VARIABLE STRUCTURE CONTROL (VSC) OF AN AUTONOMOUS UNDERWATER VEHICLE

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Abstract: The Variable Structure Control (VSC) problem of a kind of controllable regular multi-input nonlinear system is researched in this paper. For the controllable regular multi-input nonlinear system, a decentralized control strategy is selected. This paper applies the VSC method to an Autonomous Underwater Vehicle (AUV). Dynamic model of AUV with 5 directions of freedom (DOF) (without rolling) is built. In order to use the controllable regular multi-input nonlinear system VSC method, the dynamic model with 5 DOF is regularized into controllable regular form, and on this the given motion following model is regularized. AUV's position control problem and velocity control problem are researched. During the design of AUV's position controller, the pole setting method and the quadratic form optimization method are used to design switching functions, and tendency ration strategy is presented to eliminate chattering. The design of AUV's velocity controller is same to position control, but during the design of switching functions only the pole setting method is presented. Simulation results show that the controllers presented in this paper can not only effectively restrict the effect of nonlinearity and coupling, but also have strong adaptability and high control precision.

Key words：Autonomous Underwater Vehicle (AUV), variable structure control (VSC), controllable regular form.

1 Introduction

As an important means to explore oceans and defend oceans from attacking, recently Autonomous Underwater Vehicles (AUVs) develop fast. They arose high recognition in those developed countries. During past years, a lot of AUVs have been designed and constructed. During the design of AUVs, the control problem is very important. To an AUV, motion control is a basic component. To control an AUV with high precision is the precondition to realize the assignment of underwater searching, exploration, avoiding obstacles and stopping at giving point.

There are many approaches to the problem of motion control for AUV, ranging from traditional control methods to modern control methods to a variety of neural network-based architectures. Although many working systems exist, there is still a need to improve their performance and to adapt them to new vehicles.

An Autonomous Underwater Vehicle (AUV) is a seriously nonlinear system with strong coupling in every displacement. In general, it is difficult to get its precise moving equations. As an integrate control method, the merit of variable structure control (VSC) method lies in that it can use imprecise mathematics models to design the controller. Its sliding motion on a predefined hyperplane has quite strong robustness and is unchangeable to the external disturbance, so it is suitable to control AUVs.

This paper applies the VSC method to an AUV. AUV's position control problem and velocity control problem are researched and simulation experiment is carried out. In detail, the author studies the following problems:

Firstly, the VSC problem of a kind of controllable regular multi-input nonlinear system is researched. For the controllable regular multi-input nonlinear system, a decentralized control strategy is selected. Two methods of designing the switching function are presented: the pole setting and the quadratic form optimization. A kind of tendency ration strategy is presented to eliminate chattering.

Secondly, this paper applies VSC method to an AUV who uses five thrusts, two rudders and two wings to offer forces needed by it. Dynamic model of AUV with 5 directions of freedom (DOF) (without rolling) is built. In order to use the controllable regular multi-input nonlinear system VSC method, the dynamic model with 5 DOF is regularized into controllable regular form, and on this the given movement following model is regularized. AUV's position controller and longitudinal velocity controller are researched.

During the design of AUV's position controller, the pole setting method and the quadratic form optimization method are used to design switching functions, and tendency ration strategy is presented to eliminate chattering. The design of AUV's velocity controller is same to position controller, but during the design of switching functions only the pole setting method is presented.

At last, simulation experiments are done to verify the VSC designed, where the AUV has the parameter uncertainties in system modeling. Simulation experiments are carried out at two situations: in blue and whisht water and in water with current and adding noise.

Simulation results show that the controllers presented in this paper can not only effectively restrict the effect of nonlinearity and coupling, but also have strong adaptability and high control precision.

2 Controllable regular multi-input nonlinear system and its control switching mode

It is very important to the control of nonlinear system to change a nonlinear system into controllable regular form, for as it is changed into controllable form, the construction of VSC will be very easy. Moreover, changing a nonlinear system into controllable regular form provides the possibility of realizing uncoupling control.

2.1 Controllable regular form

Controllable regular multi-input nonlinear system can be described as follows[1]:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}, \quad \mathbf{x} \in R^n, \quad \mathbf{u} \in R^m \tag{2.1}$$

where

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathbf{A_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{A_m} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \boldsymbol{\alpha}_1(\mathbf{x}) \\ \vdots \\ \boldsymbol{\alpha}_m(\mathbf{x}) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \vdots \\ \boldsymbol{\beta}_m \end{bmatrix} \qquad (2.2)$$

$$\mathbf{A}_i = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I}_{(n_i-1)} \\ \hline \multicolumn{2}{c}{\mathbf{0}} \end{array}\right], \quad \boldsymbol{\alpha}_i(\mathbf{x}) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \alpha_i^0(\mathbf{x}) \end{bmatrix},$$

$$\boldsymbol{\beta}_i = \begin{bmatrix} 0 & & \cdots & & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \underset{(i)}{1} & \cdots & 0 \end{bmatrix} (i = 1, \cdots, m)$$

where $\mathbf{A}_i$, $\boldsymbol{\beta}_i$, $\boldsymbol{\alpha}_i(\mathbf{x})$ is separately a $n_i \times n_i$, $n_i \times m$, $n_i \times 1$ dimension matrix, $\sum_{i=1}^{m} n_i = n$, $\mathbf{I}_{(n_i-1)}$ is a $(n_i - 1)$ dimension identity matrix. $\mathbf{x}$, the state vector of the system, can be described as follows:

$$\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_m^T]^T$$

$$\mathbf{x}_i = [x_{i1}, x_{i2}, \cdots, x_{in_i}]^T, i = 1,2,\cdots,m$$

then the system (2.1) can be disassembled into m subsystems with n_i dimension separately. That is:

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \boldsymbol{\alpha}_i(\mathbf{x}) + \mathbf{b}_i^0 u_i \qquad (2.3)$$

where $\mathbf{b}_i^0 = [0,0,\cdots,0,1]^T$.

2.2 Control switching mode of controllable regular system

For the controllable regular system in eq. (2.1), a decentrialized sliding mode control strategy is selected. That is, the system is disassembled into m subsystems in eq. (2.3), and let each subsystem corresponds to a control element, so the original system is simplified into m single-input subsystem. For each subsystem, we design a switching function $S_i(\mathbf{x}_i)(i = 1,2,\cdots,m)$, which is only related to the state vector of this subsystem. Then we design control strategies to let that there is sliding mode at each switching surface. There will be m sliding switching surfaces with $(n_i - 1)$ dimension, and their intersection is the balance point of the system expressed in eq. (2.1), that is, $\mathbf{x} = \mathbf{0}$.

As the state vector of each subsystem of the controllable regular system is a phase variable, we can design a linear switching function for the system as follows:

$$S_i(\mathbf{x}_i) = [C_{i1}x_{i1} + C_{i2}x_{i2} + \cdots + C_{i(n_i-1)}x_{i(n_i-1)} + C_{in_i}x_{in_i}], C_{in_i} = 1, (i = 1,2,\cdots,m) \qquad (2.4)$$

Then the switching surface $S_i(\mathbf{x}_i) = 0, (i = 1,2,\cdots,m)$ is the following sliding mode differential equation:

$$S_i(\mathbf{x}_i) = [C_{i1}x_{i1} + C_{i1}\dot{x}_{i1} + \cdots + C_{i1}x_{i1}^{(n_i-2)} + C_{i1}x_{i1}^{(n_i-1)}] = 0 \qquad (2.5)$$

When the system runs into the intersection of m switching surfaces-- S_0, it will be decoupled into m independent subsystems.

3 AUV's dynamic model with 5 DOF and its regularization

3.1 Modeling

The AUV discussed in this paper has five thrusters (two longitudinal main thrusters at tail, two vertical thrusters and one lateral thruster at foreside), two forward symmetrical wings and two afterward symmetrical rudders. With these manipulative facilities, the rolling motion of the AUV is always quite small. So this paper uses a five DOF AUV hydrodynamic model and the rolling motion isn't controlled in this paper.

We transpose the derivative term of the AUV's five DOF equation[2] to the left of the equation, and the rest terms to the right. After arrangement, we express the equation in vectors as follows:

$$\mathbf{E}\dot{\mathbf{X}} = \mathbf{F}_{vis} + \mathbf{F} \qquad (3.1)$$

where

$$\mathbf{E} = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & mz_G & 0 \\ 0 & m - Y_{\dot{v}} & 0 & 0 & mx_G - Y_{\dot{r}} \\ 0 & 0 & m - Z_{\dot{w}} & -mx_G - Z_{\dot{q}} & 0 \\ mz_G & 0 & -mx_G - M_{\dot{w}} & I_y - M_{\dot{q}} & 0 \\ 0 & mx_G - N_{\dot{v}} & 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}$$

$\mathbf{F}_{vis}$ is the non-inertial hydrodynamic force, which is expressed as:

$$\mathbf{F}_{vis} = [X_{vis} \quad Y_{vis} \quad Z_{vis} \quad M_{vis} \quad N_{vis}]^T$$

where

$$X_{vis} = -m \cdot [(-vr + wq)] + [X_{qq}q^2 + X_{rr}r^2] + [X_{vr}vr + X_{wq}wq] + [X_{uu}u^2 + X_{vv}v^2 + X_{ww}w^2]$$

$$Y_{vis} = -m \cdot ur + Y_{qr}qr + [Y_{vq}vq + Y_{wr}wr] + [Y_r ur + Y_{v|r|}\frac{v}{|v|}\left|(v^2 + w^2)^{1/2}\right||r|] + [Y_0 u^2 + Y_v uv + Y_{v|v|}v\left|(v^2 + w^2)^{1/2}\right|] + Y_{vw}vw$$

$$Z_{vis} = \mathrm{m} \cdot \mathrm{uq} + Z_{rr}r^2 + Z_{vr}vr + [Z_q uq + Z_{w|q|}\frac{w}{|w|}\left|(v^2 + w^2)^{1/2}\right||q|] + [Z_0 u^2 + Z_w uw + Z_{w|w|}w\left|(v^2 + w^2)^{1/2}\right|] + [Z_{|w|}u|w| + Z_{ww}\left|w(v^2 + w^2)^{1/2}\right|] + Z_{vv}v^2$$

$$M_{vis} = [M_{rr}r^2 + M_{q|q|}q|q|] + M_{vr}vr + [M_q uq + M_{|w|q}\left|(v^2 + w^2)^{1/2}\right|q] + [M_0 u^2 + M_w uw + M_{w|w|}w\left|(v^2 + w^2)^{1/2}\right|] + [M_{|w|}u|w| + M_{ww}\left|w(v^2 + w^2)^{1/2}\right|] + M_{vv}v^2$$

$$N_{vis} = [N_{qr}qr + N_{r|r|}r|r|] + [N_{wr}wr + N_{vq}vq] + [N_r ur + N_{|v|r}\left|(v^2 + w^2)^{1/2}\right|r] + [N_0 u^2 + N_v uv + N_{v|v|}v\left|(v^2 + w^2)^{1/2}\right|] + N_{vw}vw$$

As we assume the gravity of the AUV is equal to the buoyancy and ignore the external influence, $\mathbf{F}$ is composed of forces produced by thrusters, wings and rudders. That is, $\mathbf{F}$ is the output of the controller, which is expressed as follows:

$$\mathbf{F} = \begin{bmatrix} F_x & F_y & F_z & M & N \end{bmatrix}^T .$$

Use the inverse matrix of the coefficient matrix $\mathbf{E}$ to multiply two sides of the equation (3.1) simultaneity, we get

$$\dot{\mathbf{X}} = \mathbf{E}^{-1}(\mathbf{F}_{vis} + \mathbf{F}) \tag{3.2}$$

3.2 Regularization of AUV's dynamic model with 5 DOF

AUV is a nonlinear system with strong coupling between directions of freedom, and one of our targets when designing the controller is to realize uncoupling control. It provides us with effective and feasible projects for its VSC to change the AUV's model into controllable regular form.

In the AUV's hydrodynamic model with five DOF, we introduce a state vector $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1^T & \mathbf{x}_2^T & \mathbf{x}_3^T & \mathbf{x}_4^T & \mathbf{x}_5^T \end{bmatrix}^T$, where

$$\begin{cases} \mathbf{x}_1 = \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} x_{11} \\ \dot{x}_{11} \end{bmatrix} \\ \mathbf{x}_2 = \begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} x_{21} \\ \dot{x}_{21} \end{bmatrix} \\ \mathbf{x}_3 = \begin{bmatrix} z \\ w \end{bmatrix} = \begin{bmatrix} x_{31} \\ x_{32} \end{bmatrix} = \begin{bmatrix} x_{31} \\ \dot{x}_{31} \end{bmatrix} \\ \mathbf{x}_4 = \begin{bmatrix} \theta \\ q \end{bmatrix} = \begin{bmatrix} x_{41} \\ x_{42} \end{bmatrix} = \begin{bmatrix} x_{41} \\ \dot{x}_{41} \end{bmatrix} \\ \mathbf{x}_5 = \begin{bmatrix} \psi \\ r \end{bmatrix} = \begin{bmatrix} x_{51} \\ x_{52} \end{bmatrix} = \begin{bmatrix} x_{51} \\ \dot{x}_{51} \end{bmatrix} \end{cases},$$

then the equation (3.2) can be changed into the following controllable regular form composed of five subsystems:

$$\begin{cases} \dot{\mathbf{x}}_1 = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{x}_1 + \begin{bmatrix} 0 \\ \alpha_1^0(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 \\ \dot{\mathbf{x}}_2 = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{x}_2 + \begin{bmatrix} 0 \\ \alpha_2^0(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 \\ \dot{\mathbf{x}}_3 = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{x}_3 + \begin{bmatrix} 0 \\ \alpha_3^0(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3 \\ \dot{\mathbf{x}}_4 = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{x}_4 + \begin{bmatrix} 0 \\ \alpha_4^0(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_4 \\ \dot{\mathbf{x}}_5 = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{x}_5 + \begin{bmatrix} 0 \\ \alpha_5^0(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_5 \end{cases}, \tag{3.3}$$

where $\mathbf{u}$ is the control command, and

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 \end{bmatrix}^T = \mathbf{E}^{-1}\mathbf{F}$$

$$\boldsymbol{\alpha}(\mathbf{x}) = \begin{bmatrix} \alpha_1^0(\mathbf{x}) & \alpha_2^0(\mathbf{x}) & \alpha_3^0(\mathbf{x}) & \alpha_4^0(\mathbf{x}) & \alpha_5^0(\mathbf{x}) \end{bmatrix}^T = \mathbf{E}^{-1}\mathbf{F}_{vis} .$$

So we get

$$\mathbf{F} = \mathbf{E}\mathbf{u} \tag{3.4}$$

3.3 Regularization of AUV's dynamic model of following given motion

Suppose the desired state variable is

$$\mathbf{x}_d(t) = \begin{bmatrix} \mathbf{x}_{1d}^T & \mathbf{x}_{2d}^T & \mathbf{x}_{3d}^T & \mathbf{x}_{4d}^T & \mathbf{x}_{5d}^T \end{bmatrix}^T ,$$

where

$$\begin{cases} \mathbf{x}_{1d} = \begin{bmatrix} x_d \\ u_d \end{bmatrix} = \begin{bmatrix} x_{11d} \\ x_{12d} \end{bmatrix} = \begin{bmatrix} x_{11d} \\ \dot{x}_{11d} \end{bmatrix} \\ \mathbf{x}_{2d} = \begin{bmatrix} y_d \\ v_d \end{bmatrix} = \begin{bmatrix} x_{21d} \\ x_{22d} \end{bmatrix} = \begin{bmatrix} x_{21d} \\ \dot{x}_{21d} \end{bmatrix} \\ \mathbf{x}_{3d} = \begin{bmatrix} z_d \\ w_d \end{bmatrix} = \begin{bmatrix} x_{31d} \\ x_{32d} \end{bmatrix} = \begin{bmatrix} x_{31d} \\ \dot{x}_{31d} \end{bmatrix} \\ \mathbf{x}_{4d} = \begin{bmatrix} \theta_d \\ q_d \end{bmatrix} = \begin{bmatrix} x_{41d} \\ x_{42d} \end{bmatrix} = \begin{bmatrix} x_{41d} \\ \dot{x}_{41d} \end{bmatrix} \\ \mathbf{x}_{5d} = \begin{bmatrix} \psi_d \\ r_d \end{bmatrix} = \begin{bmatrix} x_{51d} \\ x_{52d} \end{bmatrix} = \begin{bmatrix} x_{51d} \\ \dot{x}_{51d} \end{bmatrix} \end{cases},$$

then we get the following state error vector:

$$\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x}_d(t) . \tag{3.5}$$

For the i th subsystem $\mathbf{x}_i$, the error is

$$\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_{id}(t) , \tag{3.6}$$

that is,

$$\dot{\mathbf{e}}_i = \begin{bmatrix} \dot{e}_{i1} \\ \dot{e}_{i2} \end{bmatrix} = \begin{bmatrix} \dot{x}_{i1} - \dot{x}_{i1d} \\ \dot{x}_{i2} - \dot{x}_{i2d} \end{bmatrix} = \begin{bmatrix} x_{i2} - x_{i2d} \\ \dot{x}_{i2} - \dot{x}_{i2d} \end{bmatrix} = \begin{bmatrix} e_{i2} \\ \dot{x}_{i2} - \dot{x}_{i2d} \end{bmatrix} \tag{3.7}$$

Substitute eq. (3.3) into eq. (3.7), we get the following equation

$$\dot{\mathbf{e}}_i = \begin{bmatrix} 0 & | & 1 \\ \hline & 0 & \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} 0 \\ \alpha_i^0(\mathbf{x}) - \dot{x}_{i2d} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \tag{3.8}$$

The eq. (3.8) is also the controllable regular form.

4 Design of AUV's position controller

To ensure AUV to complete the assignment of searching underwater, exploring, avoiding obstacles and stopping at desired point, AUV's control capability at the ocean circumstance should include the ability of sailing at fixed deepness, to desired direction, at fixed velocity, sailing to given points and following given track, etc. So it is very important to control AUV's position with high control precision.

For an AUV's position VSC, it includes the following three steps: design of switching function, choosing the control strategy and planning AUV's motion. Sometime we must consider the problem of output restricted of controller. We will discuss these problems one by one in following text.

4.1 Design of switching function

We can design a linear switching function matrix for the system as follows:

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \end{bmatrix}^T = \mathbf{C}\mathbf{X} = [\mathbf{C}_1\mathbf{e}_1 \quad \mathbf{C}_2\mathbf{e}_2 \quad \mathbf{C}_3\mathbf{e}_3 \quad \mathbf{C}_4\mathbf{e}_4 \quad \mathbf{C}_5\mathbf{e}_5]^T \tag{4.1}$$

For the i th subsystem presented by eq. (3.8), we can express it as the following form:

$$\dot{e}_{i1} = e_{i2} \tag{4.2a}$$

$$\dot{e}_{i2} = (\alpha_i^0(\mathbf{x}) - \dot{x}_{i2d}) + u_i \tag{4.2b}$$

, and its switching surface is

$$s_i = C_{i1}e_{i1} + C_{i2}e_{i2} = C_{i1}e_{i1} + e_{i2} = 0 \quad (4.3)$$

To design the i th switching surface expressed by eq. (4.3) is equal to seek the feedback of eq. (4.2a) to let the final sliding mode has good performance, that is, to find a suitable value for C_{i1}.

Two methods of designing the switching function are presented: the pole setting and the quadratic form optimization.

4.1.1 Pole setting method

We know that to a controllable regular system, it has been proven that its poles of final sliding mode can be set arbitrarily.

Pole-setting method to design the switching function is to decide the value of C_{i1} to let the final sliding mode of the i th subsystem has the pre-given pole λ_i. On the other hand, the pole of equation $\dot{e}_{i1} = e_{i2} = -C_{i1}e_{i1}$ is $-C_{i1}$. So $\lambda_i = -C_{i1}$, and we get the value of C_{i1}.

Considering that when AUV sails underwater, it requires that the responses of surge subsystem and heave subsystem be fast, the response of sway subsystem be a little slow, and the responses of pitch subsystem and yaw subsystem be slow, we choose the following poles for AUV:

$$\begin{cases} \lambda_1 = -0.06 \\ \lambda_2 = -0.05 \\ \lambda_3 = -0.06 \\ \lambda_4 = -0.03 \\ \lambda_5 = -0.03 \end{cases}, \quad (4.4)$$

so we get:

$$\begin{cases} C_{11} = 0.06 \\ C_{21} = 0.05 \\ C_{31} = 0.06 \\ C_{41} = 0.03 \\ C_{51} = 0.03 \end{cases}. \quad (4.5)$$

4.1.2 Method of the quadratic form optimization

The realization of optimization control is that under some given condition to complete desired assignment of control to satisfy that some ability target of the system is the most excellent. Here using the concept of optimization control, we introduce the method of the quadratic form optimization to design the switching function of the VSC.

Suppose the quadratic form optimization target of i th subsystem eq. (4.2) has the following form:

$$J = \int_{t_0}^{+\infty} \mathbf{x}_i^T \mathbf{Q} \mathbf{x}_i dt \quad (4.6)$$

where

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix},$$

and $\mathbf{Q}$ is a canonical matrix with 2×2 dimension, Q_{11} is a nonzero constant, Q_{22} is a positive constant, $Q_{12} = Q_{21}$. We do the following transform:

$$\begin{bmatrix} e_{i1} \\ \gamma \end{bmatrix} = \begin{bmatrix} e_{i1} \\ Q_{22}^{-1}Q_{21}e_{i1} + e_{i2} \end{bmatrix}, \quad (4.7)$$

then the eq. (4.2a) can be transformed into:

$$\dot{e}_{i1} = -Q_{22}^{-1}Q_{21}e_{i1} + \gamma \quad (4.8)$$

and the eq. (4.6) can be transformed into:

$$J = \int_{t_0}^{+\infty} [e_{i1}^2 * (Q_{11} - Q_{12}Q_{22}^{-1}Q_{21}) + \gamma^2 Q_{22}] dt. \quad (4.9)$$

In fact, we can see that in the integral term of the eq. (4.9) the former term $e_{i1}^2 * (Q_{11} - Q_{12}Q_{22}^{-1}Q_{21})$ is error target and the later term $\gamma^2 Q_{22}$ is the energy target of the controller. The quadratic form optimization solution of eq. (4.9) can be expressed as follows:

$$\gamma = -ke_{i1}, \quad k = Q_{22}^{-1}P \quad (4.10)$$

where P is the solution of the following equation:

$$PA_{11}^* + (A_{11}^*)^T P - PQ_{22}^{-1}P + Q_{11}^* = 0,$$

where $A_{11}^* = -Q_{22}^{-1}Q_{21}$, $Q_{11}^* = Q_{11} - Q_{12}Q_{22}^{-1}Q_{21}$.

From eq. (4.7) and eq. (4.10), we get

$$(P + Q_{21})e_{i1} + Q_{22}e_{i2} = 0 \quad (4.11)$$

Suppose eq. (4.11) is the switching surface of i th subsystem. So we get

$$C_{i1} = Q_{22}^{-1}(P + Q_{21}) \quad (4.12)$$

Considering that when AUV sails underwater, it requires that the responses of surge subsystem and heave subsystem be fast, the response of sway subsystem be a little slow, and the responses of pitch subsystem and yaw subsystem be slow, for the first subsystem, that is, the surge subsystem, we choose $\mathbf{Q}_1 = \begin{bmatrix} 0.004 & 0.06 \\ 0.06 & 1 \end{bmatrix}$, then we get

$$A_{11}^* = -Q_{22}^{-1}Q_{21} = 0.06,$$

$$Q_{11}^* = Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} = 0.0004.$$

Use the function *lqr()* which is used to solve the problem of linear quadratic regulator for continuous-time systems provided by MATLAB software, we get the value of $k = 0.0032$. Then according to eq. (4.12), we get $C_{11} = 0.0632$.

On the same theory, for the remainder four subsystems, we choose the following matrices:

$$\mathbf{Q}_2 = \begin{bmatrix} 0.0028 & 0.05 \\ 0.05 & 1 \end{bmatrix}, \quad \mathbf{Q}_3 = \begin{bmatrix} 0.004 & 0.06 \\ 0.06 & 1 \end{bmatrix},$$

$$\mathbf{Q}_4 = \mathbf{Q}_5 = \begin{bmatrix} 0.001 & 0.03 \\ 0.03 & 1 \end{bmatrix},$$

then we get

$$C_{21} = 0.0529, \quad C_{31} = 0.0632, \quad C_{41} = C_{51} = 0.0316 .$$

4.2 Design of control strategy

This paper chooses exponential tendency ratio strategy[1] to design the controller. For the i th subsystem, its switching function is as follows:

$$s_i = C_{i1}e_{i1} + e_{i2} \quad , \tag{4.13}$$

and its exponential tendency ratio control strategy is as follows:

$$\dot{s}_i = C_{i1}\dot{e}_{i1} + \dot{e}_{i2} = -\varepsilon \,\mathrm{sgn}(s_i) - ks_i, \quad \varepsilon > 0, k > 0 \, . \tag{4.14}$$

Substitute the equation $\dot{e}_{i2} = (\alpha_i^0(\mathbf{x}) - \dot{x}_{i2d}) + u_i$ into eq. (4.14), we get the following control command:

$$u_i = -[\alpha_i^0(\mathbf{x}) - \dot{x}_{i2d}] + [-\varepsilon \,\mathrm{sgn}(s_i) - ks_i] - C_{i1}e_{i2} \tag{4.15}$$

Select appropriate ε and k, we can reduce chattering. For the five subsystems, we choose the same value of ε and k, that is , $\varepsilon = 0.0001$, $k = 0.0001$. Decreasing the value of ε and increasing the value of k can accelerate the tendency velocity and decrease chattering.

After calculate the control command, according to eq. (3.4) we can get the thrust command.

4.3 Planning motion of AUV

Form the eq. (4.15), we can see that if we want to get the thrust command, we must know the value of $\dot{x}_{i2d}$. But in the position controller the motion planner just provides the desired position but no accelerations or velocities requirement from current position to the desired position. So in order to get the current time's thrust command we need according to the current time's movement state to plan the next time's movement state. In the position controller, we use pre-planning method proposed in reference [4] to plan the AUV's accleleration. The following is a simple description of the method:

We choose the desired accelerations as follows:

$$\vec{a} = \mathbf{P}\vec{a}_{\max} \tag{4.16}$$

where $\vec{a}_{\max}$ is the maximal acceleration of the AUV, which mainly depends on the ability of manipulative facilities, and

$$\mathbf{P} = \begin{bmatrix} p_1 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 & 0 \\ 0 & 0 & p_3 & 0 & 0 \\ 0 & 0 & 0 & p_4 & 0 \\ 0 & 0 & 0 & 0 & p_5 \end{bmatrix}$$

where

$$\begin{cases} p_1 = \dfrac{p_x}{p_{xy}} \tanh(p_{xy}/2) \\ p_2 = \dfrac{p_y}{p_{xy}} \tanh(p_{xy}/2) \\ p_3 = \tanh(p_z/2) \\ p_4 = \tanh(p_\theta/2) \\ p_5 = \tanh(p_\psi/2) \end{cases} , \quad \begin{cases} p_x = \tilde{S}_x - C_x V_x \\ p_y = \tilde{S}_y - C_y V_y \\ p_z = \tilde{S}_z - C_z V_z \\ p_\theta = \tilde{S}_\theta - C_\theta V_\theta \\ p_\psi = \tilde{S}_\psi - C_\psi V_\psi \\ p_{xy} = \sqrt{p_x^2 + p_y^2} \end{cases} ,$$

$\tilde{S}_x, \tilde{S}_y, \tilde{S}_z, \tilde{S}_\theta, \tilde{S}_\psi$ are seperately x, y, z, θ, ψ directional draught distances, which are defined as follows:

$$\tilde{S}_i = \begin{cases} \tilde{S}_{i\max} & (S_i \ge \tilde{S}_{i\max}) \\ S_i & (-\tilde{S}_{i\max} < S_i < \tilde{S}_{i\max}) \, , i = x, y, z, \theta, \psi \\ -\tilde{S}_{i\max} & (S_i \le -\tilde{S}_{i\max}) \end{cases}$$

where $\tilde{S}_{i\max}$ and C_i are psoitive parameters needing to be decided, and they satify the following equation:

$$\tilde{S}_{i\max} - C_i V_{i\max} = 0 \, , \tag{4.17}$$

that is, when the velocity in i th direction is equal to the maxitial velocity of this direction, the AUV's displacement should be the maxitial draught distances.

According to the eq. (4.17) we can't get the values of $\tilde{S}_{i\max}$ and C_i, but we can get them by solving the following differential equation group:

$$\begin{cases} S_i'' = a_{i\max}(1 - \exp(C_i S_i - S_i))/(1 + \exp(C_i S_i - S_i)), & (t > t_0) \\ S_i = \tilde{S}_{i\max}, & (t = t_0) \\ S_i = V_{i\max} \end{cases} \tag{4.18}$$

4.4 Considition of output restricted of controller

We know that if the output of the controller is bigger than the maxitial force the manipulative facilities can provide, the effect of the controller will be degraded. When this phenomenon occurs in VSC, the situation will be worse. We know that the aim of VSC has two, one is that all motion runs into the switching surface at limited time, the other is that the switching surface is sliding mode. But if the output of the VSC is bigger than the maxitial force the AUV's manipulative facilities can provide, the above aim of VSC will not be achieved.

To solve this problem, we take the following actions. We know that althought the output of VSC is force instruction, the effect of force directly represents as the acceleration. So we can solve the problem by reducing the value of the desired acceleration $\dot{x}_{i2d}$. From the eq. (4.18), we can see that we can properly reduce the value of $a_{i\max}$ to decrease the value of $\dot{x}_{i2d}$ planned. From the eq. (4.15), we know that the value of u_i is decreased, then the output of the VSC is decreased to less than the maxitial force the manipulative facilities can provide.

Based on the above consideration, we choose the following parameters:

Table. 1 parameters for motion planning of AUV

DOF	$a_{i\max}$	$V_{i\max}$	$\tilde{S}_{i\max}$
	m/s ^{2}	m/s	m
x	0.04	0.8	12.0
y	0.022	0.3	5.0
z	0.022	0.3	5.0
θ	0.014	0.060	1.20
ψ	0.028	0.10	1.380

4.5 Simulation

Simulation experiments are carried out at two situations: in blue and whisht water and in water with current and adding noise. The velocity of current added is 0.3m/s at east-north direction. The noise added is Gaussian white noise, its average value is zero, and the error square is 0.0025. This paper just presents the simulation results at the sitilation of in water with current and adding noise. Simulation results are shown in Fig.1 and Fig.2.

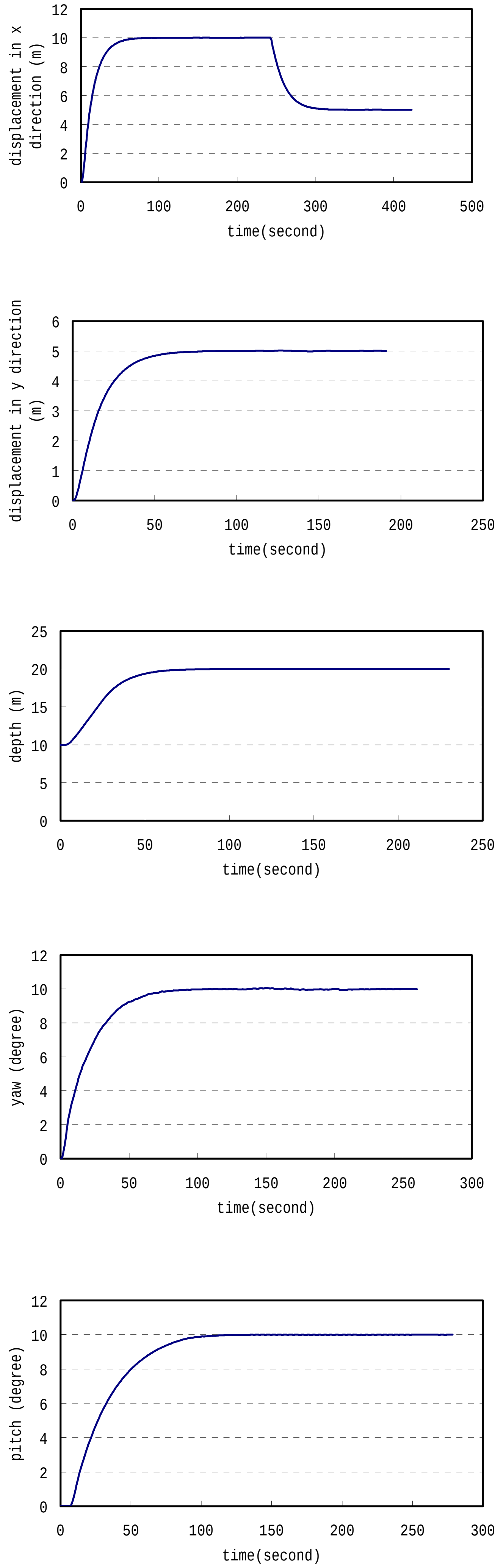

Figure 1. Simulation results of position control with VSC, using the pole setting method to design the switching function, in water with current and adding noise.

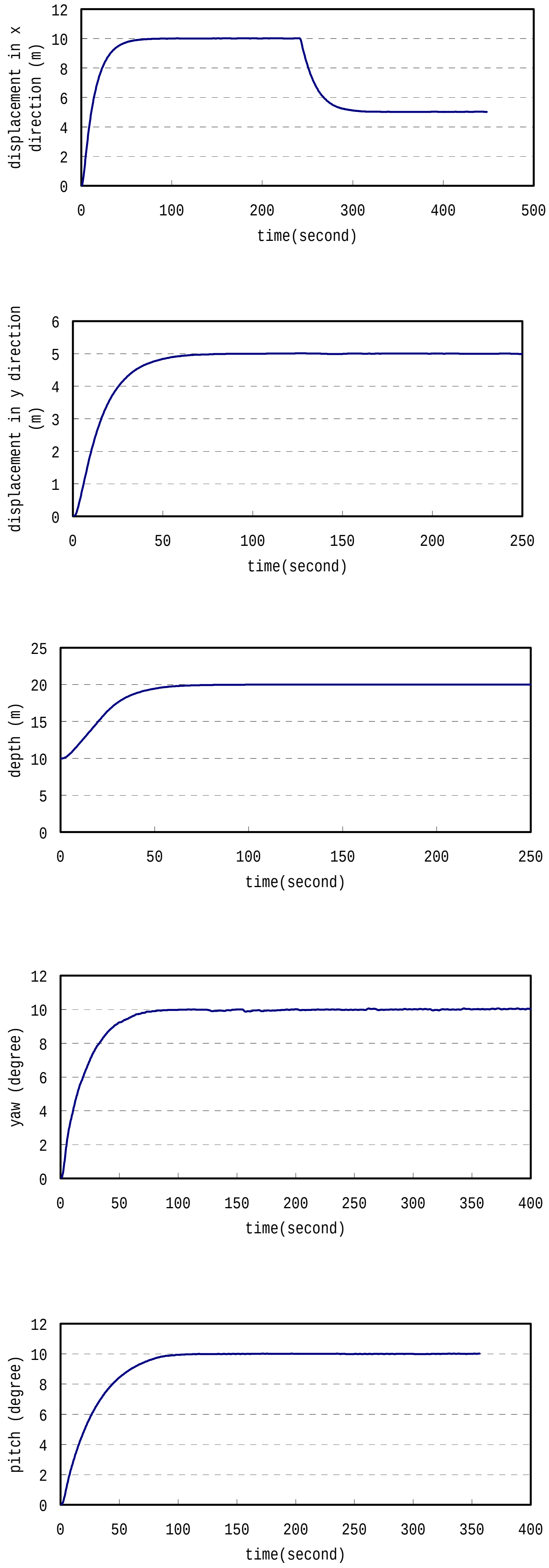

Figure 2. Simulation results of position control with VSC, using the method of the quadratic form optimization to design the switching function, in water with current and adding noise.

We can see that the position controller designed in this paper has strong adaptability and high control precision.

5 Design of AUV's velocity controller

The AUV we designed is designed to sail a long distance. During this process, we expect it to sail at a fixed longitudinal velocity. So in this paper, we design a longitudinal velocity controller.

The design of AUV's velocity controller is same to position controller, but during the design of switching functions only the pole setting method is presented.

5.1 Control strategy

When using the pole setting method to design the velocity controller, based on the same consideration with AUV's position controller, we choose the following pole for the longitudinal velocity controller. That is

$$\lambda_1 = 0.3$$

Exponential tendency ratio control strategy is used. Then we get the following control command:

$$u_1 = -[\alpha_1^0(\mathbf{x}) - \dot{u}_d] + [-\varepsilon \operatorname{sgn}(0.6e_{11} + e_{12}) - k(0.6e_{11} + e_{12})] - 0.6e \qquad (5.1)$$

where $\varepsilon = 0.0001$, $k = 0.001$.

What different to the position controller is that in the velocity controller the error state vector is

$$\dot{\mathbf{e}}_i = \begin{bmatrix} \dot{e}_{i1} \\ \dot{e}_{i2} \end{bmatrix} = \begin{bmatrix} \ddot{x}_{i1} - \ddot{x}_{i1d} \\ \ddot{x}_{i2} - \ddot{x}_{i2d} \end{bmatrix} = \begin{bmatrix} \dot{x}_{i2} - \dot{x}_{i2d} \\ \ddot{x}_{i2} - \ddot{x}_{i2d} \end{bmatrix} \qquad (5.2).$$

5.2 Planning motion of AUV

In the velocity controller, the motion planner just provides the desired velocity but no accelerations or velocities requirement from current velocity to the desired velocity. So in order to get the current time's thrust command we need according to the current time's movement state to plan the next time's movement state.

In this paper we design the acceleration of i th subsystem as follows:

$$a_i = a_{i\max} * \tanh[(V_{id} - V_i)/V_{i\max}] \qquad (5.3)$$

where a_i is the current acceleration planned of the i th subsystem, $a_{i\max}$ is the maxitial acceleration the AUV can provide in the i th DOF, V_i is the current velocity in the i th DOF, V_{id} is the desired velocity, and $V_{i\max}$ is the maxitial velocity in the i th DOF.

From the eq. (5.3), we can see that when the current velocity V_i is far less than the desired velocity V_{id}, the current accleration planned a_i is a little bigger, but when V_i is near to V_{id}, a_i will go to zero, which is coincident with the actual facts.

5.3 Simulation

The same to the simulation experiments of position controller, in the velocity controller, simulation experiments are carried out at two situations: in blue and whisht water and in water with current and adding noise. The current and noise are same to the position controller. The simulation result is show in Figure 3.

From Figure 3. we can see that the velocity controller designed in this paper exists a little steady error, but the steady error is in the allowable error range.

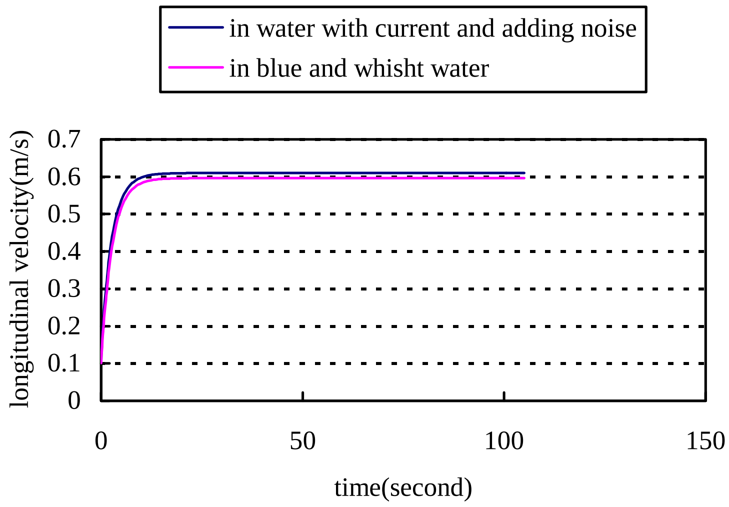

Figure 3. Simulation results of velocity control with VSC, using the pole setting method to design the switching function.

6 Conclusion

This paper applies the VSC method to an AUV with parameter uncertainties. In order to use the controllable regular multi-input nonlinear system VSC method, the dynamic model of AUV with 5 DOF is regularized into controllable regular form, and on this the given motion following model is regularized. AUV's position control problem and longitudinal velocity control problem are researched and simulation experiments are performed. During the design of AUV's position controller, the pole setting method and the quadratic form optimization method are used to design switching functions, and exponential tendency ration strategy is presented to eliminate chattering. The design of AUV's velocity controller is same to position controller, but during the design of switching functions only the pole setting method is presented. Simulation results show that the controller presented in this paper can not only effectively restrict the effect of nonlinearity and coupling, but also have strong adaptability and high control precision.

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