Time-frequency representations for wideband acoustic signals in shallow water

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Abstract—**Wideband acoustic propagation in shallow water has a unique arrival structure due to modal group speed dispersion. It is possible to unravel the modal dispersion characteristics using a proper time-frequency (TF) analysis. During the Asian Seas International Acoustics Experiment (ASIAEX) - East China Sea (ECS) experiment, signals collected by the vertical arrays have been processed using several time-frequency analysis techniques. Based upon the dispersion characteristics of acoustic propagation, theoretical time-frequency structure is used to determine the change of group delay. We present an improved time-frequency representation to provide higher resolution for TF diagrams using ECS data. The dispersion characteristics of ECS data are observed from these TF diagrams, and are verified by a theoretical model. Using this technique, higher modes can be observed with good resolution; this aids in increasing the performance of the inversion technique for long range sediment tomography.**

I. Introduction

Acoustic propagation in shallow water is greatly influenced by the properties of the bottom. When a wideband acoustic source is used in a shallow water waveguide, the acoustic propagation has a unique arrival structure due to modal group speed dispersion. Dispersion effects can be observed by time-frequency analysis of the acoustic pressure signals recorded at sufficiently large distance away from the source. Recent experiments such as the Atlantic Generating Station Site Experiment [1], the Shelf Break Primer Experiment [2], and the ASIAEX-ECS Experiment [3] collected long range propagation data from wideband explosive sources. The short-time Fourier transform (STFT) and the wavelet transform (WT) are applied to extract the time-frequency characteristics of these signals. Group velocity dispersion characteristics have been successfully utilized for the inversion of geoacoustic properties.

STFT and WT are well known time-frequency analysis techniques to discover the time-varying spectral information of a signal. They are best suited for signals with linear time-frequency structure [4]. However, the WT often can be used to improve the resolution problems identified with the STFT. Instead of the constant resolution of STFT, the resolution of WT depends upon the frequency [5]. Although the morelet wavelet analysis gives better time-frequency resolution than the STFT using the Primer data and the ECS data, it does not give good frequency resolution at higher frequencies [2,3]. Hence it is very difficult to extract group speed values for higher modes at higher frequencies. In order to observe higher modal dispersion, an improved TF technique is required to provide a high resolution TF diagram.

Matching pursuit was originally introduced in the signal-processing community as an algorithm "*that decomposes any signal into a linear expansion of waveforms that are selected from a redundant dictionary of functions* [6]." It can be used to provide excellent spectral localization of a signal, if the selected waveform matches correctly the characteristics of the signal. Since the theoretical group speed curves can be derived from the acoustic propagation model, we are able to design a dictionary of functions which match the change of group delay of dispersive acoustic signals. This offers new opportunities for the improved processing algorithms and spectral interpretation methods.

This article is organized as follows. In Sec. II, we first review the classical time-frequency representations including the STFT and the WT, then we present an improved time-frequency representation based on the matching pursuit. A dictionary of functions for the dispersive acoustic signals is derived to well match the wideband acoustic dispersive signals. In Sec. III, the time-frequency representations discussed in Sec. II are applied to analyze the ECS data The conclusions of the present study are presented in Sec. IV.

II. Time frequency representations

A. STFT and Spectrogram

STFT is the most commonly used and easy to understand time-frequency representation [5],

$$STFT_x^{(\gamma)}(t,f) = \int x(\tau)\gamma^*(\tau - t)e^{-j2\pi f\tau}\,d\tau\,. \qquad (1)$$

The STFT at time t is the Fourier transform of the signal $x(\tau)$ multiplied by a shifted "analysis window" $\gamma^*(\tau - t)$ centered around t (the superscript * denotes complex conjugation). Because of the multiplication by the relatively short-time window, the STFT is simply a "local spectrum" of the signal $x(\tau)$ around the "analysis time" t. Thus, it is obvious that good time resolution of STFT requires a short window. On the other hand, good frequency resolution requires a long analysis window. It has a trade-off between time resolution and frequency resolution. The quadratic version of STFT is called the spectrogram,

$$SPEC_x^{(\gamma)}(t,f) \equiv \left|STFT_x^{(\gamma)}(t,f)\right|^2 . \tag{2}$$

It is an intuitively reasonable assumption to represent a time-frequency energy distribution, since energy is a quadratic signal representation. The spectrogram is a member of the Cohen's class TFRs which satisfy the property of time-frequency shift covariance. It is a real valued and positive function, and it is matched to signals with linear time-frequency characteristics as it preserves the signal's constant time-frequency shifts [7]. If $\int |\gamma(t)|^2 dt = 1$, the integration of the spectrogram over the whole time-frequency plane will give you back the total signal energy .

B. WT and Scalogram

The WT was originally introduced as a time-scale representation [8]. The time-frequency version of WT can be obtained by replacing the analysis scale a as f_0 / f ,

$$WT_x^{(\gamma)}(t,f) = \int x(\tau)\sqrt{f/f_0}\,\gamma^*\left(\frac{f}{f_0}(\tau - t)\right)d\tau \tag{3}$$

where $\gamma(t)$ is called the analysis wavelet and the parameter f_0 is the center frequency of $\gamma(t)$. The scalogram is the quadratic structure of the WT defined as

$$SCAL_x^{(\gamma)}(t,f) \equiv \left|WT_x^{(\gamma)}(t.f)\right|^2 . \tag{4}$$

The scalogram is a TFR in Affine class that is covariant to scale changes and time translations [4]. When wideband acoustic signals travel in a dispersive waveguide, this time-frequency scaling property is a useful property to model and approach the modal dispersion characteristics of wideband acoustic arrivals, which the lower frequency components of a particular mode have a longer arrival times, and its higher frequency components arrival earlier. Compared the scalogram with the spectrogram, the analysis wavelet of the scalogram is able to dilate or compress corresponding to each spectral component. Thus, the resolution of the scalogram is depended upon frequency. It has been used to discover the time-frequency structure of the wideband acoustic signals. The drawback of the scalogram is its spectral resolution becomes poorer as frequency increases. It is very difficult to extract the time-frequency structure of higher modes.

C. The Matching Pursuit Algorithm

The matching pursuit algorithm decomposes any signal into a linear expansion of waveforms that are selected from a redundant dictionary of functions. These waveforms are chosen in order to best match the signal structures. Each element of dictionary is generated by multiplying a moving window function as

$$\Psi_{(\tau,\theta)}(t) = \gamma(t-\tau)\cdot e^{j\theta(t)} \tag{5}$$

where $\theta(t)$ is chosen to best match the instantaneous frequency structure of signal's component, and τ is the translation of the element of dictionary. If the window function $\gamma(t)$ is Gaussian then $\Psi_{(\tau,\theta)}(t)$ are called Gabor atoms. The procedure of the matching pursuit first projects the analysis signal onto each element of the dictionary as

$$MPA_x^{(\gamma)}(\tau,\theta) = \int x(t)\Psi_{(\tau,\theta)}(t)dt . \tag{6}$$

If the selected waveforms best match the signal structures, the resulting highest energy of Eq. (6) will indicate the time-frequency localization of signal's component.

A dictionary of functions based on wave propagation in a range-independent, constant sound-speed waveguide can be designed to well match the experimental datal. In shallow water environments, the acoustic field can be represented in terms of normal modes due to the acoustic sea surface/sediment interaction. For the ideal waveguide, the group velocity for each mode can be derived as [9, 10]

$$U_m = c\sqrt{1-\frac{f_{0m}^2}{f^2}} \tag{7}$$

where c is the sound speed in water column and f_{0m} is the cutoff frequency of the mth mode. The instantaneous frequency distribution for each mode can be obtained from the group speed dispersion in Eq. (7). Since the instantaneous frequency of a time-varying signal is defined as the instantaneous change in the phase of that signal, a dictionary of functions is derived as

$$\Psi_{(\tau,f_{0m},c,R)}(t) = \gamma(t-\tau)\cdot \exp(j\frac{2\pi f_{0m}}{c}\sqrt{c^2t^2-R^2}) \tag{8}$$

where $\gamma(t)$ is a window function centered at the modal arrival time τ , and R is the source-receiver range. The constant values of c and R are determined from the environmental parameters of the experiment work.

Substituting $\Psi_{(\tau,\omega_{0m},c,R)}(t)$ into Eq. (6), a time-frequency representation is obtained as

$$TFR_x^{(\gamma)}(\tau,f_{0m}) = \int x(t)\gamma(t-\tau)\exp(j\frac{2\pi f_{0m}}{c}\sqrt{c^2t^2-R^2})dt . \tag{9}$$

The TF version of Eq. (9) can be obtained through the substitution $f_{0m} = \frac{f\sqrt{c^2t^2-R^2}}{ct}$. The quadratic version TFR (QTFR) is defined as the squared magnitude of the TF version of Eq. (9)

$$QTFR_x^{(\gamma)}(t,f) \equiv \left|TFR1_x^{(\gamma)}(t,f_{0m})\right|^2\Big|_{f_{0m}=\frac{f\sqrt{c^2t^2-R^2}}{ct}} \quad . \qquad (10)$$
$$= \left|\int x(t')\gamma(t'-t)\exp(j\frac{2\pi f_{0m}}{c}\sqrt{c^2t'^2-R^2})dt'\right|^2\Big|_{f_{0m}=\frac{f\sqrt{c^2t^2-R^2}}{ct}}$$

III. TIME-FREQUENCY ANALYSIS OF ECS DATA

The ASIAEX- East China Sea (ECS) Experiment, with focused on the acoustic bottom interaction experiment, was conducted in May-June 2001 in the East China Sea. Wideband explosive sources (WBS) were deployed in the experimental area from a Chinese Research vessel (Shi Yan 2) during the experiment. These shots were deployed in a circular pattern of radius 30 km. The acoustic signals from these explosive sources were received at the vertical line array (VLA) hung from R/V Melville near the center of the circular pattern. Data were sampled on this array at the rate of 2048 samples/s. The GPS positions of the WBS charges are shown in Fig. 1. The sediment properties in this experiment sites have a sharp contrast on either side of a sediment front. The presence of high-speed sand and low-speed mud/sand on either side of the front has been confirmed by core data from ASIAEX-2000. The time-frequency analysis of received explosive signals is used to reveal the dispersion behavior of the acoustic modes. In a shallow water waveguide, high frequencies generally arrive earlier whereas low frequencies arrive latter at sufficiently long range. The lower frequency data and the higher modes have more interaction with the bottom and hence are more important in geoacoustic inversions [2].

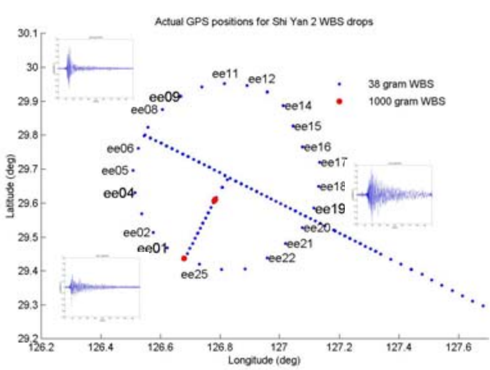

Fig. 1. Actual GPS positions for the ECS data.

The matching pursuit QTFR in Eq. (10) is applied to analyze the ECS data. For the ECS experiment, the average sound speed and the source-receiver range are chosen as $c = 1500$ *m/s* and $R = 30$ *km*, respectively. They are reasonable for the environmental parameters of ECS experiment. Since the sampling frequency of ECS data is 2048 *Hz*, the TF analyses of ECS data with one-second duration are calculated to reveal the TF information for frequency between 0 and 512 *Hz*. The minimum frequency of each mode, f_{0m}, is sampled as $f_{0m} = 2,4,\cdots,512$ *Hz* and the translation τ is sampled as $\tau = 1/256, 2/256, \cdots, 1\,sec$ to generate a TF diagram with 256x256 pixels.

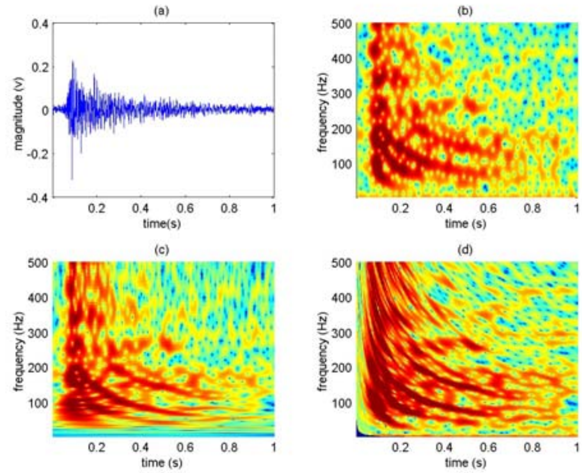

Fig. 2. Time-frequency analysis using several time-frequency representations: (a) ECS data from shot ee01 channel 1, (b) the spectrogram, (c) the scalogram, (d) the matching pursuit TF diagram.

As an example of the processed data, Fig. 2 shows the time-frequency analysis of the ECS data using the spectrogram, scalogram, and the matching pursuit TF diagram. The dispersion characteristics of acoustic propagation are investigated with different time-frequency resolution. Compared with the spectrogram, which has the uniform resolution for the whole TF plane, the spectrogram has better frequency resolution (but poor time resolution) for low frequencies and better time resolution (but poor frequency resolution) for high frequencies. The result of matching pursuit algorithm clearly shows the modal dispersion, including higher modes, because the chosen waveforms in the dictionary are well matched the data.

An isovelocity waveguide with free surface and perfectly rigid bottom is used to compare with the TF diagrams of ECS data. The group velocity as given by Eq. (7) can be used to derive the ideal TF structure of the acoustic arrivals. These theoretical dispersive curves for sound speed $c = 1500$ *m/s*, water depth $D = 100$ *m*, and the source-receiver range $R = 30$ *km*, are plotted as the continuous lines in Fig. 3. The matching pursuit QTFR compares well with these theoretical curves for mode 1 to mode 8. It should be noted that the mode 7 does not appear in this channel. Differing from the previous analysis using the scalogram [2, 3], the matching pursuit algorithm produces TF diagram with higher resolution for both low and high modes.

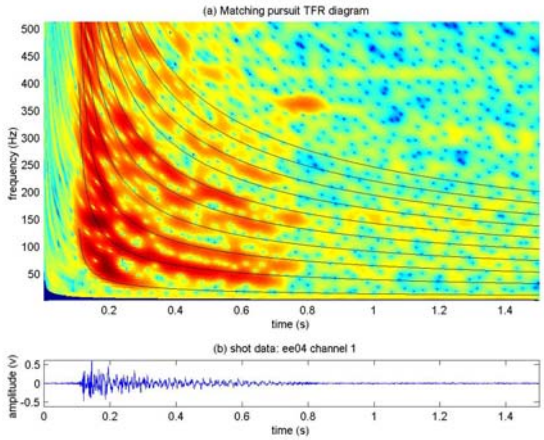

Fig. 3. (a) Matching pursuit TF diagram for the ECS data. The continuous lines represent the theoretical dispersive curves for an isovelocity profile. (b) Time domain plot of ECS data.

The modal structures for the theoretical isovelocity waveguide is shown in Fig. 4. The left panel in the figure shows mode 1 to mode 3 for signal's component with *125 Hz*, whereas the right panel shows mode 4 to mode 6 for frequency *225 Hz*. It should noted that the *m*th mode has *m* zeros in its stationary pattern of vibration in the depth axis. The positions of 14 hydrophones at the ECS vertical line array are denoted as the notation "x" in this figure. Figure 5 shows the matching pursuit time-frequency diagrams of ECS shot data ee04 at channel 1, 4, 9 and 13. Compared ECS data with theoretical model, the mode 2 has a zero node around the position of channel 9 in the theoretical model, the matching pursuit TF diagrams using ECS data also show that the mode 2 disappears at channel 9. Mode 3 has two zeros around the positions of channel 2 and channel 12 in the theoretical model; the results of the matching pursuit TF diagrams show that mode 3 disappears at channel 4 and channel 13. The behavior of mode 4 to mode 6 of ECS data also obeys the stationary pattern of vibration as shown in Fig. 4. The time-frequency analysis of ECS data satisfies the modal structure and is verified by the theoretical model.

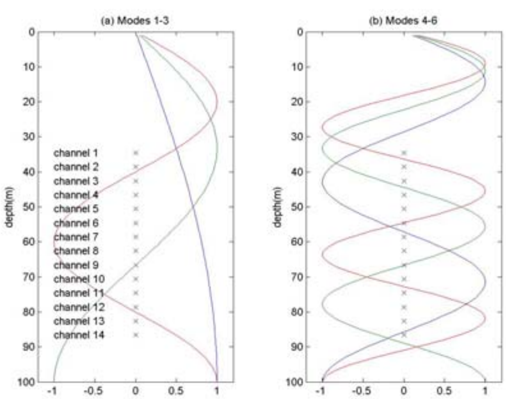

Fig. 4. Theoretical mode shapes for an isovelocity waveguide at frequency *125 Hz & 225 Hz*.

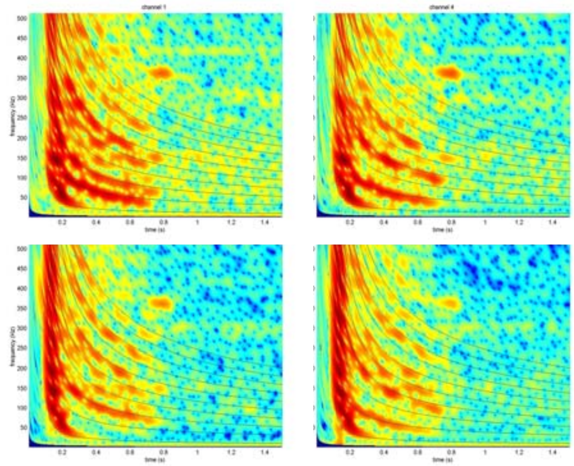

Fig. 5. Matching pursuit TF diagrams for ECS shot ee04 data at channel 1 (upper left), channel 4 (upper right), channel 9 (lower left), and channel 13 (lower right). The continuous lines represent the theoretical dispersive curves for an isovelocity profile.

IV. Conclusion

When wideband acoustic signals travel in a dispersive waveguide, this time-frequency scaling property of the scalogram is a useful property to approach signals' modal dispersion characteristics. However, the resolution of scalogram is depended upon frequency, it is very difficult to extract the time-frequency structure of higher modes. Instead of using the scalogram, we derived a dictionary of functions from the theoretical model to well match the dispersion characteristics of wideband acoustic arrivals. Since we project this dictionary of functions onto the signal using a constant-length window function, the matching pursuit algorithm provides a uniform resolution diagrams over the entire time-frequency plane. Therefore, the time-frequency representation shown in Eq. (10) is able to observe the dispersion characteristics for higher modes.

Matching pursuit with the proper design waveforms offers new opportunities for improved processing algorithms and spectral interpretation methods. We build our atoms in the dictionary, Eq. (8), to well match dispersive acoustic signals. Using this technique, our time-frequency diagrams of ECS data extract successfully the dispersion characteristics of explosive arrivals for at least 8 modes. Higher modes can be observed with good resolution; this aids in increasing the performance of the inversion technique for long range sediment tomography.

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