Pectoral Fin Motions for Vehicle Control

Thomas Gieseke, Dana Hrubes, Adrienne Yandell, Jeffery Walker, and Naomi Kato

***Abstract*—This document details the design and fabrication of a three degree of freedom articulated fin. The mechanism was developed to support the advancement of biologically inspired fin propelled undersea vehicles. Parallel efforts conducted at universities and laboratories in Japan are producing measures of forces and moments produced by articulated fins. Hardware and methodologies were needed to validate and support those efforts at a U.S. facility. Furthermore, a method to experimentally evaluate control methodologies was needed as part of the larger cooperative effort. The device described in this document answers the needs of this effort. It is a relatively large mechanical device capable of articulating an appendage through three rotations (each independently) following an arbitrarily specified profile. It is fabricated to sit on an instrumented platform to measure the full set of three forces and three moments during its operation.**

***Index Terms*—Oscillating foil appendages, undersea vehicles, unsteady hydrodynamics, vehicle control.**

I. Introduction

The United States Navy has long recognized the need to operate undersea vehicles in chaotic undersea environments. One of the key elements of Marine Corps operation doctrine is "ship to objective maneuvers," which requires U.S. forces rapidly transit from a staging area off-shore through various hazards in the littoral waters (including mine fields and obstacles) to a selected beach landing area. This type of freedom of maneuvers can only be achieved with adequate intelligence of the location and nature of all hazards along the route. In order to collect this information, stealthy robust marine systems are needed to survey near-shore areas collecting critical intelligence. Typically, these near-shore areas are very difficult to navigate due to waves, currents, and debris. Any vehicle that is tasked to operate in these areas must have robust control and propulsion systems that are insensitive to strong body force perturbations (as would be anticipated from a breaking wave or tidal currents).

Manuscript received July 20, 2003. This work was supported in part by the Office of Naval Research under Grant N0001403WX20065 (Dr. Robert Gisiner).

T. J. Gieseke is with the Naval Undersea Warfare Center, Newport, RI 02841 USA (401-832-1807; fax: 401-832-6202; e-mail: giesekétj@ npt.nuwc.navy.mil).

J. D. Hrubes is with the Naval Undersea Warfare Center, Newport, RI 02841 USA (401-832-2766; fax: 401-832-6202; e-mail: hrubesjd@ npt.nuwc.navy.mil).

A. Yandell is a student at the Massachusetts Institute of Technology, Cambridge, MA 02139 USA. She was at the Naval Undersea Warfare Center as an NREIP intern during this project (e-mail: ayandell@mit.edu).

J. A. Walker is with the University of Southern Maine, Portland, ME 04103 USA (207-228-8166; fax: 207-228-8116; Email: walker@usm.maine.edu).

N. Kato was with Tokai University. He is now with the Graduate School of Engineering at Osaka University, Yamadoaka, Suita, 565-0871 Japan (81-6-6879-7579; fax: 81-6-6879-7594; Email: kato@naoe.eng.osaka-u.ac.jp).

Nature has successfully solved these propulsion challenges and a myriad of biological species can be found living in oceanic environments where chaotic flow conditions can be found (Denny, 1988). The ability of these species to successfully navigate, feed, and generally thrive reflects some uniquely superior elements of their genetic design. A key genetic design element observed repeatedly in these fish species is the presence of pectoral fins to offer stability and control, particularly in species that are capable of hovering.

Fish that are capable of rapid acceleration and sustained cruising speeds use a recognizable oscillatory caudal fin motion with associated body bending or undulations. Considerable research has been conducted in efforts to understand and mimic the subtleties of these basic flapping motions (Ahlborn et al, 1997; Barrett et al., 1999; Kumph et al., 1999; Triantafyllou et al., 2000; Anderson et al., 2001, Anderson & Kerrebrock, 1999). Robotic fish (robo-tuna and similar vehicles) at MIT and Charles Stark Draper Laboratory embody key elements of this type of propulsion. These vehicles are build around flexible streamlined bodies with oscillatory caudal fins.

Fish that maneuver with great ability, however, generate turning moments with oscillating fins and not undulating bodies (Walker, 2000). Such maneuvering specialists have inspired the development of robots with flapping foils and other fish-like control devices attached to rigid cylinders, for example, those at NUWC (Bandyopadhyay et al., 1997, Bandyopadhyay et al., 1999a, Bandyopadhyay et al., 1999b) and elsewhere (Hobson et al., 1999; Kato, 2000). It is not surprising that these studies have arrived at the conclusion that fish-based control surfaces are better suited for maneuvering than for propulsion

A three-motor-driven mechanical pectoral fin (3MDMPF) has been recently developed by Kato and his team (Kato, 1999a and Kato 1999b). Gimbal structures were incorporated to allow three DC servo motors to independently generate rowing motion, feathering motion, and flapping motion, respectively, and to produce vertical motion of the fin. Its hydrodynamic characteristics were investigated experimentally and theoretically. It was found that the flapping motion 3MDMPF is effective in increasing the thrust force. Furthermore, a pair of 3MDMPFs were installed in a autonomous underwater vehicle to test its maneuverability in both the horizontal plane and the vertical plane. Its high maneuverability was demonstrated such as ascending and descending both in forward and backward directions.

Kato and his team are continuing to develop these mechanisms for application to an autonomous vehicle as part of a collaborative effort with the U.S. As part of this development extensive testing has been conducted on the performance of fins undergoing various motions.

A long range objective this study is to develop a rigid underwater vehicle built for precision maneuverability, including hovering and balancing, while being perturbed due to rapidly changing external flows (consistent with operation in littoral waters). To achieve this goal, the existing body of research must be extended so that we can answer the question of what is the optimal fin shape, motion, number, and locations to meet the specified maneuvering requirements? As a template for the design of this vehicle, we will use data from highly maneuverable fish that use one or more pairs of fins as active control surfaces.

II. Approach

A. *Teamed Effort*

The overall approach to achieve the stated objectives is to conduct a series of experiments and calculations leading up to the production of a prototype biologically inspired vehicle. A joint numerical and experimental study is currently being conducted to evaluate the underwater precision maneuverability of a cylindrical body, in particular the stop and hover motions, by using fore and aft pairs of pectoral fins, similar to their usage in a fish. The study involves the following major tasks:

1) Review available information on the motion of pectoral fins of several types of swimming in low speeds or hovering under disturbances such as water currents or waves.
2) Experimentally assess the kinematics of pectoral fin motion during hovering.
3) Experimentally assess 3MDMPF performance in a controlled laboratory environment.
4) Construct a test body utilizing fore and aft pairs of 3MDMPF for maneuvering control.
5) Develop automatic control algorithms for guidance and control of the test body in three dimensional underwater space.
6) Demonstrate the high maneuverability of the cylindrical underwater vehicle equipped with 3MDMPFs following a prearranged trajectory around undersea obstacles.

The Naval Undersea Warfare Center is supporting this effort through the development of control strategies, and operation of test ranges. Biological systems are limited in the types of strokes they can achieve because they are incapable of rotating their appendages about their central axis (spinning them). Consequently it is not possible for them to complete variations on basic flapping strokes to achieve propulsion forces in arbitrary directions. To compensate for this physical limitation, they use different classes of strokes. Each stroke class is uniquely suited to producing forces in a specific direction under specific flow conditions. For example, a fish may select between a drag-based rowing motion and a lift-based flapping motion in different situations.

One of the control strategies advocated by NUWC offers some advantages over the approach observed in biological systems. It has been postulated that if the tip of the pectoral fin model were to scribe a circle (the motion made by the major axis of the fin forming a cone) and the feathering continually adjusted, force vectors could be produced in an arbitrary direction (confined to the plane perpendicular to the axis of the cone of motion) by properly adjusting the feathering perturbation. The normal mode of operation (before generation of force) would be for the fin to be optimally streamlined during all phases of the motion. To create a force in a desired direction, the feathering would be perturbed at the correct phase during the circular motion and then returned to the nominal streamlined profile during the balance of the cycle. This control methodology requires continuous rotation of the fin about its major axis (no motion limits).

To complement work conducted by Dr. Kato in Japan and to explore additional control methodologies (such as the one outlined above), it was determined that construction of a three degree of freedom fin articulation mechanism with provision for continuous rotation in all axes, along with an apparatus to enable measurements of loading was required in a U.S. lab. This document describes the mechanism developed for this purpose.

B. *Drive Mechanism*

An articulated fin drive mechanism was required to produce motions with three degrees of freedom of rotation. The rotational motions are defined as; feathering: rotation about the axis of the fin; flapping: motion changing the angle between the central axis of the device and the axis of the fin, i.e. the angle at the “knee” of the linkage; and rotation: motion of the entire mechanism about its central axis.

In rotation and feathering, it is desired that there be no limits to the extent of motion. The fin must be capable of continuous rotation about the device axis and the fin axis. In the flapping motion, plus and minus 90 degrees is desired.

A mechanism built around two nested bevel gear assemblies has been developed to generate these motions. Figure 1 shows an image of this system. The device was fabricated in the Naval Undersea Warfare Center shops. Components were sized based on availability of hardware and ease of manufacture. Much of the design details were completed by the shop staff during fabrication.

The two yokes were fabricated using an advanced part construction technique, the advancement of which is funded by the office of Naval Research. Prometal Rapid Manufacturing offers a part printing service which constructs steel alloy parts from a layered structure of powdered material which is then heat-treated to form a solid piece. This approach allowed for rapid fabrication of complex parts for use in this device.

Rotation output is directly driven by motor 1 through a spur gear mounted to the outermost yoke. Flapping output is driven through the synchronized motion of motors 1 and 2. To achieve a desired flapping motion, motor 2 must be driven at the desired flapping output less the output of motor 1. Feathering output is achieved by synchronized motion of all three motors. To achieve the desired feathering output, motor 3 must be driven at the desired feathering less the output of motor 2 and twice the output of motor 1.

Three equal torque/power motors are required to drive this device. The device is currently driven by R34 Pacific Scientific servo motors capable of speeds up to 4000rpm and torques of 8.3Nm. Three single axis servo motor controllers are incorporated as a drive system for the device. The output shafts for these three motors pass through reduction gear boxes and a single spur gear reduction for a total speed reduction of 25:1 on two axes and 75:1 on the third axis.

Software to translate the desired motion to motor control commands has been developed so that a user can specify a desired rotation history as a "cam" profile. These commands are translated to required motor outputs and the desired motion profile is produced.

Because all axes are mechanically coupled, careful tuning of the device is required to assure stability of the closed-loop servo control systems. Parameters used in this tuning are strong functions of the system inertia. Although the system is fully functional for the paddle design and motions desired for this study, its full capabilities have not been realized. It is anticipated that more careful system tuning will enable more radical motion profiles to be achieved.

C. *Instrumentation*

As part of the servo motor system, angular deflection of the three servo motors are regularly monitored. These outputs are easily translated to absolute rotations via a simple algebraic transformation.

To measure loads produced during motion of the paddle as it is oscillated in a test tank, an instrumentation test platform was designed. The basic design was to use a set of preloaded button force sensors, integrated into the support structure. This load balance system is designed to measure the 3 forces and 3 moments externally applied to the test platform by supporting the platform by at least 6 independent load cells. This force measurement device consists of five major components: a lower support platform (1), an upper support platform (2), six (or more) interconnecting members (3) containing load cells (4), and an instrumentation package (5). A representative arrangement containing these components is shown in Figure 2.

The lower platform is secured to a rigid supporting structure and the loads to be measured (linear forces **F** and moments **M**) are applied with respect to a central point on the upper platform. Under the load, the six inter-connecting members go into tension and compression in a way which is uniquely determined by the three loading forces, the three loading moments, the connecting member configuration, and any pre-load on the connecting members. The load cells produce a voltage output which is proportional to the member tensions.

The upper and lower plates are designed to be rigid under the design loads of the system, light, and fit geometrically into the confined space for the application. For the flapping foil load balance the upper plate supports the fin, gear-box and motor system. The lower plate is the surrounding structure and the interconnecting members in the flapping foil system are the button load cells. The suggested load cell design approach requires that each connecting member be unable to resist forces not aligned with their longitudinal axis. Interconnecting members with ball joints on each end meet this requirement. The button load cell configuration does not currently meet this constraint, however in the next phase of this study this problem will be corrected.

The relationship between connecting member tension/compression and the applied load is determined by examining the static force balance on the device. Each member, i, connects a point $\mathbf{x_{1i}}=(x_1,y_1,z_1)_i$ on the bottom plate to a point on the upper plate $\mathbf{x_{2i}}=(x_2,y_2,z_2)_i$. Because the members transmit forces along their axis (defined as the vector from $\mathbf{x_{1i}}$ to $\mathbf{x_{2i}}$, $\mathbf{v_i}$) with a magnitude equal to the strain (tension) in the member, the net force imposed on the top plate by a connection member is the projection of the tension vector into the three component directions.

$$(F_x, F_y, F_z)_i = T_i\left(\frac{\mathbf{v}_i}{\|\mathbf{v}_i\|}\right) \quad (1)$$

where F_x is the force in the x direction, F_y is the force in the y direction, F_z is the force in the z direction, and T_i are the tensions in the connecting members (i=1 to 6).

The net moment imposed on the top plate by a connection member about a reference point (the point of load application) is the tension vector crossed into the distance between the connection point $\mathbf{x_{2i}}$ and the reference point $\mathbf{x_r}=(x_r,y_r,z_r)$, e.g

$$\mathbf{d_i} = \mathbf{x_{2i}} - \mathbf{x_r} \quad (2)$$

$$(M_x, M_y, M_z)_i = T_i\left(\frac{\mathbf{v}_i}{\|\mathbf{v}_i\|}\right)\times(\mathbf{d}_i) \quad (3)$$

where M_x is the moment about the x axis, M_y is the moment about the y axis, and M_z is the moment about the z axis. Combining equations 1.0 and 3.0 and defining a matrix of calibration coefficients **A**, the inversion equation can be simply written as

$$(F_x, F_y, F_z, M_x, M_y, M_z) = \mathbf{TA} \quad (4)$$

It is possible to optimize the configuration of load cells and connecting members for maximum sensitivity to the anticipated range of loading. However, because there is considerable uncertainty in the anticipated load ranges produced by the oscillating foil the initial device construction was selected for convenience. Currently 2250N and 4500N transducers have been incorporated into the design. These transducers offer adequate overload protection, are capable of supporting the foil drive system (approximately 60kg) and will provide information necessary for more optimal selection of load cells in the future.

There are two methods by which the coefficient matrix in equation 4 can be developed. A careful study of the mechanism design can be conducted and based on the device geometry, the coefficients can be analytically determined. Alternately a calibration procedure can be conducted to estimate the 36 (or more) unknown coefficients. Combinations of loads and moments can be applied to the device and the sensor outputs monitored. Through application of the correct loading combinations the complete linear calibration matrix can be calculated using least squares methods.

The flapping foil design incorporates a total of eight transducers. A circular main central plate holds the flapping mechanism and drive motors. This plate is sandwiched by 6 transducers attached to a structure mechanically fixed to ground. These six transducers are placed in pairs and the three pairs are spaced at 120 degree intervals around the circumference of the central plate. The transducers in each pair are placed orthogonal to each other with axes contained in a plane normal to the central plate and passing through the plate centerline. The V shaped couplets of sensors are aligned so that one presses 45 degrees down onto the top of the plate and the other presses upward 45 degrees onto the bottom of the plate. The set of 6 transducer effectively clamps the central plate in place preventing it from moving linearly in any direction or rotating about two axes. Ideally, these six primary transducers are not capable of resisting rotation about the main axis of the device. Two secondary transducers were added to resist this motion, effectively providing a torque measurement. A least squares approach is used in developing the calibration matrix, thus accounting for all uncertainties in positioning and the measurement redundancy.

III. RESULTS

A. Calibration

A calibration procedure was used to generate the coefficient matrix relating force inputs to transducer outputs. A calibration device was constructed to allow application of various combinations of linear forces and moments. This configuration incorporated the fin actuation system mounted atop a unistrut frame, a frame with eye fittings as a replacement for the paddle, an adjustable cable and pulley system, and weights. By adjusting the attachment point on the frame, the pulley location and the weights used, various force and moments can be applied to the fin mechanism. In total 140 loading combinations were applied. With weights ranging up to 10kg and lever arms up to 0.5m. A coordinate system for the calibration (and motion) has been defined as z along the central axis of the device (positive down), x along the primary axis of motion for the tip, and y perpendicular to the primary axis of motion.

Figures 3 shows the applied forces in the x direction and those predicted from the calibration matrix. The mean calibration error (rms) for this force (and all others) is as shown in table 1. The data is presented as a percentage of the span of loading applied during calibration.

TABLE 1
CALIBRATION ERRORS

Force	Error %
Fx	5.2
Fy	4.6
Fz	8.4
Mx	7.1
My	3.8
Mz	31.0

B. Generated Motions

The mechanism described in this document is capable of generating arbitrary motions with three degrees of freedom (within the mechanical limits of the system). Three representative motions were selected to evaluate the basic system capabilities; stroke producing forces mostly from drag (two-dimensional rowing), a motion generating lift and drag (two-dimensional flapping), and a complex motion (three-dimensional figure-8). For all motions a 0.81m paddle (flat cross-section) with a 15cm chord was attached to the mechanism. Approximately 35cm of the paddle was immersed in a tank of water. For both two dimensional motions, the paddle was commanded to flap from -9 to 9 degrees in a sinusoidal pattern. At the end of the stroke, the paddle motion paused and the paddle feathering angle was adjusted to the angle desired for the return stroke. The length of this pause was selected to prevent control system errors. In the case of the flapping motion, the angle of attack was fixed at 45 degrees during both power phases of the motion. The high angle of attack was selected to generate both relatively large lift and drag forces during the motion. For the pure drag motion the fin surface was normal to the direction of motion in one direction and tangential to the direction of motion in the other direction. The cycle speed in the two dimensional motions was set such that the paddle went through its flapping range of motion in .4 seconds. The associated peak tip velocity was 1.0m/s (peak Reynolds number based on chord 3.5e5). Because there is no ambient mean flow the normal dimensionless oscillation frequencies are not defined. The figure-8 motion was designed to complete its flapping cycle in the same time as the pure flapping motion. Figures 4 and 5 show the flap, rotation, and feathering angles associated with the three motions. For ease of presentation, the motions are presented as dimensionless form and the bounds are noted in the captions.

C. Forces Generated

The load balance system developed for the articulated fin mechanism measures all forces produced during the fin motion. These forces include both the hydrodynamic forces and the inertial forces needed to accelerate and decelerate the paddle. To separate these two effects, the fin was initially driven in air and the loads seen by the sensors were observed. For the speeds considered, the unsteady components of these forces were below the sensors ability to measure accurately. However, they did offer a means to measure transducer offset. Figures 6 through 8 show the linear forces produced during rowing, flapping and figure-8 respectively. Because certain issues with load balance system have not been fully resolved and the accuracy of force and moment data is questionable, the force in the z direction and all moments have not been presented.

D. System Limitations

Although the system presented in this document was shown to be capable of producing arbitrary force motions while simultaneously measuring forces, several limitations with the current design were noted.

The most troublesome issue was that the load balance system was not capable of measuring moments about the z axis. It was determined that problems with transducer mounts made it impossible for these moments to be transferred to the relevant transducers. A suitable correction has been devised and is currently being implemented.

Of secondary importance was the lower than desired speeds achievable with the mechanism. These relatively low speeds yielded small hydrodynamic and inertial forces which were difficult to measure accurately.

Of lesser importance was the load range selected for the transducers. Because of the large weight of the system, high load capable transducers had to be selected for the system. Because only low foil motion rates could be achieved the resulting load ranges were approximately 1% of the transducer capacities. These low sensitivities let to larger errors due to drift and electronic noise.

IV. Conclusion

In order to evaluate novel fin-based control algorithms for implementation in multiple vehicles, a robust test platform was required. To meet this requirement a three degree of freedom fin mechanism was constructed and evaluated at the Naval Undersea Warfare Center. This device was shown to be capable of producing arbitrary three degree of freedom motions at rates of the order 1hz. Integral to this design was a six degree of freedom load balance system. Although functional for the purposes of this demonstration, significant limitations and technical issues were observed. However, these problems are well understood and simple modifications should yield a fully functional system capable of measuring hydrodynamics loads during transient fin motions.

Plans are in place to correct deficiencies with this mechanism and to use to evaluate closed-loop control algorithms and advanced three-dimensional fin articulation schemes.

References

[1] Denny, M. W. (1988). *Biology and the Mechanics of the Wave-Swept Environment*. Princeton University Press, Princeton.

[2] Ahlborn, B., Chapman, S., Stafford, R., Blake, R. W. & Harper, D. G. (1997). Experimental simulation of the thrust phases of fast-start swimming of fish. *Journal of Experimental Biology* **200,** 2301-2312.

[3] Barrett, D. S., Triantafyllou, M. S., Yue, D. K. P., Grosenbaugh, M. A. & Wolfgang, M. J. (1999). Drag reduction in fish-like locomotion. *Journal of Fluid Mechanics* **392,** 183-212.

[4] Kumph, J.M., M.S. Triantafyllou, D. Nugent and M.D. Santos, 1999, "Fast-Starting and Maneuvering Vehicles: Robopike and Robomuskie," *Proceedings of 11th International Symposium on Unmanned, Untethered Submersible Technology*, pp.439-445

[5] Anderson, E. J., McGillis, W. R. & Grosenbaugh, M. A. (2001). The boundary layer of swimming fish. *Journal of Experimental Biology* **204,** 81-102.

[6] Triantafyllou, M. S., Triantafyllou, G. S. & Yue, D. K. P. (2000). Hydrodynamics of fishlike swimming. *Annual Review of Fluid Mechanics* **32,** 33-53.

[7] Anderson, J.M. and P.A. Kerrebrock, 1999, "The Vorticity Control Unmanned Undersea Vehicle(VCUUV) Performance Results,"*Proceedings of 11th International Symposium on Unmanned, Untethered Submersible Technology*, pp.360-369

[8] Walker, J. A. "Does a rigid body limit maneuverability?" J. Exp. Biol. 203, pp. 3391-3396 , 2000

[9] Bandyopadhyay,P.R., J.M. Castano, J.Q. Rice, R.B. Philips, ,W.H. Nedderman, and W.K. Macy, 1997, "Low-Speed Maneuvering Hydrodynamics of Fish and Small Underwater Vehicles," *Journal of Fluids Engineering, Transactions of the ASME*, Vol.119, pp.136-144

[10] Bandyopadhyay,P.R., S.N. Singh, 1999a, "Biologucally-Inspired Bodies Under Surface Waves-Part 2: Theoretical Control of Maneuvering," *Journal of Fluids Engineering, Transactions of the ASME*, Vol.121, pp.479-487

[11] Bandyopadhyay,P.R., W.H. Nedderman, and J.L. Dick, 1999b, "Biologucally-Inspired Bodies Under Surface Waves-Part 1: Load Measurements," *Journal of Fluids Engineering, Transactions of the ASME*, Vol.121, pp.469-478

[12] Hobson, B., Murray, M. & Pell, C. (1999). Pilotfish: Maximizing agility in an unmanned-underwater vehicle. *Proceedings of the Eleventh International Symposium on Unmanned Untethered Submersible Technology* **,** 41-51

[13] Kato,N., 2000, "Control Performance of Fish Robot with Mechanical Pectoral Fins in Horizontal Plane", *Journal of Oceanic Engineering, Transactions of the IEEE*, Vol.25, No.1

[14] Kato, N., 1999a, "Three-Motor-Driven Mechanical Pectoral Fin," *Proceedings of 11th International Symposium on Unmanned, Untethered Submersible Technology*, pp.467-476

[15] Kato, N., 1999b, "Hydrodynamic Characteristics of Mechanical Pectoral Fin, *Journal of Fluids Engineering, Transactions of the ASME*, Vol.121, pp.605-613

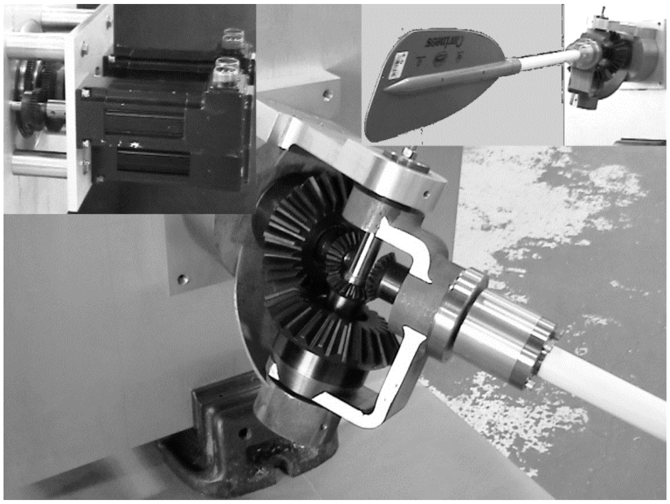

Fig. 1. Nested bevel gear assembly providing three degree of freedom motion.

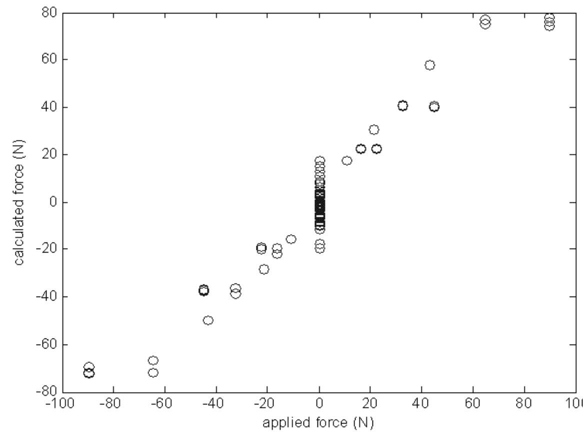

Fig. 3. Calibration data for force in direction x.

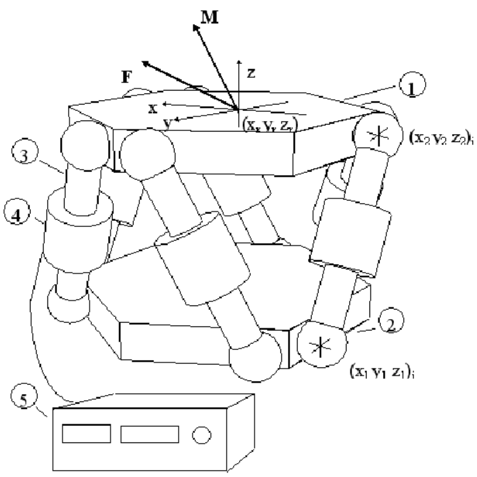

Fig. 2. Representative load balance configuration.

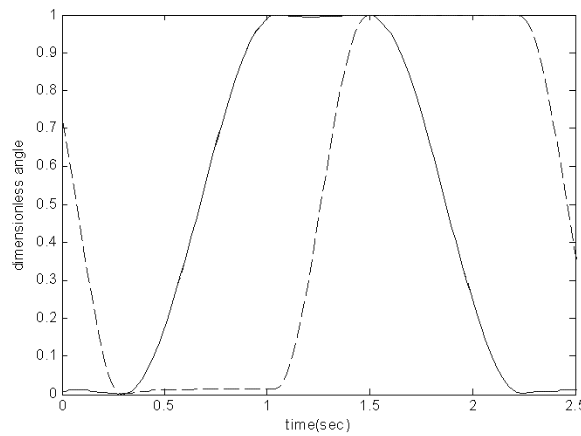

Fig. 4. Dimensionless angular motions for rowing *(- flapping, -9 to 9 degrees; - - feathering 0 to 90) degrees* and flapping *(- flapping, -9 to 9 degrees; - - feathering -45 to 45).*

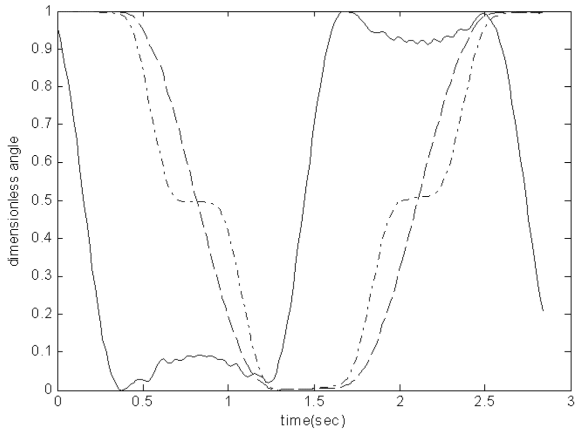

Fig. 5. Dimensionless angular motions for figure-8*; -- flapping, -9 to 9 degrees; - - rotation, -30 to 30 degrees; - · feathering -90 to 90 degrees.*

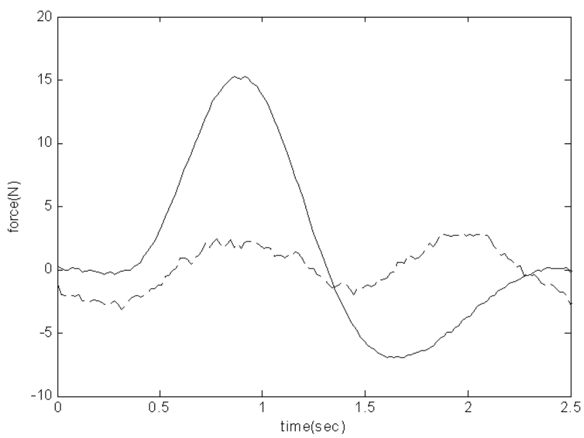

Fig. 6. Horizontal forces generated during rowing motion; dotted line: force in y; solid line force in x.

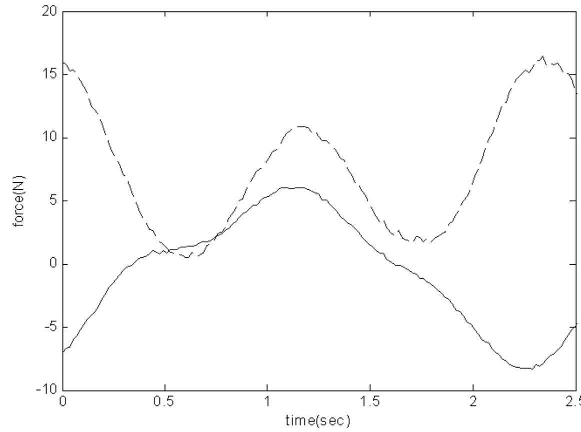

Fig. 7. Horizontal forces generated during flapping motion; dotted line: force in y; solid line force in x.

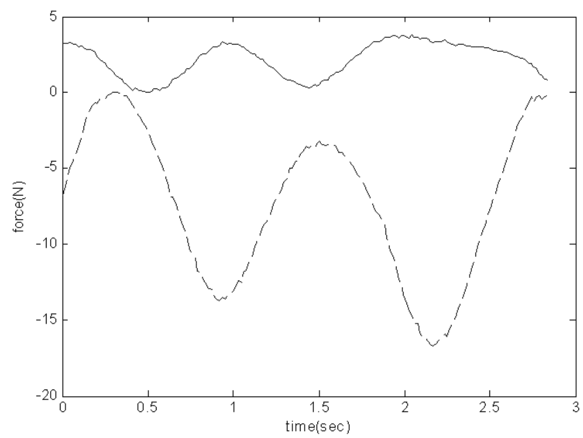

Fig. 8. Horizontal forces generated during figure-8 motion; dotted line: force in y; solid line force in x.