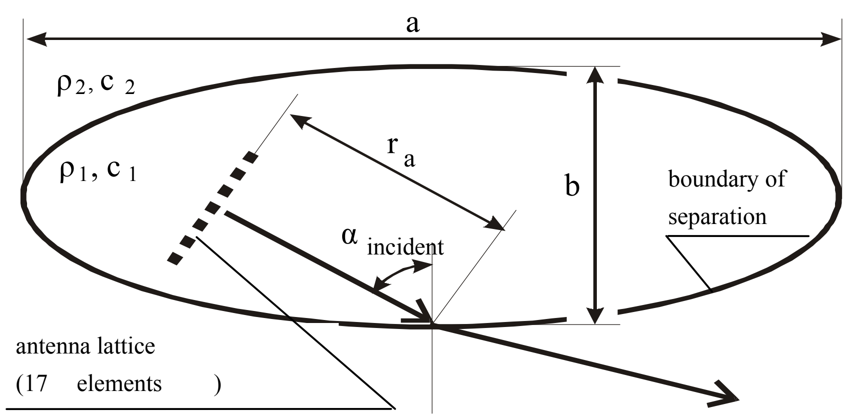


Fig.1 Geometrical scheme of task. Placement of separation surface of two media and sources forming “beam” incident onto frontier

Correspondingly [2] the K coefficients and K are found in this way:

$$K_{\text{perc\_n.}} = \frac{\rho_1 \left[h_0^{(1)}(k_1 R_{0n}\big|_S) + K_{\text{отр.}} h_0^{(2)}(k_1 R_{0n}\big|_S) \right]}{\rho_2 h_0^{(1)}(k_2 R_{npn}\big|_S)},$$

$$K_{refl\_n} = \frac{\begin{pmatrix} \rho_1 c_1 \dfrac{\partial R_{prn}}{\partial n} h_0^{(1)}(k_1 R_{0n}\big|_S) \cdot h_1^{(1)}(k_2 R_{prn}\big|_S) - \\ - \rho_2 c_2 \dfrac{\partial R_n}{\partial n} h_1^{(1)}(k_1 R_{0n}\big|_S) \cdot h_0^{(1)}(k_2 R_{prn}\big|_S) \end{pmatrix}}{\begin{pmatrix} \rho_1 c_1 \dfrac{\partial R_{prn}}{\partial n} h_0^{(2)}(k_1 R_{0n}\big|_S) \cdot h_1^{(1)}(k_2 R_{prn}\big|_S) + \\ + \rho_2 c_2 \dfrac{\partial R_n}{\partial n} h_1^{(2)}(k_1 R_{0n}\big|_S) \cdot h_0^{(1)}(k_2 R_{prn}\big|_S) \end{pmatrix}}, \quad (4)$$

here $$h_0^{(1)}(k_1 R_{0n}) = \frac{e^{ik_1 R_{0n}}}{R_{0n}},$$

$$h_0^{(2)}(k_1 R_{0n}) = \frac{e^{ik_1 R_{0n}}}{R_{0n}}.$$

Oscillation velocity in the arbitrary point of space would be computed accordingly such equation:

$$\overline{V}(\bar{r},t) = -grad\Phi_n(\bar{r},t).$$

Pressure in the arbitrary point would be calculated in such way:

$$P(\bar{r}) = i\omega\rho\Phi(\bar{r}).$$

Group velocity of energy propagation (transfer of energy) in the space should be determined from the formula:

$$C_{gr} = \frac{I}{\langle\omega\rangle} \quad (5),$$

where:

$I = \frac{1}{T}\int_0^T P(\bar{r}) \cdot V(\bar{r})dt$ - intensity of power stream (average magnitude for period $T = \frac{2\pi}{\omega}$, magnitude of power stream).

$\omega = \frac{1}{2}(\rho|V|^2 + \frac{P^2}{\rho c^2})$ - full density of energy.

Fig. 2-3 represents some results of numerical reckoning of the group velocity inside W volume and outside the volume. Fig. 4-8 presented results of the spreading pressure, oscillatory velocity and intensities inside and outside of the volume.

III. Experiment

Short – base system consisted of four hydrophones situated in the angles of horizontal tetragon. Each side of this regular tetragon was equal to 0.6 m and oriented at the angles of 0^0 and 90^0 towards eradiation source. The system was placed near the shore – line at the depth of 28 m on bottom. One autonomous radiator was located on the bottom in 15 km from another that was positioned on the shelf selvage at 76 m depth. The phase – manipulated signal was being emitted to have 511 constituent symbols and the symbol length equalled to five periods of carrier frequency of 260 Hz. Ten emission sets were generated with the period of replica repetition being equalled to the sequence length.

The result of processing of amplitude information for given experiment is shown in figure 1 as a sequence of correlative responses of all emission sets with computation concerningly maximal magnitude. Main maxima on the picture are adequate to acoustic signal arrivals of water wave with average velocity of about 1500 m /s as well, there was observed a forerunner with essentially lower amplitude having velocity of 1800 m/s that evidently resulted from ground wave, produced by expansion of mighty signal (around 7kPa/m) from wide section of the waveguide into the narrow one. Figures 9-13 show the same sequence in the scale that is magnified on amplitude and in time.

As such type of theoretical model the wave equation or the equation of Helmholtz might be applied. Stringent solution of these equations results in magnitudes being mostly close to real characteristics of fields in the sea.

In the theoretical model of a closed restricted volume the most complicated problem is to identify rigorously geometrical shape of separation frontier of water and surrounding media (bottom and surface of the sea).

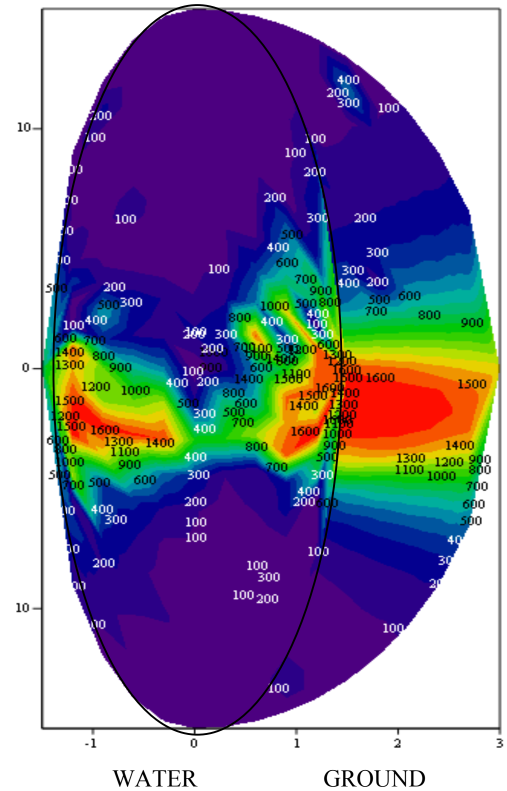
The present work suggests to imagine the real restricted sea area as an ellipsoid with non – homogeneous frontiers. Phase velocities and media density are assigned on the frontier of separation.

IV. Conclusion

Stringent solution of Helmholtz equation as Green's function provided acquisition of complete representation of acoustic field in every point of restricted volume in the sea ("sea lens") and marine bottom.

Pressure, pressure gradient in some small area allowed to calculate the direction of moving particles of water medium. Provided by the authers of presented research the experimental tests in marine restricted basin (sea bay) presumed and confirmed the theoretical calculation of pressure and pressure gradient.

Experimentally tested data of energy spread velocity in the bottom halfspace (near interface) confirmed information obtained by American scientists in the same exploration [1]. Also, the results of these experiments state firmly the identity of the data and applicability of theoretical model under consideration in presented report to ponder on "model-to-data" comparison reliability.



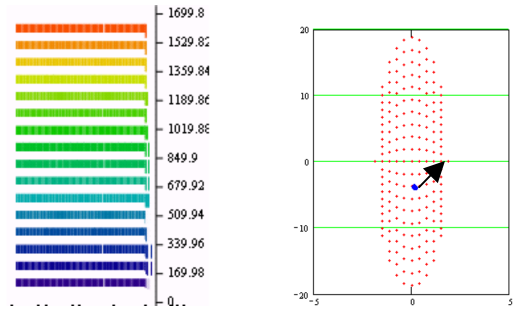
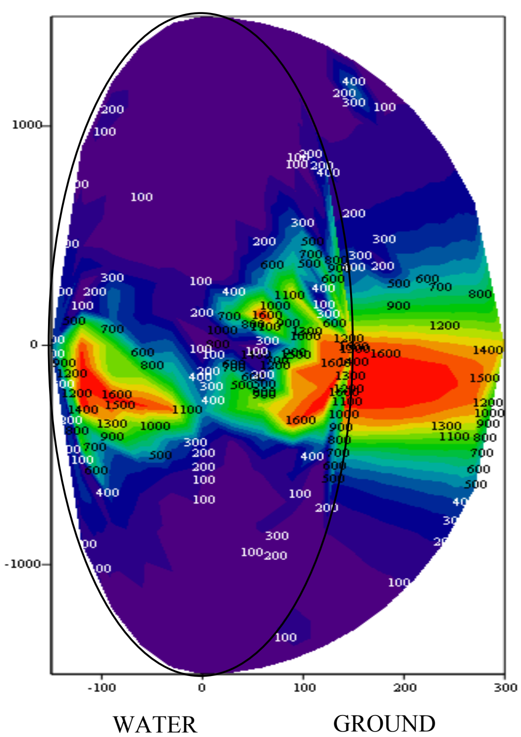


Fig. 2. Graph of the distribution of group velocity inside lens of elliptical shape filled up with water having parameters $\rho = 1000$ kg/m^3, C = 1500m/s.
The outside of lens is ground with parameters $\rho = 2000$ kg/m^3, C = 1700 m/s.
Angle of the acoustic beam dropping onto the boundary of separation $\alpha = 67$ degrees. Maximum value of group velocity in the lens equals:
C_{group} (max) = 1590 m/s, minimum value is C_{group} (min) = 0.4 m/s. Frequency 100000 Hz.



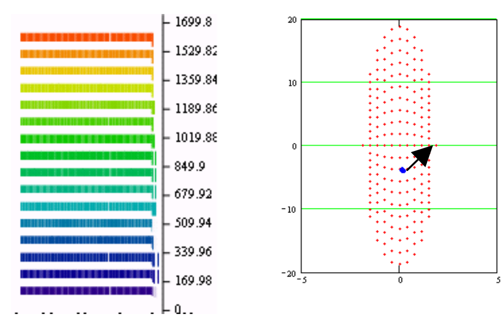


Fig. 3. Graph of the distribution of group velocity inside lens of elliptical shape filled up with water having parameters $\rho = 1000$ kg/m^3, C = 1500 m/s. The outside of lens is ground with parameters $\rho = 2000$ kg/m^3, C = 1700 m/s.
Angle of the dropping of acoustic beam onto separation frontier $\alpha = 67$ degrees. Maximum value of the group velocity in lens C_{group} (max) = 1610 m/s, minimum is C_{group} (min) = 0.4 m/s. Frequency 1000Hz

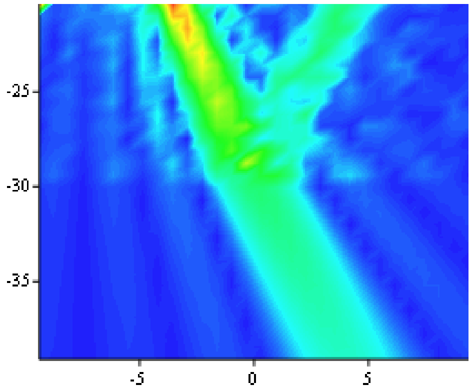


Fig. 4. Graph of acoustic pressure at 80 kHz frequency and 20° incident angle

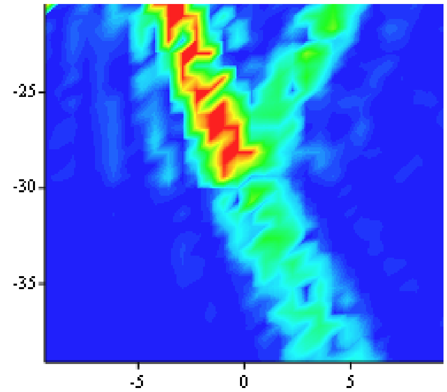


Fig. 5. Graph of intensity component being normal to separation frontier at 80 kHz frequency and incident angle of 20°

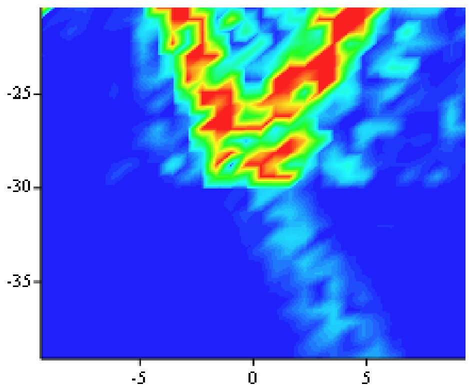


Fig. 6. Graph of intensity component being tangent to separation frontier at 80 kHz frequency and 20° incident angle

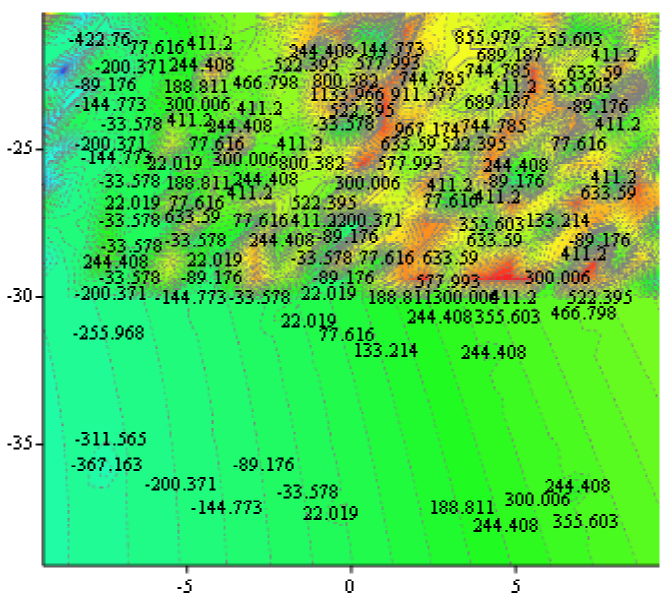


Fig. 7. Graph of group velocity component being tangent to separation frontier at 80 kHz frequency and incident angle of 20°

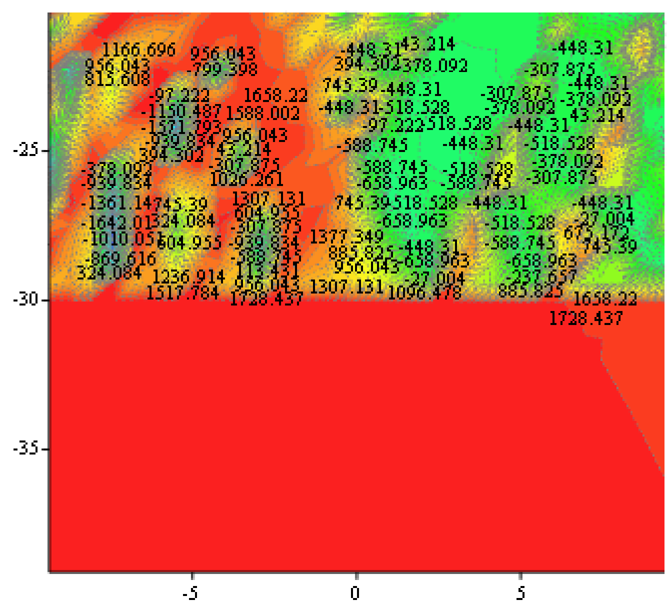


Fig. 8 Graph of group velocity component being normal to separation frontier at 100 kHz frequency and 20° incident angle

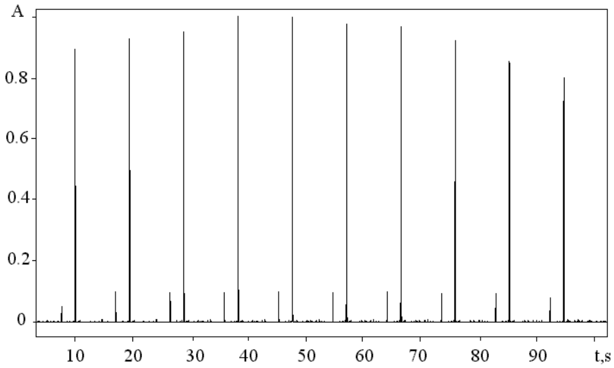


Fig. 9. Reciprocal correlation function in the point of reception (sequence of ten pulses).

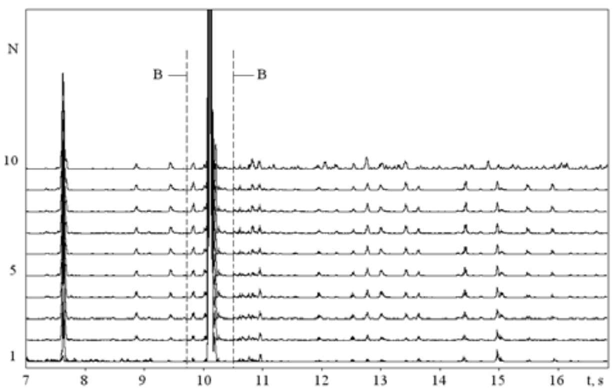


Fig. 10. Reciprocal correlation function in the point of reception (the same as in fig.1. in enchanced scale accodingly amplitude and time).

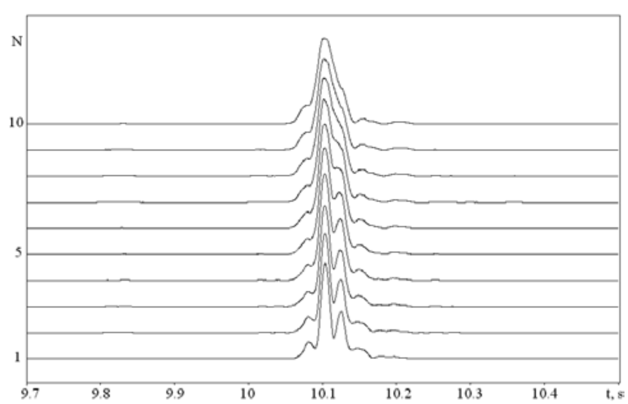


Fig. 11. Reciprocal correlation function in the point of reception (fragment B-B from fig.2.).

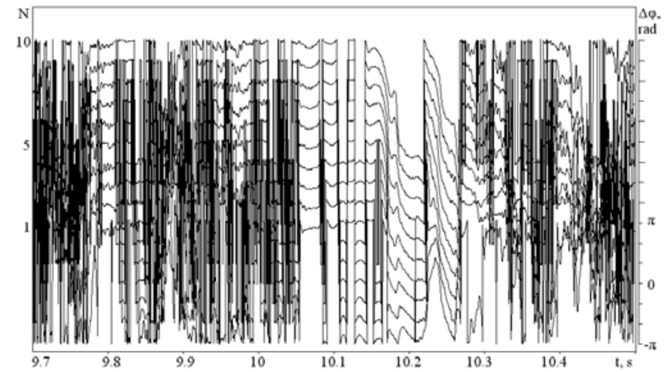


Fig. 12. Difference of phases measured on two elements of base oriented to the signal source.

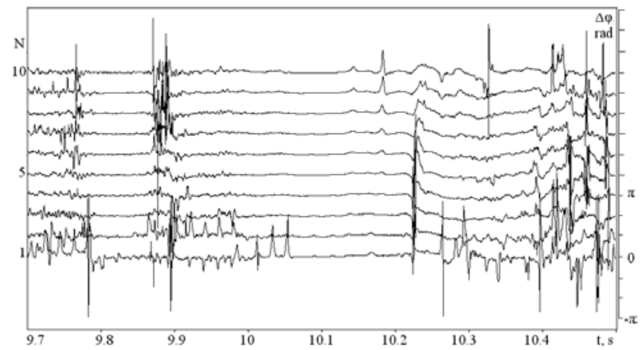


Fig. 13. Difference of phases measured on two elements of base oriented under 90° angle to the trajectory of propagation.

References

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