EM375 Project Handout
CALCULATING CONSTITUTIVE EQUATIONS FOR THE ELASTOMER

BACKGROUND: The distance the projectile travels after leaving the slingshot is primarily a function of how much energy is imparted to it from the slingshot. The energy in the slingshot can be obtained by integrating the force-deflection or stress-strain curve for the elastic tubing as it is stretched. To do this we need a relationship that relates the stress in the tubing with its elongation or strain. Earlier this semester you obtained load vs. extension data for the large and small rubber tubing. We now need to use that data to determine the material properties. Once we have those properties, we will be able to calculate (a different project handout) how much energy is stored in the rubber when it is stretched.

You are already familiar with the common constitutive relation called "Hooke's Law," which establishes a *linear* relation between stress and strain and has a single material property, *Young's modulus of elasticity*. Unfortunately, elastomers are nonlinear, and they do not follow Hooke's Law. The data reduction described here will enable you to determine the material properties for TWO DIFFERENT NONLINEAR MODELS; **The Mooney Formula**, and the **Strain Hardening Relationship**. To find these properties you will use a linearizing transformation, followed by linear regression.

THE MOONEY FORMULA: The first relationship considered is the two-term Mooney Formula. In this formula C_1 and C_2 are the material constants we need to identify. They have units of stress.

$$s_n = 2\left[l - \frac{1}{l^2}\right]\left[C_1 + \frac{C_2}{l}\right]$$

Even though "necking" will occur, the stress, s_n is calculated by dividing force by the original cross sectional area. The stretch ratio, l, is the ratio of stretched length divided by original length, $l = L/L_O$. To find the constants in the Mooney Formula from test data and linear regression, rewrite (transform) the Mooney Formula as:

$$\frac{s_n}{2\left[l - \frac{1}{l^2}\right]} = C_1 + \frac{C_2}{l}$$

which plots as a straight line for $\frac{s_n}{2\left[l - \frac{1}{l^2}\right]}$ versus $\frac{1}{l}$. The slope gives constant C_2 and the intercept gives constant C_1.The zero stress-zero elongation point will not be plotted. Why not?

STRAIN HARDENING RELATIONSHIP: The second relationship we consider is a strain hardening relationship of the form:

$$s_n = s_O e_n^a$$

In this equation, the material constants we are trying to identify are s_O (with units of psi) and a (a dimensionless strain hardening exponent). e_n is the strain, DL/L_O. If we transform the equation by taking natural logarithms, we get:

$$\ln(s_n) = \ln(s_O) + a \ln(e_n)$$

This relationship plots as a straight line on the transformed axes. The slope gives α. The intercept gives the natural logarithm of s_O, so you have to "untransform" the intercept to get s_O. Note that the strain parameter, e_n, is different from the elongation parameter, l, but they are related. How?

DATA REDUCTION: The data reduction is to be done using MathCAD. It is recommended that you develop the worksheet for one size of tubing first. Once it is working correctly, change the input data (and unit conversions???) for the other size of tubing, and save the file with a new name.

1. Use coordinate transformation and linear regression to determine the Mooney constants C_1 and C_2, and the Strain Hardening constants s_O and a. Remember units!

2. For each size of tubing, create the following plots (you should end up with 4 plots in total):

    a) The transformed data and linear regression overlaid with the bands for 90% confidence interval.

    b) The raw data (load vs. extension) overlaid with the theoretically regenerated curves.

SUBMIT: BEFORE you move on to calculate the energy in the bands, you must get your data reduction approved by your instructor. Submit to your instructor the following:

A very short memorandum that tabulates all the calculated material properties (with units). You must also include a clear statement as to which model (strain hardening or Mooney) you think best fits your data. Include a justification.

Attach as enclosures:

1. MathCAD printout for the small tubing

2. MathCAD printout for the large tubing