Numerical Calculation of Hydrodynamic Characteristics of Rigid and Flexible Oscillating Tuna-Tail

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Abstract—**The unsteady hydrodynamic characteristics of rigid and flexible oscillating tuna-tail are analyzed numerically by boundary element method. The tuna-tail is considered as a 3-D lunate fin oscillating with the mode combined heaving and pitching in uniform flow. A boundary element method using the hyperboloidal quadrilateral panels is applied to numerically solve the unsteady flow. At each time step, the fin is re-meshed and pressure Kutta condition is satisfied on the trailing edge. The hydrodynamic characteristics periodically change with time. The mean thrust coefficient, mean lateral force coefficient and the propulsive efficiency in one oscillating period can be obtained. The effects of the oscillating amplitude and the phase difference between heaving and pitching are investigated. The numerical results for rigid and flexible fins are compared and analyzed, and they have a good agreement with the experimental data and other calculation.**

I. Introduction

In recent years, the propulsion system imitating the aquatic animal such as tuna or dolphin has been tried to apply on autonomous underwater vehicle. Thus the theoretical research on the propulsive principle of aquatic animal becomes more important and attracted more scholars to make the efforts on it. Lighthill[1] investigated the caudal fin as a flat wing with 2-D airfoil theory and this method was extended to 3-D wing by Chopra[2]. Wu[3] studied the swimming propulsion problem by the potential flow theory. Karpouzian, Spedding & Cheng[4] applied the lifting line theory to the lunate tail. Liu & Bose [5] calculated numerically the performance of caudal fin with quasi-vortex-lattice method. Cheng[6] used 3-D waving plate model to analyze the hydrodynamic characteristics of several kinds of fish caudal fin. Sendberg & Ramamurti[7] analyzed numerically the unsteady fluid dynamics of the tuna-tail and oscillating airfoil using a finite element flow solver based on unstructured grids. Doi & Yhomatsu[8] studied unsteady viscous flow field around a oscillating plate with the method solving 3-D unsteady N-S equation.

In this paper, the unsteady hydrodynamic characteristics of rigid and flexible oscillating tuna-tail are calculated by boundary element method. The tuna-tail is considered as a lunate fin oscillating with the mode combined heaving and pitching in uniform flow. The deformation of flexible fin is assumed according to the lateral force. The effects of pitching amplitude and the phase difference between heaving and pitching on the hydrodynamic characteristics of the fin are investigated. The hydrodynamic characteristics of the rigid fin and flexible fin are compared.

II. Fundamental Theory

For a lifting body moving with a constant velocity $\boldsymbol{V}_0(x,y,z,t)$ in an inviscid, incompressible, and irrotational flow, the flow field perturbed by it can be characterized by a perturbation velocity potential $\phi(t)$, which satisfies the Laplace equation[9]

$$\nabla^2\phi(t)=0 \tag{1}$$

According to Green's identity, at an arbitrary time t the perturbed potential at an arbitrary field point $P(x,y,z,t)$ can be expressed as an integral on the boundary surface of flow field S, which is composed of body surface S_B, wake surface S_W and outer control surface S_∞ that closes flow field:

$$4\pi E\phi(P,t)=\iint_S[\phi(Q,t)\frac{\partial}{\partial n_Q}(\frac{1}{R(P,Q)}) - \frac{\partial\phi(Q,t)}{\partial n_Q}\frac{1}{R(P,Q)}]dS \tag{2}$$

Where

$$E=\begin{cases}0 & for\ \ P\ inside\ S\\ 1/2 & for\ \ P\ on\ S\\ 1 & for\ \ P\ outside\ S\end{cases}$$

R(P,Q) = distance between field point *P(x,y,z,t)* and boundary point $Q(x_0,y_0,z_0,t)$

$\frac{\partial}{\partial n_Q}$ = normal derivative to S at point Q

Considering following boundary conditions:

$$\nabla\phi(t)\to 0, \qquad as \quad S_\infty\to\infty \tag{3}$$

$$\frac{\partial\phi(t)}{\partial n_Q} = -\boldsymbol{V}_0(x,y,z,t)\cdot\boldsymbol{n}_Q \quad on \quad S_B \qquad (4)$$

$$p^+ - p^- = 0$$
$$(\frac{\partial\phi(t)}{\partial n_{Q_1}})^+ - (\frac{\partial\phi(t)}{\partial n_{Q_1}})^- = 0 \qquad on \quad S_W \qquad (5)$$

Thus the integral equation (2) can be written as

$$2\pi\phi(P,t) = \iint_{S_B}\phi(Q,t)\frac{\partial}{\partial n_Q}(\frac{1}{R(P,Q)})dS$$
$$+\iint_{S_W}\Delta\phi(Q_1,t)\frac{\partial}{\partial n_{Q_1}}\frac{1}{R(P,Q_1)}dS \qquad (6)$$
$$+\iint_{S_B}(\boldsymbol{V}_0(t)\cdot\boldsymbol{n}_Q)(\frac{1}{R(P,Q)})dS$$
$$on \quad S_B$$

Where $\Delta\phi(Q_1,t)$ is the potential jump across the wake surface which can be expressed by

$$\Delta\phi(t) = \phi(t)^+ - \phi(t)^- \qquad on \quad S_W \quad (7)$$

For the unsteady problem it changes with the time and along a streamline. However, at an arbitrary time t the $\Delta\phi(t)$ most adjacent to the trailing edge of blade can be determined by Kutta condition. In the present method the following pressure Kutta condition is employed to determine the potential jump $\Delta\phi(t)$ across the wake surface.

$$(\Delta p)_{TE}(t) = p^+_{TE}(t) - p^-_{TE}(t) = 0 \qquad (8)$$

Therefore, the integral equation (6) can be uniquely solved by numerically iterative method.

The perturbed velocities $\boldsymbol{V}(x,y,z,t)$ on the body surface are evaluated by differentiating the velocity potential on the body surface. So that the total velocity is

$$\boldsymbol{V}_t(x,y,z,t) = \boldsymbol{V}_0(x,y,z,t) + \boldsymbol{V}(x,y,z,t) \qquad (9)$$

By Bernoulli's theorem the pressure on the body surface can be expressed as

$$p(t) = p_0 + \frac{1}{2}\rho[|\boldsymbol{V}_0(t)|^2 - |\boldsymbol{V}_t(t)|^2] - \rho\frac{\partial\phi(t)}{\partial t} \qquad (10)$$

The hydrodynamic characteristics are obtained by integrating the pressure on the body surface. Here a three points backward difference scheme is applied to calculate the value of $\frac{\partial\phi(t)}{\partial t}$.

III. Calculation Model

For a lunate caudal fin we take a coordinate system shown in Fig.1. When tuna is swimming in a constant high speed $\boldsymbol{V}_0$, its caudal fin does movement combined heaving in z direction with pitching around y-axis rapidly with small amplitudes. It develops almost all thrust for propulsion. In this case, the movement equations of caudal fin can be written as:

$$\begin{cases} Z(t) = A_z\sin(2\pi ft) \\ \theta(t) = \theta_0\sin(2\pi ft - \phi_0) \end{cases} \qquad (11)$$

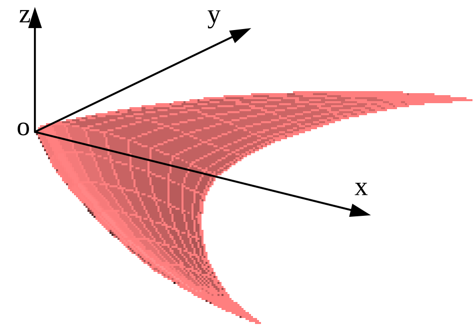

Fig.1 Coordinate system of caudal fin

Corresponding heaving velocity $V_z(t)$ and pitching angle velocity $\omega(t)$ are

$$\begin{cases} V_z(t) = 2\pi fA_z\cos(2\pi ft) \\ \omega(t) = 2\pi f\theta_0\cos(2\pi ft - \phi_0) \end{cases} \qquad (12)$$

Here, A_z and θ_0 is respectively the amplitudes of heaving and pitching; f is the frequency of oscillation; ϕ_0 is the phase difference between heaving and pitching. Advanced velocity is expressed as

$$V_0 = A_0 C_0 \qquad (13)$$

Where C_0 is the character chord of the fin, A_0 is a constant.

Generally the length of tuna is about seven times of the character chord of tail. While A_0=7, advanced velocity is one body length per second. As shown in Fig.2, inflow velocity for the fin is

$$V_1(t) = \sqrt{V_0^2 + V_z(t)^2} \qquad (14)$$

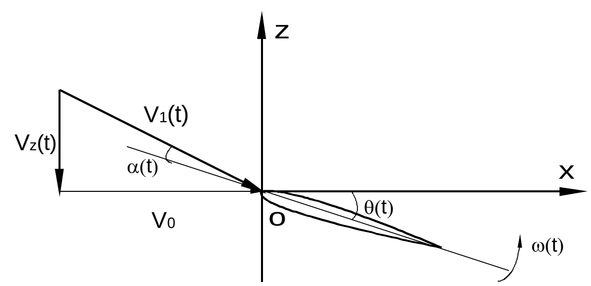

Fig.2 Movement of oscillating caudal fin

When we use the boundary element method introduced above to analyze the fin, we fixed the coordinate system on caudal fin. It moves together with the fin forward and heaving. At arbitrary time step in numerical calculation the fin has to be re-meshed. The velocity of the point on the fin surface is

$$\boldsymbol{V}_p(x,y,z,t) = \boldsymbol{V}_0 + \boldsymbol{V}_z(t) + \boldsymbol{V}_\theta(x,y,z,t) \qquad (15)$$

In above equation $\boldsymbol{V}_\theta$ is the pitching velocity of the point on the fin surface.

By pressure coefficient C_p the thrust coefficient C_x , lateral force coefficient C_z and the moment coefficient C_{my} about y-axis of the fin can be estimated with integrals

$$\begin{cases} C_x = \iint_{S_B} C_p n_x ds \\ C_z = -\iint_{S_B} C_p n_z ds \\ C_{my} = \iint_{S_B} C_p (x n_z - z n_x) ds \end{cases} \quad (16)$$

Here, n_x, n_z are the normal unit vector of the fin surface. These coefficients vary frequently with time. We can measure them by averaged coefficient or corresponding averaged coefficient in one period C_{xm}, C_{zm} and C_{mym}. Therefore, the propulsive efficiency of caudal fin can be derived as

$$\eta = \frac{C_{xm} \cdot V_0}{K_P \cdot f} \quad (17)$$

Here, K_P is the consumed power coefficient defined as

$$K_P = \int_{z(0)}^{z(T)} C_z \cdot dz + \int_{\theta(0)}^{\theta(T)} C_0 C_{my} \cdot d\theta \quad (18)$$

For flexible fin the movement mode is same as the rigid fin. Its shape is changing with the movement. Because the fin corresponds to a thin wing, it will only deform in z-direction under the action of lateral force. From Fig. 9 we know that lateral force of the fin fluctuates in one period. So that the periodically deformation of fin is assumed. The heaving movements of fore end point P_f and the corresponding pitching movement of aft end point Pa are expressed as

$$\begin{cases} Z_f(t) = A_f \sin(2\pi f t) \\ \theta_f(t) = \theta_{f0} \sin(2\pi f t - \phi_f) \end{cases} \quad (19)$$

Where, Z_f is heaving distance, A_f is heaving amplitude, θ_f is pitching angle of aft end point, θ_{f0} is pitching amplitude and ϕ_f is the phase difference between heaving and pitching. According to the lateral force fluctuation, ϕ_f should be equal to $\pi/2$ approximately. Between the points P_f and P_a the deformed distance of chord line of the fin in z-direction is assumed to satisfy the following parabolic equation

$$z(x) = a x^2 \quad (20)$$

But the chord length and the thickness of the flexible fin are maintained be constant. Fig. 3 is the trail curves of the fore end point and the aft end point in a typical situation. A deformation history of the flexible fin is shown in Fig. 4.

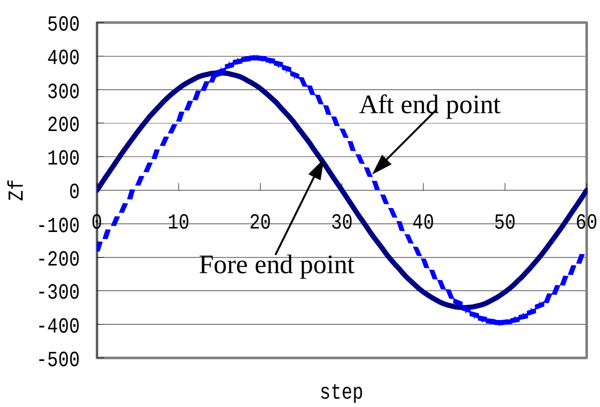

Fig.3 Movement of fore and aft end points

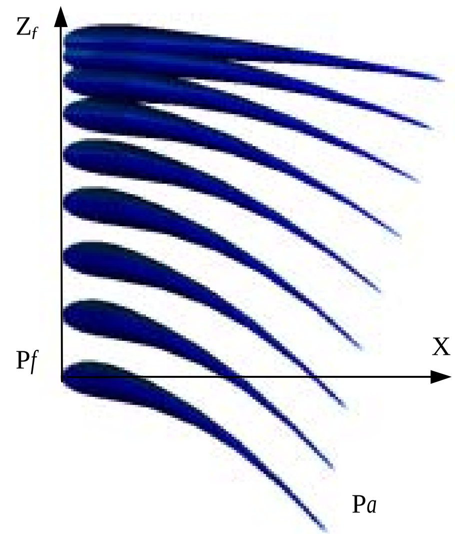

Fig.4 Deform of flexible fin

For lifting problem wake vortices always appears in downstream. Here the linear wake on which the induced velocity of fin has no effect is assumed. The panel arrangement and wake of caudal fin are shown in Fig.5.

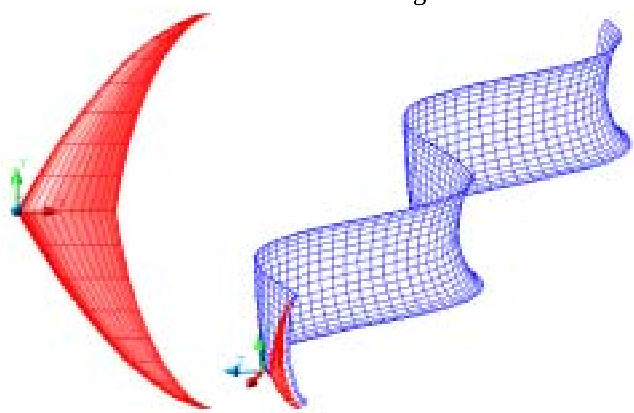

Fig.5 Wake and panel arrangement of caudal fin

IV. NUMERICAL RESULT

At first a rigid rectangular wing oscillating periodically and a flexible rectangular wing moving wavily are calculated to verify present method. The section of rigid wing is NACA0012. Its heaving amplitude $A_z=0.4C_0$ and pitching amplitude $\theta_0=0$. The induced frequency is defined as $k=2\pi f C_0/V_0$. The aspect ratio of fin is taken as 8.0. From Fig. 6 we can see that present calculated result has a good agreement with the result in reference [10].

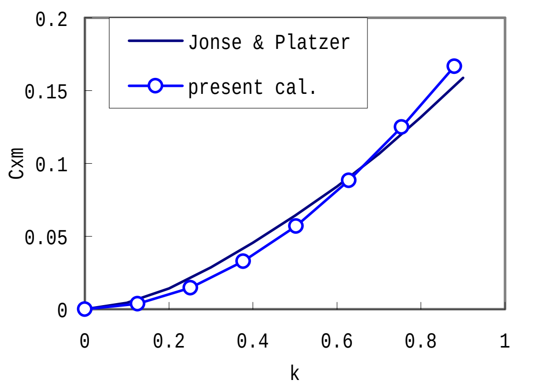

Fig.6 Calculated result for rigid rectangular wing

On the other hand, the aspect ratio of flexible rectangular wing is 1.333. Its section is NACA0018. The movement of this wing follows equation

$$Z_f(x,t) = Z_0(x)\sin(x\phi_f - 2\pi ft) \qquad (21)$$

Where, $Z_0(x)$ is the oscillating amplitude at chordwise position x. In reference [11], the values of $Z_0(x)$ at six points along chordwise are given. Their movements are shown in Fig.7. The calculated thrust by present method is compared with measured data[11] in Fig.8. They have same changing tendency, but the difference between them is evident. Especially for very low speed it is large. The reason of this difference is considered due to the wing does wavy movement but not standard oscillating movement.

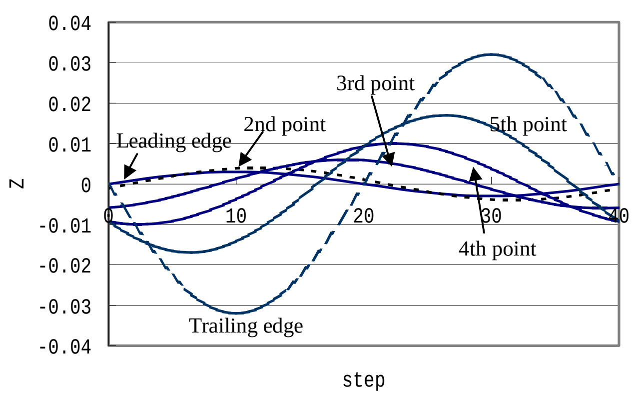

Fig.7 Movements of several points on the wing

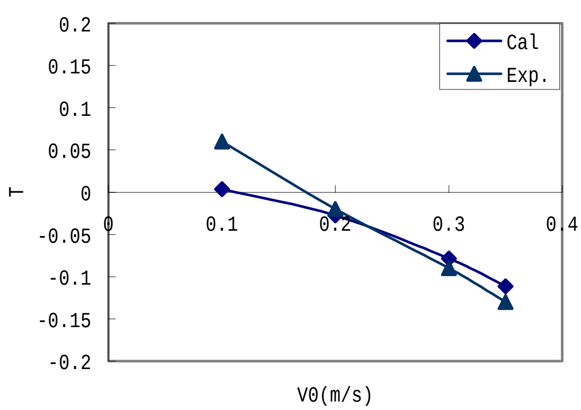

Fig.8 Calculated thrust compared with measured data

With the present method, we analyzed the caudal fin with the contour of black tuna tail. The section is NACA0018 foil. Generally we should take the calculating time step Δt that makes the corresponding angle step $2\pi f\Delta t$ be smaller than 9 degrees. For case of θ_e=20°, A_z=C_0, ϕ_0=90°, f=0.5, V_0=6C_0, we take Δt=6°/2πf. Then after three period iterations we obtained the convergent numerical results. Fig.9 is varying hydrodynamic coefficients in one period. These hydrodynamic coefficients vary periodically with time. The averaged values of the lateral force coefficient and moment coefficient in one period approach to zero. But the averaged thrust is positive.

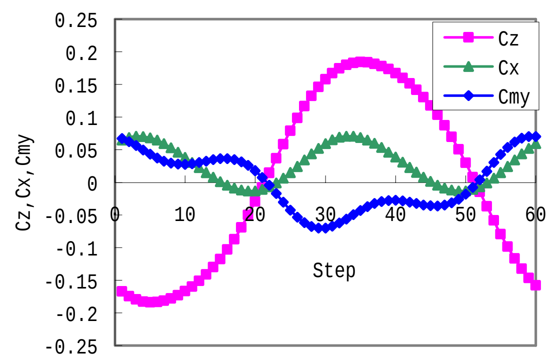

Fig.9 Varying hydrodynamic coefficients with time

The hydrodynamic characteristics of the lunate fin are shown in Fig.10 as the functions of Strouhal number St=2fA_z/V_0. Mean lateral force coefficient C_{zm} is an equivalent averaged coefficient in the meaning of consumed power. Clearly mean thrust coefficient C_{xm} and mean lateral force coefficient C_{zm} increase with the decreasing of 1/St and the propulsive efficiency has a maximum value at 1/St=5.

In some research the hydrodynamic characteristics of the oscillating fin are considered to depend on St only. However, since the viscous effect and unsteady flow effect are included, the characteristics should be different at same St for different frequency.

Fig.11 is the effect of pitching amplitude on the hydrodynamic characteristics of caudal fin. In the calculated situation the propulsive efficiency achieve an optimum value at θ_0=25°. In Fig.12 we can see that the hydrodynamic characteristics of caudal fin varies with the phase difference between heaving and pitching. And among several calculated cases the efficiency at ϕ_0=45° is the highest.

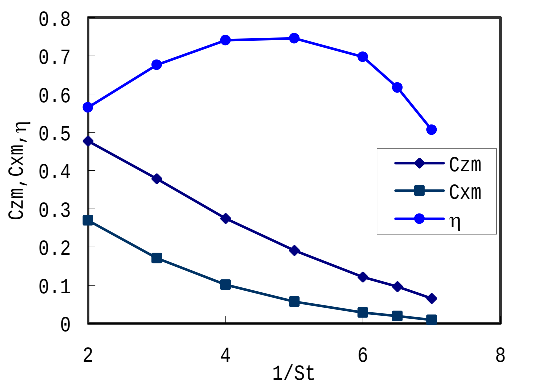

Fig.10 Effect of Strouhal number on hydrodynamics
(θ_0=20°, A_z=C_0, ϕ_0=90°, f=0.5)

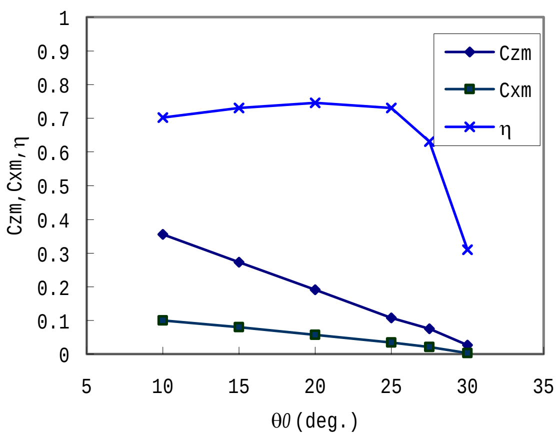

Fig.11 Effect of pitching amplitude on performance
(V_0=5C_0, A_z=C_0, ϕ_0=90°, f=0.5)

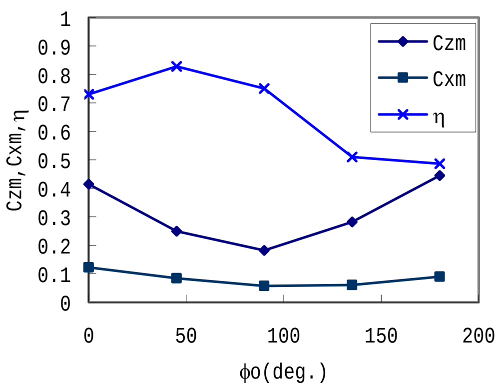

Fig.12 Effect of phase difference on performance
(θ_0=20°, A_z=C_0, V_0=5C_0, f=0.5)

For flexible lunate fin, we take the same contour and sections as rigid fin. The deformed magnitude is decided by pitching amplitude θ_{f0}. That means the phase difference between fore end point and aft end point is coursed by flexibility. In Fig.13 it is shown that with the change of Strouhal number St the hydrodynamic characteristics of the flexible fin varies with same tendency as rigid fin. However, the changing of the efficiency is slower than that of the rigid fin.

In Fig.14 and 15, the hydrodynamic characteristics of the flexible fin are compared with those of the rigid fin. They are compared at same pitching amplitude. According to the assumption for the deformation, the bigger pitching amplitude of the flexible fin means bigger deformation. For the same working condition that means the flexible fin is softer. From Fig.15 we know that corresponding to a given working condition there is a most proper flexibility to make the efficiency be highest. When the pitching amplitude is less than 15°, the mean thrust coefficient and lateral force coefficient are smaller than those of the rigid one due to the flexible deformation. But the efficiency of the former is higher than the later one. When the pitching amplitude is greater than 15°, contrarily the thrust coefficient and lateral force coefficient of the flexible fin become larger than those of the rigid one, and its efficiency becomes smaller. It is clear that the flexible lunate fin has relatively soft hydrodynamic characteristics. The highest efficiency of the rigid fin is greater than that of the flexible fin, but the higher efficiency range of the flexible fin is wider than that of the rigid fin.

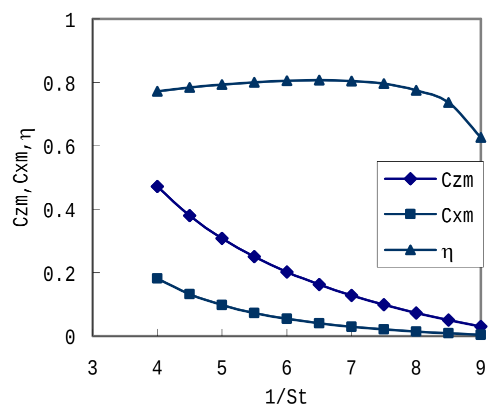

Fig.13 Effect of Strouhal number on hydrodynamics
of flexible caudal fin

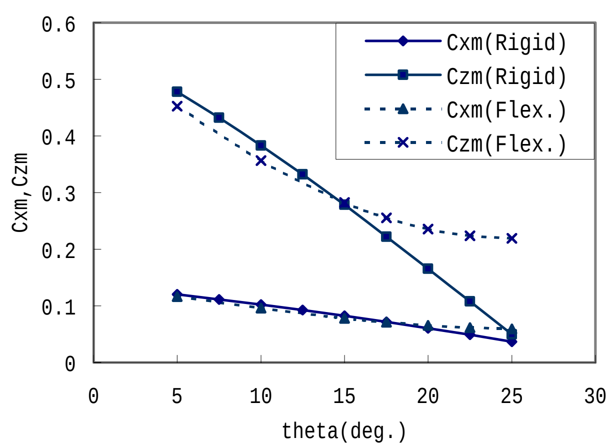

Fig.14 Force coefficients of flexible and rigid fin

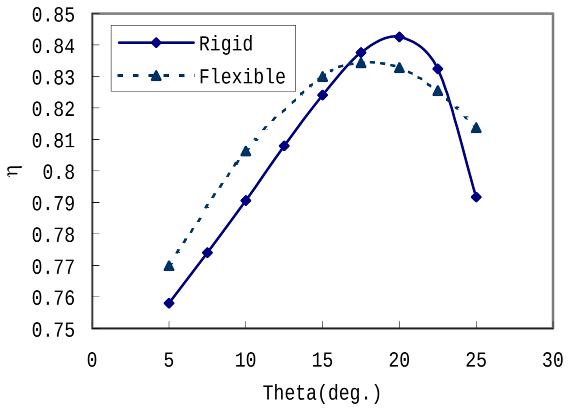

Fig.15 Efficiency of flexible and rigid fin

V. CONCLUDING MAKES

According to above discussion we can obtained following conclusions:

(1) Present method is effective for the analysis of the hydrodynamic characteristics of the rigid and flexible oscillating tuna-tail. The numerical results of present method are reasonable, but the difference between calculation and experiment or other analytical results is evident.

(2) For the rigid and flexible fin, the thrust and lateral force coefficients increase with the deceasing of 1/St, and the efficiencies have the maximum values at certain 1/St.

(3) The thrust and lateral force coefficients of the rigid fin are decrease when the pitching amplitude becomes larger. The efficiency takes highest value for a proper pitching amplitude.

(4)The thrust coefficient and lateral force coefficient of the flexible fin decrease with the increasing of pitching amplitude. For a certain situation there is an optimum flexibility that makes the efficiency reaches the highest value.

(5) For the flexible fin the phase difference should be near to $\pi/2$. But the rigid fin can move with various phase difference. Its hydrodynamic characteristics are sensitive to phase difference. Generally when phase difference equals to about 45^o, the performance is better.

(6) From comparison of the rigid and the flexible fin, we know that the later has relative soft hydrodynamic characteristics. The former highest efficiency is greater than the later one, but the high efficiency range of the later is wider than that of the former.

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