Identification of Parametric Models for Ultrasonic Reflection Experiments Performed on Sediments at Oblique Incidence

Abdellatif Bey Temsamani[1], Luc Peirlinckx[2], Steve Vandenplas[3] and Leo Van Biesen[4]
[1,2,3,4] Dept. ELEC, Vrije Universiteit Brussel, Pleinlaan 2, B-1050, Belgium
[2] Post-doctoral researcher of the Fund for Scientific Research (FWO-Vlaanderen)

Abstract- **Reflection of bounded ultrasonic beams at oblique incidence from a viscoelastic material in controlled measurement conditions is described using a combination of complex harmonic plane waves (CHW model). Previous work was carried out in this field at normal incidence. The shear properties of materials cannot be determined by the normally incident experiment using an immersion technique. This paper gives an extension by modelling wave propagation at oblique incidence using the magnitude as well as the phase of the incident ultrasonic field. A maximum likelihood estimator (MLE) is constructed for the estimation of the model parameters, which takes the measurement noise on both the input and the output signals into consideration. From the estimated model parameters, the absorption and dispersion properties and velocities for both shear and longitudinal waves as well as the density of the sample medium are obtained simultaneously. In order to garanty that only the material properties are identified, the calibration of the measurement set-up is incorporated in the estimation approach.**

I. INTRODUCTION

This work is elaborated in the framework of acoustic characterization of marine sediments. In order to simulate a sediment, the experiments are performed on a viscous porous material boundary. The material used in the modelling approach is a composite (B2900, [1]). This material was selected since it exhibits a mechanical behaviour, which is a combination of elastic and viscous effects. So, it could simulate approximately the behaviour of a fine sediment.

Instead of a classical approach by means of homogeneous waves, more general complex harmonic waves (CHW) are used to examine both the incident and reflected acoustic field using a description of the bounded ultrasonic beam as a finite superposition of inhomogeneous waves. In a recent paper [2], this formalism was applied to characterize a Plexiglas plate at normal incidence. Even though this latter can significantly reduce the complexity associated with the wave propagation modelling, any deviation from this condition may introduce a considerable error. The most general way is then to model the wave propagation at oblique incidence. This allows also to determine the shear properties of viscoelastic materials, which cannot be determined at normal incidence. However, attention has to be paid to the choice of the angle of incidence. In fact, this latter is chosen to be smaller than the angle of total reflection, where a significant displacement of the acoustic field takes place [3]. On the other hand, the surface waves cannot be excited at the experimental angle in the case of the used material.

The paper is organized as follows. In section II, the wave propagation model is established at oblique incidence and the calibration procedure is discussed. In section III, a frequency domain maximum likelihood estimator (MLE) is developed to give the estimation of the model parameters. Experimental results are presented in section IV. Finally, the conclusions are drawn in section V.

II. MODEL APPROACH

The experimental set-up is illustrated in Fig. 1. The incident bounded beam propagates from the emitter to the sample (thickness h) at an angle θ. The sample thickness is chosen so that the first arrived pulse can be separated in time and space from the other rays. The reflected beam is measured with a small probe, which scans the acoustic field

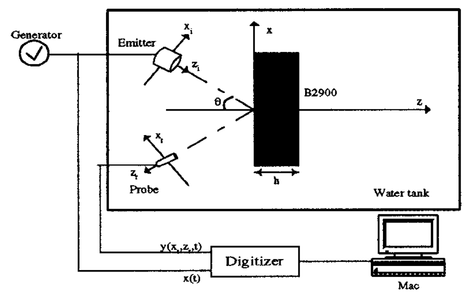

Fig. 1 Experimental configuration

in a plan perpendicular to the propagation axis. The incident angle is controlled by a goniometer with a very high accuracy (±0.5°). Both the generated pulse $x(t)$ (input) and the received signal $y(x_r,z_r,t)$ (position-dependent output) are digitised and Fourier-transformed into $X(\omega)$ and $Y(x_r,z_r,\omega)$, respectively, with ω the angular frequency.

A. *Bounded beam representation in a linear visco-elastic medium in terms of CHW combination:*

1) General approach:

The general mathematical representation of the particle displacements of an heterogeneous wave in a viscoelastic medium can be expressed in terms of two potential functions of the following form:

$$\phi_m(x,z,\omega) = A_m.\exp(i(\mathbf{k_m}.\mathbf{r}-\omega t)) \qquad m = d \text{ or } s \tag{1}$$

where

$$\mathbf{k_m}.\mathbf{k_m} = \kappa_m^2 = (k_{0m}+i\alpha_{0m})^2, \quad m = d \text{ or } s \tag{2}$$

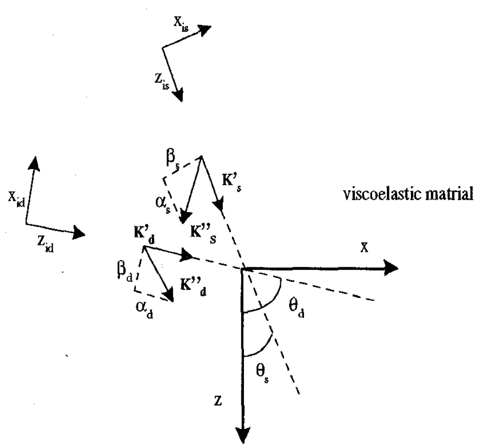

Fig. 2: Geometry of dilatational and shear CHW by means of bivectors

In general A_m are complex constants while $\mathbf{k_m}$ are complex vectors. They can be written as $\mathbf{k_m}'+i\mathbf{k_m}''$, where the real-valued vector $\mathbf{k_m}'$ defines the angle with respect to the z direction, and has as module the propagation coefficient k_m. $\mathbf{k_m}''$ can be decomposed into a component α_m along $\mathbf{k_m}'$ and a component β_m orthogonal to $\mathbf{k_m}'$ (Fig. 2). For each heterogeneous wave, the vector $\mathbf{k_m}$ satisfies the dispersion relation (2), where κ_d and κ_s are the complex wave numbers for the dilatational (m=d) and shear (m=s) component, respectively, related to the medium frequency dependent velocity $\frac{\omega}{k_{0m}}$ and attenuation α_{0m}. They are described by the rational form given in [4] by:

$$\kappa_m(\omega) = \frac{\omega}{\vartheta_m(\omega)} \quad where$$

$$\vartheta_m(\omega) = C_{0m}\sqrt{\frac{1+\sum_{l=1}^{n}\alpha_{lm}(j\omega)^l}{1+\sum_{l=1}^{d}\beta_{lm}(j\omega)^l}} = C_{0m}.P_m(\omega,\alpha,\beta). \tag{3}$$

The complex velocity $\vartheta_m(\omega)$ depends on the dispersionless velocity C_{0m} and on the numerators and denominators coefficients α and β of the rational form, repectively. This latter describes the broadband absorption-dispersion functions much better than the classical models [4].

In a coordinate system (x_{im},z_{im}) obtained from the x-z system by an in plane rotation over an angle θ_m so that the z_{im} axis is pointing toward the direction of propagation, the potential functions for a bounded beam take on the following forms:

$$\phi_m(x_{im},z_{im},\omega) = \sum_{n=-N}^{N} A_{nm}.\exp(\beta_{nm}x_{im}).\exp(-\alpha_{nm}z_{im}).\exp(i(k_{nm}z_{im}-\omega t)), \tag{4}$$
$$(m = d,s).$$

where:

$$\begin{cases} k_{nm}^2-\alpha_{nm}^2-\beta_{nm}^2 = k_{0m}^2-\alpha_{0m}^2 \\ k_{nm}.\alpha_{nm} = k_{0m}.\alpha_{0m} \end{cases} \qquad (m = d \text{ or } s) \tag{5}$$

Equation (4) is a description of plane harmonic waves propagating in the z_{im} direction with a phase velocity ω / k_{nm}. The coefficient β_{nm} represents the heterogeneity (exponential behaviour of the amplitude variation along the wave front), while α_{nm} corresponds to the attenuation of the wave in the propagation direction.

The determination of the complex amplitude A_{nm}, the heterogeneity β_{nm} and the order N were the subject of the recent work given in [2].

2) Calibration procedure:

Since both the calibration and reflection measurements are carried out in the water path, only dilatational waves are considered to model the propagation of the incident beam, and can insonify the material in the reflection mode.

The calibration measurements are carried out in the water path by scanning the generated bounded beam perpendicular to it's main axis of propagation at a distance z_{cal} from the emitter with a small probe. In term of the x_i-z_i coordinates, the general function relating the input-output measured data is obtained, using the following expression:

$$\frac{Y_{cal}(x_i,z_i,\omega)}{X_{cal}(\omega)} = \sum_{n=-N}^{N} A_n \exp(i(k_n z_i - \omega t))\exp(-\alpha_n z_i)\exp(\beta_n x_i).H_T(\omega).H_R(\omega) \tag{6}$$

with:

$$\begin{cases} k_n^2 - \alpha_n^2 - \beta_n^2 = K_w^2 - \alpha_w^2 \\ k_n.\alpha_n = K_w.\alpha_w \end{cases} \tag{7}$$

for all values of the summation index n and where ω / K_w and α_w are the dispersion and absorption in water. $H_T(\omega)$ and $H_R(\omega)$ represent the transmitting and receiving characteristics of the transducers, along with the associated electronics, respectively.

Since water is considered as a non absorptive medium, $\alpha_w = 0$. This means that (7) becomes:

$$\begin{cases} k_n^2 - \alpha_n^2 - \beta_n^2 = K_w^2 \\ \alpha_n = 0 \end{cases} \tag{8}$$

and so, the propagation of the incident beam in the water path is given by:

$$\frac{Y_{cal}(x_i,z_i,\omega)}{X_{cal}(\omega)} = \sum_{n=-N}^{N} A_n.\exp(i(k_n z_i - \omega t)).\exp(\beta_n x_i).H_T(\omega).H_R(\omega) \tag{9}$$

B. Bounded beam representation at oblique incidence:

As presented in [5,6], and based on the assumption of linearity, the amplitude and phase distribution of the reflected field for any angle of incidence θ, can be found by multiplying each heterogeneous wave by its complex valued reflection coefficient $R_n(k_n \sin(\theta) - i\beta_n \cos(\theta))$. In addition to the dispersion relation, the continuity conditions at the water-material boundary, require invariability of the x coordinate, and the generalized Snell-Descartes laws have to be satisfied:

$$\begin{cases} k_r \sin\theta_r = k_n \sin\theta_n \\ -\beta_r \cos\theta_r = -\beta_n \cos\theta_n \end{cases} \tag{10}$$

where the subscript r, refers to the reflected field. In this case $\theta_r = \pi - \theta_n$, and so, from (10), the sign of the heterogeneity coefficients for the reflected beam are changed. In terms of the x_r-z_r coordinates, as shown in Fig.1, the first reflected beam from the material interface is given by:

$$\frac{Y_m(x_r,z_r,\omega)}{X_m(\omega)} = \sum_{n=-N}^{N} A_n.R_n.\exp(i(k_n z_r - \omega t)).H_T(\omega).H_R(\omega) \tag{11}$$

Hence, the global propagation model can be written as:

$$\frac{Y_m(x_r,z_r,\omega)}{X_m(\omega)} = H(x_r,z_r,\omega)\frac{Y_{cal}(x_i,z_i,\omega)}{X_{cal}(\omega)} \tag{12}$$

Note that the coordinate systems (x_i,z_i) and (x_r,z_r) are the same. The calibrated field model $H(x_r,z_r,\omega)$ is given by:

$$H(x_r,z_r,\omega) = \frac{\sum_{n=-N}^{N} A_n.R_n.\exp(ik_n z_r)}{\sum_{n=-N}^{N} A_n.\exp(ik_n z_i).\exp(\beta_n x_i)} \tag{13}$$

III. MAXIMUM LIKELIHOOD ESTIMATOR

From (12), it is readily seen that the nonparametric estimates $H^{np}(x_r,z_r,\omega) = \frac{Y(x_r,z_r,\omega)}{X(\omega)}$ and $H^{np}_{cal}(x_i,z_i,\omega) = \frac{Y_{cal}(x_i,z_i,\omega)}{X_{cal}(\omega)}$, can be regarded as the input and output of a linear system, consistent with:

$$H^{np}(x_r,z_r,\omega) = H(x_r,z_r,\omega)H^{np}_{cal}(x_i,z_i,\omega) \tag{14}$$

where $H(x_r,z_r,\omega)$ is the calibrated field model defined by (13). The nonparametric estimates $H^{np}(x_r,z_r,\omega)$ and $H^{np}_{cal}(x_i,z_i,\omega)$ are obtained from averaging the input-output measurements during the calibration and measurement of acoustic fields viz.:

$$H^{np}(x,z,\omega) = \exp\langle\frac{1}{M}\sum_{i=1}^{M}\log(\frac{Y(x,z,\omega)}{X(\omega)})\rangle \tag{15}$$

where M is the number of measurements performed. Since the generation and the acquisition of the signals are not perfectly synchronized, the H_{log} averaging technique is applied [7]. During the averaging procedure, the variances of the input-output data $\sigma^2_{H^{np}}(x_r,z_r,\omega)$ and $\sigma^2_{H^{np}_{cal}}(x_i,z_i,\omega)$, are determined as well.

Taking into account both the input and output noises, the cost function to be minimized in ML sense is given in [8] as:

$$C = \sum_{l=1}^{L}\sum_{f=1}^{F}\left|e(x_l,\omega_f)\right|^2; \tag{16}$$

where the error function is expressed by:

$$e(x_l,\omega_f) = \frac{H^{np}_{cal}(x_{il},z_i,\omega_f)H(x_{rl},z_r,\omega_f) - H^{np}(x_{rl},z_r,\omega_f)}{\sqrt{\sigma^2_{H^{np}_{cal}}(x_{il},z_i,\omega_f)\left|H(x_{rl},z_r,\omega_f)\right|^2 + \sigma^2_{H^{np}}(x_{rl},z_r,\omega_f)}}; \tag{17}$$

and where $\{\omega_f, f=1,...,F\}$ and $\{x_l, l=1,...,L\}$, denote the set of angular frequencies and the set of transverse distances taken into account, respectively.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. Experimental results:

In this section, the bounded beam propagation theory, developed in the previous section, is applied to the parametric modelling of the ultrasonic reflection experiments on a viscoelastic plate at oblique incidence. The modelling approach takes into account the beam properties of the incident field generated by a transducer of finite aperture. Fig.1 shows the experimental arrangement. A pulse generator (5055PR, Panametrics) is used to excite a 38.1mm diameter transducer (V389, Panametrics). The source velocity pattern of the V389, has been characterized using the CHW formalism in the frequency band from 400 to 800 kHz, as described in section II. The reflected ultrasonic fields are measured by a 1.34mm diameter probe. The position of the probe is controlled by a positioning system. Fig. 3(a), (b) displays the wave fields measured for the calibration and reflection purposes. Fig. 4(a) shows the calibrated measured field. The transverse distance used for the measurements ranges from -50 to 50mm with a scanning interval of 2mm. The frequency band ranges from 400 to 800 kHz. This choice is made because at low frequencies, the waist width of the incident beam is too large, and then different rays constituting the beam affect the geometrically reflected pulse in the time domain, which introduces considerable peaks in the frequency domain.

B. Estimation results:

Using the measured calibration and reflection data, as the input and output, respectively, the ML estimator is applied to determine the parameters of the investigated model. The magnitude and phase of the measured and estimated fields are depicted as a function of the frequency in Fig.5, where the transverse distance x_r=0 mm.

The measured and estimated fields can also be plotted as a function of the transverse distance, as shown in Fig.6, for the central frequency component of the emitter (500kHz). Note that the fitting error is generally increasing as the radial distance increases. This may be due firstly to the fact that the model is more reliable in the region near the

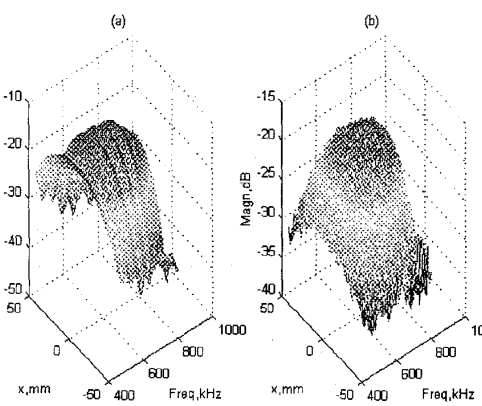

Fig. 3 (a) Measured acoustic field at the water path (b), Reflected acoustic field from water-material interface.

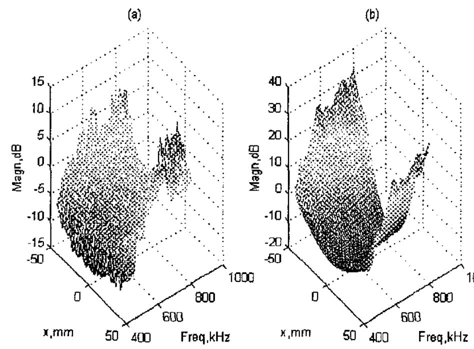

Fig. 4 (a) Calibrated Measured acoustic field. (b), Estimated profile in frequency and space domains.

propagation axis of the transmitter. This is explained by the divergence, which is an inherent characteristic of the described formalism, caused by the different phase velocities of the heterogeneous waves used in the decomposition [5]. Secondly, the signal-to-noise ratio for both the calibration and reflection measurements decreases in the region far away from the centre.

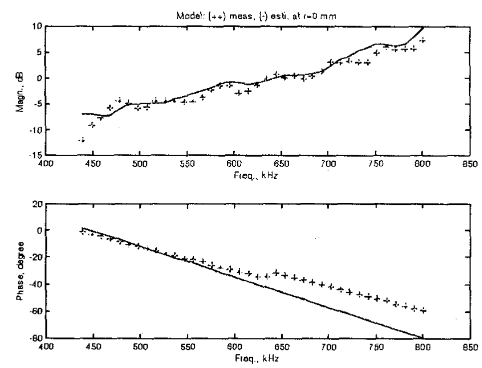

Fig. 5 Magnitude and phase of the measured (++) and estimated (-) fields as a function of frequency at x=0mm.

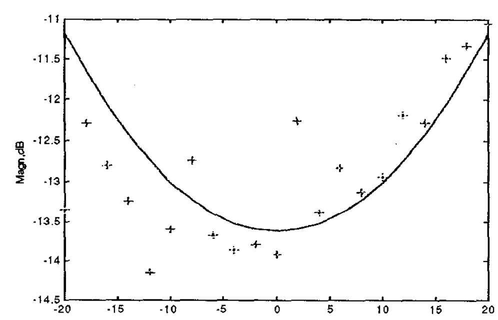

Fig. 6 Magnitude of the measured (+) and estimated (-) field as function of the transverse distance at 500kHz.

Table 1 gives the estimated model parameters together with the associated standard deviations (STD). The estimated parameters are calculated with a water velocity of 1500 $m.s^{-1}$ and a density of 1000 $kg.m^{-3}$ (the temperature equals 24°C). The standard values are found in [1] and are calculated using the plane wave propagation with diffraction correction model at normal incidence. The shear coefficients α and β are calculated using the relation between the longitudinal and shear properties following the model developed in [9]. Using the estimated longitudinal velocity and the coefficients for the rational absorption-dispersion model, the attenuation corresponding to the longitudinal and shear waves in the material are shown in Fig.7. Moreover, a comparison is made with the calculated velocities and attenuation of the longitudinal an shear waves in material, using the standard values [1].

V. CONCLUSION

A complex harmonic wave (CHW) combination formalism has been presented for the parametric modelling of ultrasonic reflection experiments from a water-viscoelastic material boundary. Maximum likelihood estimators are developed for the estimation of the model parameters, which take into account both the frequency and the spatial information. It is shown that this approach is capable of estimating material properties only from the first reflected bounded beam, which is also applicable for sediments characterisation. The standard deviations for the CHW model are bigger than those obtained with the plane wave model. This is evident since also multiple reflections are used to estimate the material parameters by the plane wave model. And since the measurements are carried out at normal incidence, no big model errors are introduced. However, it is concluded that the estimated parameters are more or less in the range of the standard values, and encourage to develop the investigated model to take into account multiple reflections experiments.

ACKNOWLEDGMENTS

This work was sponsored by the MAST-3 SIGMA project (No MAS3-CT97-0100), and by the Fund for Scientific Research (FWO-Vlaanderen; contract G.03220.96).

References:

[1] Luc Peirlinckx, *Identification of Parametric Plane Wave Propagation Models for Underwater Acoustic Reflection and Transmission Experiments*, PhD thesis, Free University of Brussels (1994).

[2] A. B. Temsamani, , L. Peirlinckx and L. P. Van Biesen, " Experimental Validation of Parametric Models for Ultrasonic Beam Propagation", will be published in the Proceedings of the Fourth European Conference on Underwater Acoustics, Rome (Italy) 21-25 September 1998.

[3] H. L. Bertoni and T. Tamir, "Unified Theory of Rayleigh-Angle Phenomena for Acoustic Beams at Liquid-Solid Interfaces" Appl. Phys. 2, 157-172 (1973).

[4] L. Peirlinckx, P. Guillaume, R. Pintelon and L. P. VanBiesen, "A Global System Identification Approach for the Accurate Parametric Modeling of Ultrasonic Reflection and Transmission Experiments" *IEEE Trans. Ultrason. Ferro. Freq. Control*, 43(4), 628-639 (1996).

[5] K.Van Den Abeele and O. Leroy, "complex harmonic wave scattering as a framework for investigation of bounded beam reflection and transmission at plane interfaces and its importance in the study of vibrational modes", J.Acoust.Soc.Am., 93(1), pp. 308-323, 1993.

[6] J. M. Claeys and O.Leroy, "Reflection and Transmissin of Bounded Sound Beams on Half-Spaces and through Plates", J.Acoust.Soc.Am., 72(2), pp. 585-590, 1982.

[7] P. Guillaume, R. Pintelon and J. Schoukens, "Nonparametric frequency response function estimators based on non-linear averaging techniques," IEEE Trans. Instrum. Meas., 41, 739-746 (1992).

[8] J. Schoukens and R. Pintelon, *Identification of Linear Systems: A Practical Guideline to Accurate Modelling* , Pergamon Press, New York, 1991.

[9] Walter Madigosky and Ralph Fiorito, "Modal Resonance Analysis of Acoustic Transmission and Reflection Losses in Viscoelastic Plates, " J.Acoust. Soc. Am, 65(5), 1105-1115 (1979).

Table 1 : The set of the estimation results for the B2900 material

Model Parameters	Estimated Values	Standard Values
Incidence Angle (°)	29.97 (0.04)	30 (-)
Density ($kg.m^{-3}$)	1601.6 (3.6)	1837 (2)
Longitudinal Velocity ($m.s^{-1}$)	2740 (14)	2736 (1.57)
Shear Velocity ($m.s^{-1}$)	1544 (1)	1540 (-)
α_{L1}	1.1e-6 (0.1e-6)	1.09e-6 (0.01e-6)
α_{L2}	1.2e-15 (0.5e-15)	1.10e-15 (0.03e-15)
β_{L1}	9.9e-7 (0.9e-7)	1.02e-6 (0.01e-6)
α_{S1}	1.14e-6 (0.04e-7)	1.13e-6 (0.01e-6)
α_{S2}	1.5e-15 (0.2e-16)	1.85e-15 (0.05e-15)
β_{S1}	9.95e-7 (0.04e-7)	1.02e-6 (0.01e-6)

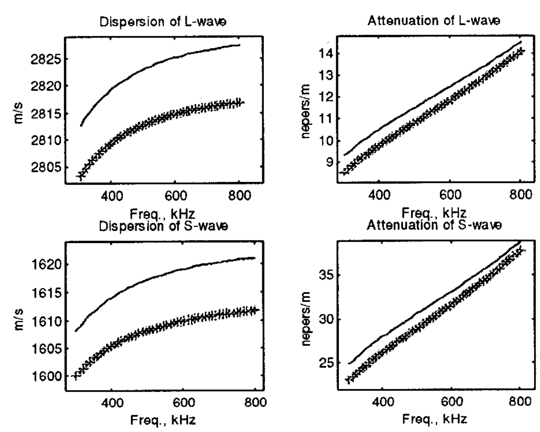

Fig.7 Longitudinal and shear dispersion and attenuation properties of the B2900 material at the frequency band [400:800] kHz. (+) calculated with plane wave, (-) calculated with CHW approach.