

FRACTIONAL BANDWIDTH ALGORITHMS

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Abstract

Autonomous underwater vehicles are gradually being recognized as key assets in future combat systems.¹ Central to this attitude is the realization that teams of vehicles acting in concerted fashion can accomplish tasks that are either too costly or simply outside the range of capabilities of single vehicles. The VSW-MCM target reacquisition problem is the primary driver of underwater multi-agent research.² Because of the VSW's inherent high vehicle attrition rate and unreliable communication, it is felt that vehicle coordination must be done off-site. The plume-monitoring problem is another important driver, where in this case the spatial and temporal scales prohibit the use of a single vehicle. In this paper, we present algorithms that address these problems.

Introduction

Fractional bandwidth is a set of two principles that guides the design of multi-agent algorithms. The approach arose in response to the sobering facts that real vehicles fail and N vehicles fail even more, and that multi-vehicle communication is unreliable if available at all.³ Further:

Centralized coordination fails when communications fail. Consider an algorithm where a central node controls the vehicles. The central node continually determines the optimal deployment of the vehicles and communicates those requests. This approach is robust to loss of vehicles but it breaks down catastrophically when communication is lost.

Parallel execution fails when vehicles fail. The other extreme is an algorithm where each

vehicle follows a pre-mission plan. This is robust against communication loss, but fails when vehicles fail.

These extremes show that vehicle coordination cannot rely entirely on communication, yet it cannot be free from it. Fractional bandwidth is a set of two principles for achieving it:

- a. each vehicle must individually be capable of accomplishing the entire mission;
- b. communication is a performance accelerator.

In this article, we apply these principles to two problems: target reacquisition and plume monitoring.

VSW-MCM Reacquisition

Background

Underwater mine countermeasure (MCM) is currently accomplished by diver-dolphin teams. Because it is hazardous to personnel, one of the objectives of the US Navy is to replace these diver-dolphin teams by unmanned underwater vehicles (UUVs).^{2,4}

A UUV-based MCM has three phases: detection/reacquisition/neutralization. Very-shallow water MCM (VSW-MCM) is the focus of current Navy efforts.⁵

This article is concerned with the reacquisition problem. Simply stated, a large number of near-shore targets of known location must be visited as quickly as possible. Obstacles to achieving this are the harsh conditions inherent to the VSW,² the

high rate of UUV attrition, the magnitude of the area to be covered, and time constraints.⁴

Given existing small-UUV technology, VSW-MCM expects to require coordination of ~ 75 vehicles. The coordination framework under consideration is centralized control from a remote platform.⁴ The remote platform gives each vehicle a list of targets and launches them. When they return, the vehicles update the central node's target registry. The central node adjusts for vehicle attrition by keeping track of losses and assigns the missed targets to other vehicles.

The approach we propose is complementary. It achieves on-site vehicle coordination while being robust against attrition and bad communication. Its value is particularly high when the attrition rate is large, or when centralized control is not available.

Model

Following a sweep by a separate vehicle, a team of N UUVs reacquires the T targets. The objective is to visit these targets optimally.⁶

The target field is a square of side L . The vehicles are modeled by point objects moving at constant speed while approaching a target and stationary while classifying it. An exponential random process models the mean time between failures of individual vehicles. A finite communication range models communication unreliability, i.e. vehicles within communication range alone hear the transmissions.

Greedy algorithm

The greedy algorithm is the simplest implementation of the fractional bandwidth. Other algorithms are presented elsewhere.⁷ The structure of the greedy algorithm is the following:

(a) Each vehicle keeps a map of all targets, i.e. their position and a status flag. The status flag takes one of three values: "available," "selected," and "done." All targets are initially "available."

(b) Each vehicle looks at its map and selects the nearest "available" target. If no targets are "available," the vehicle selects the nearest "selected" target. If none is left, the vehicle ends its mission.

(c) After target selection, each vehicle broadcasts its choice. Vehicles within communication range receive this data and update their map as follows: if the current value of the received target's status is "available," it upgrades to "selected," if it was "selected," it stays "selected," if it is "done," it stays "done." This step is crucial to coordination, as two vehicles cannot knowingly select the same target (unless no targets remain).

(d) Each vehicle travels to its target. When within the search radius, it executes the classification.

(e) When done with its target, each vehicle updates the status of the target to "done," and then broadcasts the identity and updated status of the target. Each vehicle within communication range updates its maps as follows: if the current value of the received target's status is "available" or "selected," it is upgraded to "done," if it was "done," it remains "done."

(f) Return to step 2 until all targets are "done."

The effectiveness of this algorithm is surprisingly high, especially considering the low rate of communication (~ 15 bits/target). Although inter-vehicle communication is parsimonious, it provides effective coordination since two vehicles within range do not go after the same target. It also provides long-range coordination since the locality of the target selection algorithm favors growth by domains (see movies at www.math.duke.edu/~daniel/search.html).

The algorithm is robust against loss of vehicles (Figure 1). The probability of mission failure is less than 1% up to an attrition rate of .67/mission. The algorithm performs well even as vehicles are lost because live vehicles pick up the targets of dead vehicles.

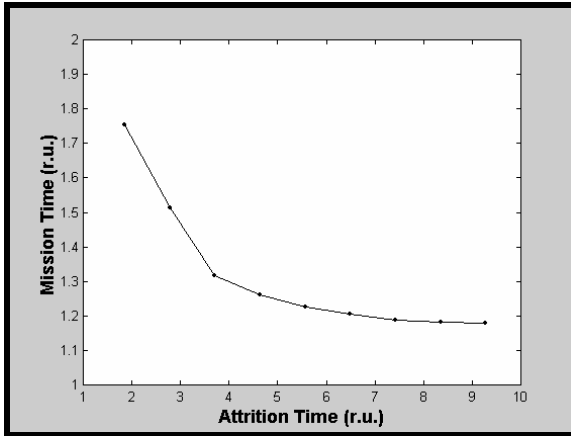


Figure 1. Dependence of the mission time on vehicle attrition. Mission time and attrition time in units of the time taken without attrition. The mission ends when one of the vehicles has acquired all the targets. Simulation performed on a 100x100 target field, with 50 targets, 10 vehicles, unit speed, and infinite range.

The performance of the algorithm is sensitive to the communication range (Figure 2). Full coordination arises when the communication range is comparable to the size of the target field. Mission time increases exponentially fast down to 10% of the field size at which point the vehicles are effectively ex-communicado. The rate of communication loss depends on inter-target spacing and is independent of inter-vehicle spacing.

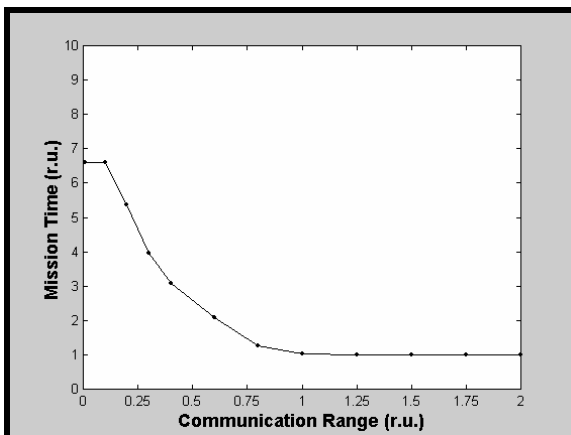


Figure 2. Dependence of mission time on comm range. Mission time in units of time taken with infinite range. Comm range in units of field size. Simulation using 100x100 target field, 50 targets, 10 vehicles, unit speed, and zero attrition.

Mission time increases as the square root of the number of targets (Figure 3) and inversely with the number of vehicles (Figure 4). Without attrition and with perfect communication, the vehicles search the target field uniformly. Because individual vehicles select the target nearest to them, the transit time between targets scales as $1/\sqrt{T}$. Mission time scales as $\sqrt{T}/N$ because each vehicle acquires on average T/N of the T targets.

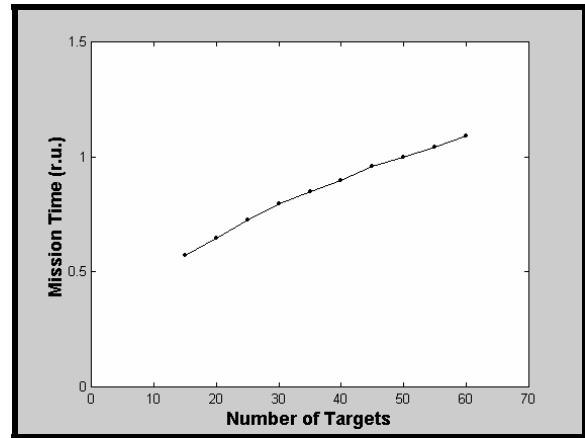


Figure 3. Dependence of mission time on number of targets. Regression gives an exponent of ~ 0.5 . Mission time in units of time taken with 50 targets. Simulation using 100x100 target field, 10 vehicles, unit speed, zero attrition, and infinite comm range.

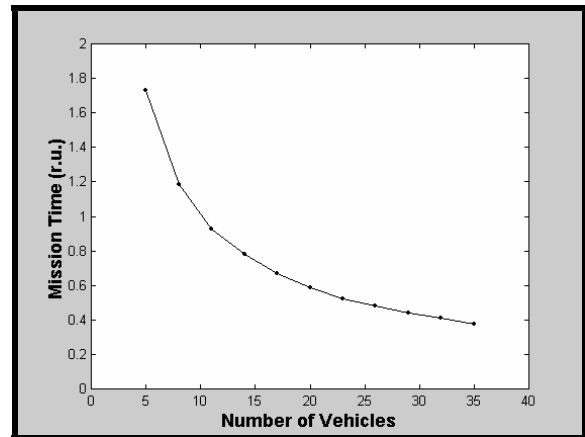


Figure 4. Dependence of mission time on number of vehicles. The exponent (-0.8) indicates sub-optimal coordination. Mission time in units of time taken with 10 vehicles. Simulation using 100x100 target field, 50 targets, unit speed, zero attrition, and infinite comm range.

Plume Monitoring

Background

Underwater plumes are difficult to forecast. The major reason for this deficiency is that initiation, transport, and dissipation of plumes are complex events that can change rapidly in space and time. Remote monitoring tools such as satellites are ineffective for most plumes because both the source of the plume and its basin are located below the surface. Traditional in-situ monitoring methods, such as towed and moored systems, are also ineffective. Towed systems do not provide sufficient data at appropriate spatial or temporal resolution to develop predictive capabilities; multi-point moored sensing arrays provide better resolution but have long deployment times. UUVs do not have these disadvantages. Used in teams, they should provide the needed high-resolution data.

Snake algorithm

The algorithm we propose is an image processing technique called topological snakes. The snake concept is that of a chain of nodes *endowed with dynamics*. Given proper dynamics, the chain finds selected objects in an image and wraps itself around them.

Snakes minimize a certain functional. In image processing, the functional generally depends on the first and second derivatives of the path and on the gradient of the image density. The path that minimizes this functional satisfies an equation that reflects the mechanical balance between the elasticity and rigidity of the curve and the gradient of the image.⁸ One way to solve this equation is to perform a time-dependent gradient flow:

$$\partial_t X = \alpha X_{ss} - \beta X_{ssss} + \nabla P \quad (1)$$

whose steady-state minimizes the functional. ∇P is a mixture of terms that push the vehicles towards the edge of the plume, spread them evenly, and account for their speed.⁹⁻¹¹ This PDE is the motivation for the proposed plume monitoring algorithm.

The link from image processing to multi-vehicle control is that each vehicle is a node in the chain. The control law is found by finite-differencing the PDE, i.e. each vehicle uses snake-like update equations to compute its desired heading.¹² The equation is fourth order in space, hence the discretization introduces an interaction between each vehicle and its four nearest neighbors.

Results

After initial deployment at arbitrary locations, the vehicles quickly converge to the edge of the plume and maintain a lock on it (Figure 5). With minor modifications, the vehicles can converge to lines of arbitrary density.

The algorithm is robust to vehicle loss. After loss of a link, the nearest links simply reconnect at the point of rupture.

The algorithm is bandwidth efficient. It only needs position and concentration from the nearest neighbors. The relative scale of the plume features relative to the speed of the vehicles determines the rate of transmission.

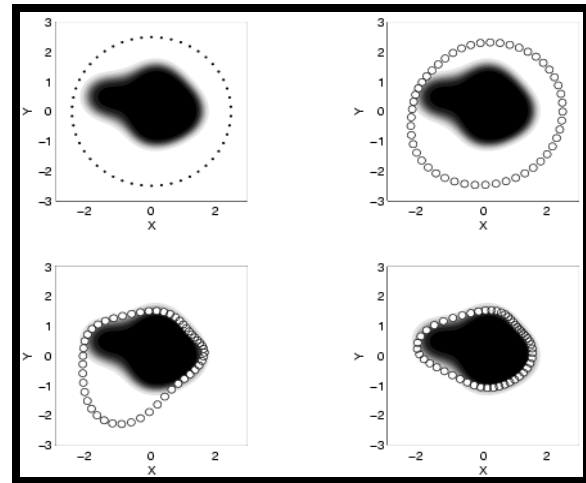


Figure 5. Snake algorithm. Each dot is a moving vehicle. The vehicles are launched outside the plume (top left); they converge to the edge (top right and bottom left), and maintain a lock on it (bottom right).

We have designed a variant that uses no bandwidth; it still produces convergence to the perimeter of the plume (Figure 6). Since

the vehicles do not know about each other, they tend to clump instead of spreading out. This variant is important because it works even if only one vehicle remains. It also opens the door to an algorithm where data acting as a performance accelerator, i.e. one that defaults to the zero-bandwidth case when no data is available and pushes the vehicles apart when it is.

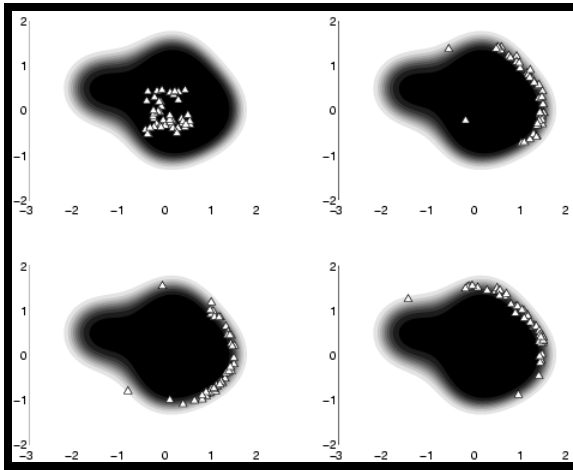


Figure 6. Zero-bandwidth algorithm.

4. CONCLUSION

We have presented vehicle coordination algorithms that work despite loss of vehicles and communication. These algorithms are example of the "fractional bandwidth" principles. Fractional bandwidth is valuable when centralized control is not available and when the attrition rate is high. Our simulations indicate that by sharing minimum information, vehicles can substantially augment their mission effectiveness. We are in the process of migrating these algorithms to a live multi-UUV system.

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REFERENCES

1. B. Fletcher. UUV Masterplan: A Vision for Navy UUV Development. *Proceedings Oceans 00 Conference*. 2000.
2. M.R. Blackburn, R.T. Laird, and H.R. Everett. Unmanned Ground Vehicles: Lessons Learned. *SPAWAR Technical Report 1869*. 2001.
3. B. Hobson et al., 2003,. Field Results of Multiple UUV Deployments, UUST this issue.
4. NAVSEA Indian Head. Request for Proposal: Man Transportable Robotic System, *NAVSEA Indian Head Contract Office*. 2002.
5. See for example: R.D. Rhinehart, VSW/SZ MCM Reconnaissance Concept Assessment and Development, *ONR Ocean Atmosphere Space Annual Reports*. 2001; A.H. Rupert, Tactile Technologies for VSW/SZ MCM Missions. *ONR Ocean Atmosphere Space Annual Reports*. 1999; D. Darling and J. Jalbert. Morpheus AUV Development. *ONR Ocean Atmosphere Space Annual Reports*. 2001; D.R. DelBalzo et al. Environmental Effects on MCM Tactics Planning. *Proceedings Oceans 02 Conference*. 2002; T.C. Liu, H. Schmidt. AUV-Based Seabed Target Detection and Tracking. *Proceedings Oceans 02 Conference*. 2002.
6. A.J. Healey, Y. Kim. Control And Random Searching With Multiple Robots, *Proceedings IEEE CDC 00 Conference*. 2000; A. Healey. Application of Formation Control for Multiple Vehicle Robotic Minesweeping. *Proceedings IEEE CDC 01 Conference*. 2001.
7. B. Cook, D. Marthaler, C. Topaz, and A.L. Bertozzi, and M. Kemp, 2003, *Fractional Bandwidth Reacquisition Algorithms for VSW-MCM*, in Multi-Robot Systems Workshop, Washington DC.
8. M. Kass, A. Witkin, and D. Terzopoulos, 1988, *Snakes: Active Contour Models*, International J. of Computer Vision, p.321.
9. L. Cohen, 1991, *On Active Contour Models and Balloons*, Comput. Vision, Graphics, and

Image Processing: Image Understanding, 53-2.

10. L. Cohen and I. Cohen, 1993, *Finite-Element Methods for Active Contour models and Balloons for 2-D and 3-D Images*, IEEE Transactions on Pattern Anal. and Machine Intelligence 15-11.

11. H. Levine, W.J. Rappal and I. Cohen, 2001, *Self Organization in Systems of Self Propelled Particles*, Phys Rev E 63, p. 17101.

12 D. Marthaler and A. L. Bertozzi, 2003, *Collective Motion Algorithms for Determining Environmental Boundaries*, submitted to Autonomous Robots, special issue on Swarming.