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A thresholding method based on two-dimensional Renyi's entropy

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Abstract

In this paper, we present a new thresholding technique based on two-dimensional Renyi's entropy. The two-dimensional Renyi's entropy was obtained from the two-dimensional histogram which was determined by using the gray value of the pixels and the local average gray value of the pixels. This new method extends a method due to Sahoo et al. (Pattern Recognition 30 (1997) 71) and includes a previously proposed global thresholding method due to Abutaleb (Pattern Recognition 47 (1989) 22). Further, our method extends a global thresholding method due to Chang et al. (IEEE Trans. Image Process. 4 (1995) 370) to the two-dimensional setting. The effectiveness of the proposed method is demonstrated by using examples from the real-world and synthetic images.
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1. Introduction

Image segmentation is an important and fundamental task in many digital image processing systems. Image segmentation by thresholding is the simplest technique and involves the basic assumption that the objects and the background in the digital image have distinct gray level distributions. If this assumption holds, the gray level histogram contains two or more distinct peaks and threshold values separating them can be obtained. Segmentation is performed by assigning regions having gray levels below the threshold to the background, and assigning those regions having gray levels above the threshold to the objects, or vice versa.

Threshold selection methods [1–48] can be classified into two groups, namely, global methods and local methods. A global thresholding technique thresholds the entire image with a single threshold value obtained by using the gray level histogram (which is an approximation of the gray level probability density function) of the image. Local thresholding methods partition the given image into a number of subimages and determine a threshold for each of the subimages. Global thresholding methods are easy to implement and are computationally less involved. As such they serve as popular tools in a variety of image processing applications such as automatic target recognition [49], segmentation of synthetic radar images [50], video indexing through scene cut [51], image sequence segmentation [52], text enhancement [53], automatic grading system for beef marbling [54,55], measuring the canopy of tomato seedling for quality control [56], for winter pruning of long wood grape vines [57], detection of big rock in mining industry [58], locating addresses on envelopes for mail sorting [59], and biomedical image analysis [60–62]. For a survey on threshold selection techniques the reader is encouraged to see Refs. [36,42,62].

In Ref. [18], Kapur et al. raised the following concern in the conclusion of their paper: "What happens if two different pictures have the same gray-level histogram and thus

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the same threshold? Will it be suitable for both?" They also suggested that a second-order statistic or some local properties with the entropic concept of thresholding might give a better insight into this problem. In Ref. [1], Abutaleb extended the entropy based thresholding method of Kapur et al. using the two-dimensional entropy. The two-dimensional entropies were obtained from a two-dimensional histogram which was determined by using the gray value of the pixels and the local average gray value of the pixels. This extension due to Abutaleb was refined by Brink [5] by maximizing the smaller of two entropies instead of maximizing the sum of the two entropies of the background class and object class. Liu and Li [63] proposed a two-dimensional Otsu thresholding method [37] which performed better than one-dimensional method when images are corrupted with noise. Pal and Pal [29] have also extended one-dimensional entropic thresholding approach to two-dimensional entropic approach on the basis of two-dimensional histogram. Recently, Sahoo et al. [39] proposed a one-dimensional entropy based method, known as the entropy crossover method, and outlined its two-dimensional extension. In Jansing et al. [16] the extension that was outlined in Ref. [39] was implemented using features such as gray level of the pixel and the fractal error. While the extension of one-dimensional approaches to two-dimensional histogram results in much better segmentation, it gives rise to the exponential increment of computational time. We only consider the entropy-based methods for comparison. The computational complexity of Brink's method [5], using a "maximum" optimization procedure, is bounded by $O(L^4)$, and, after improved by Chen et al. [8] is still bounded by $O(L^{8/3})$, where L is the number of gray levels. But the computational complexity of the method of Kapur et al. [18] is only bounded by $O(L^{3/2})$. In Ref. [48], Wu et al. give a fast recurring two-dimensional thresholding algorithm to improve the computation time of the thresholding method due to Abutaleb [1]. Similarly, Gong et al. [12] give a fast recurring two-dimensional thresholding algorithm to improve the computation time of the two-dimensional Otsu's thresholding method [28].

In this paper, we propose an automatic global thresholding technique based on two-dimensional Renyi's entropy. The two-dimensional Renyi's entropy was obtained from the two-dimensional histogram which was determined by using the gray value of the pixels and the local average gray value of the pixels. This new method extends a method due to Sahoo et al. [38] and includes a previously proposed well known global thresholding method due to Abutaleb [1]. Further, this method also extends to two-dimensional setting a thresholding method due Chang et al. [7]. This paper is organized as follows: in Section 2, we describe the mathematical setting of the threshold selection and the newly proposed thresholding method. In Section 3, we report the effectiveness of our thresholding method when applied to some real-world and synthetic images. In Section 4, we present some concluding remarks about our method.

2. The proposed method

Let $f(m,n)$ be the gray value of the pixel located at the point (m,n). In a digital image $\{f(m,n) \mid m \in \{1,2,\ldots,M\}, n \in \{1,2,\ldots,N\}\}$ of size $M \times N$, let the histogram be $h(x)$ for $x \in \{0,1,2,\ldots,255\}$. For the sake of convenience, we denote the set of all gray levels $\{0,1,2,\ldots,255\}$ as G. Global threshold selection methods usually use the gray level histogram of the image. The optimal threshold is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image.

Let t be a threshold value and $B = \{b_0, b_1\}$ be a pair of binary gray levels with $b_0, b_1 \in G$. Typically b_0 and b_1 are taken to be 0 and 255, respectively. The result of thresholding an image function $f(m,n)$ at gray level t is a binary function $f_t(m,n)$ such that

$$f_t(m,n) = \begin{cases} b_0 & \text{if } f(m,n) \leqslant t, \\ b_1 & \text{if } f(m,n) > t. \end{cases}$$

In general, a thresholding method determines the value $t^\star$ of t based on a certain criterion function. If $t^\star$ is determined solely from the gray level of each pixel, the thresholding method is point dependent [36].

In order to compute the two-dimensional histogram of a given image we proceed as follow. Calculate the average gray value of the neighborhood of each pixel. Let $g(x,y)$ be the average of the neighborhood of the pixel located at the point (x,y). The average gray value for the 3×3 neighborhood of each pixel is calculated as

$$g(x,y) = \left\lfloor \frac{1}{9} \sum_{i=-1}^{1} \sum_{j=-1}^{1} f(x+i, y+j) \right\rfloor,$$

where $\lfloor r \rfloor$ denotes the integer part of the number r. While computing the average gray value, disregard the two rows from the top and bottom and two columns from the sides. The pixel's gray value, $f(x,y)$, and the average of its neighborhood, $g(x,y)$, are used to construct a two-dimensional histogram using

$$h(k,m) = Prob(f(x,y) = k \text{ and } g(x,y) = m),$$

where $k, m \in G$. For a given image, there are several methods to estimate this density function. One of the most frequently used methods is the *method of relative frequency*. The normalized histogram is approximated by using the formula

$$\hat{h}(k,m) = \frac{number\ of\ elements\ in\ the\ event\ (f(x,y)=k)\ and\ (g(x,y)=m)}{number\ of\ elements\ in\ the\ sample\ space} = \frac{number\ of\ pixels\ with\ gray\ value\ k\ and\ average\ gray\ value\ m}{number\ of\ pixels\ in\ the\ image}.$$

The total number of frequences (occurrences), $n_{(i,j)}$ of the pair (i,j), divided by the total number of pixels, $N \times M$,

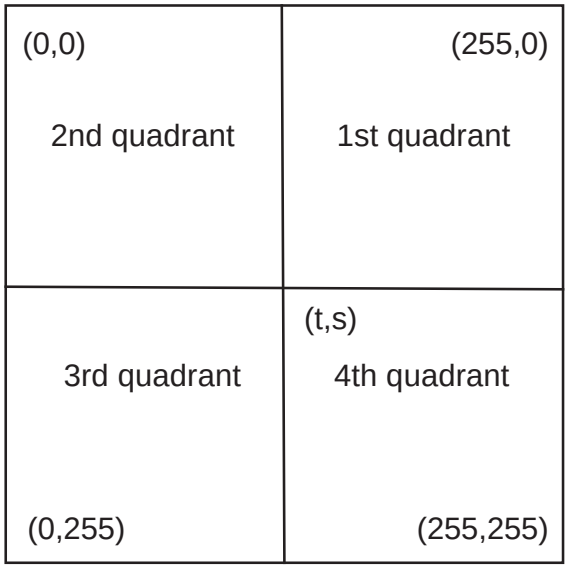

Fig. 1. Quadrants in the 2D histogram due to thresholding at (t,s).

defines a joint probability mass function, $p_{(i,j)}$. Thus

$$p_{(i,j)} = \frac{n_{(i,j)}}{N \times M},$$

where $i, j = 0, 1, \ldots, 255$.

The threshold is obtained through a vector (t,s) where t, for $f(x,y)$, represents the threshold of the gray level of the pixel and s, for $g(x,y)$, represents the threshold of the average gray level of the pixel's neighborhood. The frequency of occurrence of each pair of gray values is calculated. From this a surface can be drawn that will have two peaks and one valley. The object and background correspond to the peaks and can be separated by selecting the vector (t,s) that maximizes the sum of two class entropies. Using this vector (t,s), the histogram is divided into four quadrants (see Fig. 1 above).

We denote the first quadrant by $[t+1, 255] \times [0,s]$, the second quadrant by $[0,t] \times [0,s]$, the third quadrant by $[0,t] \times [s+1, 255]$, and the fourth quadrant by $[t+1, 255] \times [s+1, 255]$. Since two of the quadrants, first and third, contain information about edges and noise alone, they are ignored in the calculation. Because the quadrants which contain the object and the background, second and fourth, are considered to be independent distributions, the probability values in each case must be normalized in order for each of the quadrants to have a total probability equal to 1. Our normalization is accomplished by using a posteriori class probabilities, $P_2(t,s)$ and $P_4(t,s)$. We assume that the contribution of the quadrants which contains the edges and noise is negligible, hence we further approximate $P_4(t,s)$ as $P_4(t,s) \approx 1 - P_2(t,s)$.

The a priori Renyi's entropy of order α of an image is defined as (see Ref. [64])

$$H_T^\alpha = \frac{1}{1-\alpha} \ln \sum_{i=0}^{255} \sum_{j=0}^{255} p(i,j)^\alpha,$$

where $\alpha (\neq 1)$ is a positive real parameter. Since $\lim_{\alpha \to 1} H_T^\alpha = H_T$, Renyi's entropy H_T^α is a one parameter generalization of the Shannon entropy H_T, where

$$H_T = -\sum_{i=0}^{255} \sum_{j=0}^{255} p(i,j) \ln p(i,j).$$

Renyi's entropies associated with object and background distributions are given by

$$H_b^\alpha(t,s) = \frac{1}{1-\alpha} \ln \sum_{i=0}^{t} \sum_{j=0}^{s} \left(\frac{p_{(i,j)}}{P_2(t,s)} \right)^\alpha$$

and

$$H_w^\alpha(t,s) = \frac{1}{1-\alpha} \ln \sum_{i=t+1}^{255} \sum_{j=s+1}^{255} \left(\frac{p_{(i,j)}}{1-P_2(t,s)} \right)^\alpha.$$

Here we have assumed that the off-diagonal probabilities are negligible and $H_w(t,s)$ is computed by using $1 - P_2(t,s)$ instead of $P_4(t,s)$. The optimal threshold pair $(t^*(\alpha), s^*(\alpha))$ can be obtained by finding the pair (t,s) which maximizes $H_b^\alpha(t,s) + H_w^\alpha(t,s)$, that is

$$(t^*(\alpha), s^*(\alpha)) = Arg \max_{(t,s) \in G \times G} [H_b^\alpha(t,s) + H_w^\alpha(t,s)].$$

For a priori chosen α, we will use only the optimal threshold $t^*(\alpha)$ to threshold an image.

The limiting case of the proposed extension is Abutaleb's method [1]. To see this, compute the limiting value of $H_b^\alpha(t,s)$ and $H_w^\alpha(t,s)$ as $\alpha \to 1$. When $\alpha \to 1$, then

$$\begin{aligned}
&\lim_{\alpha \to 1} H_b^\alpha(t,s) \\
&= \lim_{\alpha \to 1} \frac{\frac{d}{d\alpha} \left[\ln \sum_{i=0}^{t} \sum_{j=0}^{s} \left(\frac{p_{(i,j)}}{P_2(t,s)} \right)^\alpha \right]}{\frac{d}{d\alpha}(1-\alpha)} \\
&= \lim_{\alpha \to 1} \frac{\frac{1}{\sum_{i=0}^{t} \sum_{j=0}^{s} \left(\frac{p_{(i,j)}}{P_2(t,s)} \right)^\alpha} \sum_{i=0}^{t} \sum_{j=0}^{s} \left(\frac{p_{(i,j)}}{P_2(t,s)} \right)^\alpha \ln \left(\frac{p_{(i,j)}}{P_2(t,s)} \right)}{-1} \\
&= -\sum_{i=0}^{t} \sum_{j=0}^{s} \left(\frac{p_{(i,j)}}{P_2(t,s)} \right) \ln \left(\frac{p_{(i,j)}}{P_2(t,s)} \right) \\
&= H_b(t,s)
\end{aligned}$$

and

$$\lim_{\alpha\to 1} H_w^{\alpha}(t,s) = \lim_{\alpha\to 1} \frac{\frac{d}{d\alpha}\left[\ln \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^{\alpha}\right]}{\frac{d}{d\alpha}(1-\alpha)}$$

$$= \lim_{\alpha\to 1} \frac{\frac{1}{\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^{\alpha}} \sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^{\alpha}\ln\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)}{-1}$$

$$= -\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)\ln\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)$$

$$= H_w(t,s).$$

Hence,

$$\lim_{\alpha\to 1}[H_b^{\alpha}(t,s) + H_w^{\alpha}(t,s)] = H_b(t,s) + H_w(t,s).$$

Therefore, when $\alpha \to 1$, the threshold value equals to the same value found by Abutaleb's method [1]. Thus this proposed method includes Abutaleb's method as a special case.

If the parameter α equals to 2, then our method extends a global thresholding method due to Chang et al. [7] to two-dimensional setting. When $\alpha = 2$, then $H_b^{\alpha}(t,s)$ and $H_w^{\alpha}(t,s)$ becomes

$$H_b^2(t,s) = \frac{1}{1-2}\ln\sum_{i=0}^{t}\sum_{j=0}^{s}\left(\frac{p_{(i,j)}}{P_2(t,s)}\right)^2$$

$$= -\ln\sum_{i=0}^{t}\sum_{j=0}^{s}\left(\frac{p_{(i,j)}}{P_2(t,s)}\right)^2$$

and

$$H_w^2(t,s) = \frac{1}{1-2}\ln\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^2$$

$$= -\ln\sum_{i=t+1}^{l-1}\sum_{j=s+1}^{l-1}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^2.$$

The sum of $H_b^2(t,s)$ and $H_w^2(t,s)$ is

$$H_b^2(t,s) + H_w^2(t,s)$$

$$= -\ln\sum_{i=0}^{t}\sum_{j=0}^{s}\left(\frac{p_{(i,j)}}{P_2(t,s)}\right)^2 - \ln\sum_{i=t+1}^{255}\sum_{j=s+1}^{255}\left(\frac{p_{(i,j)}}{1-P_2(t,s)}\right)^2$$

$$= -\ln(G_b(t,s)G_w(t,s)) + 2\ln(P_2(t,s)(1-P_2(t,s))),$$

where

$$G_b(t,s) = \sum_{i=0}^{t}\sum_{j=0}^{s} p_{(i,j)^2} \quad \text{and} \quad G_w(t,s) = \sum_{i=t+1}^{255}\sum_{j=s+1}^{255} p_{(i,j)^2}.$$

The expression

$$-\ln(G_b(t,s)G_w(t,s)) + 2\ln(P_2(t,s)(1-P_2(t,s)))$$

can be used as a criterion function to obtain the optimal threshold. Thus, this method can be considered as an extension of the method due to Chang et al. [7] to a two-dimensional setting.

3. Analysis of test results

In this section, we discuss the experimental results obtained using the proposed method. This discussion includes the choice of the optimal threshold and the presentation of the optimal threshold values of some real-world and synthetic images. These images are *cameraman.tif*, *kids.tif*, *shadow.tif*, *moon.tif*, *rice.tif*, *shot1.tif*, *eight.tif*, *circuit.tif*, *ic.tif*, *tire.tif*, *bonemarr.tif*, *blood1.tif*, and *bacteria.tif* and they are shown in Figs. 12–14. The optimal threshold value was computed by the proposed method for these thirteen images. Table 1 lists the optimal threshold values that are found for these images for α values equal to 0.3, 0.5, 0.7, 0.8, 1.0 and 2.0, respectively.

The original images together with their histograms and the thresholded images obtained by using the optimal threshold value $t^{\star}(0.7)$ and $t^{\star}(1.0)$ are displayed side by side in Figs. 2–14. Note that the thresholded image obtained by

Table 1
The optimal threshold values for various values of α

Test image	$t^{\star}(0.3)$	$t^{\star}(0.5)$	$t^{\star}(0.7)$	$t^{\star}(0.8)$	$t^{\star}(1.0)$	$t^{\star}(2.0)$
camaraman.tif	99	99	99	95	119	189
kids.tif	32	32	32	29	45	26
shadow.tif	134	128	128	126	197	206
moon.tif	130	142	152	130	197	223
rice.tif	135	135	135	135	51	195
shot1.tif	126	126	120	120	161	237
eight.tif	143	140	145	154	228	210
circuit.tif	90	89	83	81	49	182
ic.tif	128	87	87	84	20	253
tire.tif	111	111	93	93	162	19
bonemarr.tif	122	121	121	121	216	253
blood1.tif	131	131	138	138	160	201
bacteria.tif	103	103	103	101	199	201

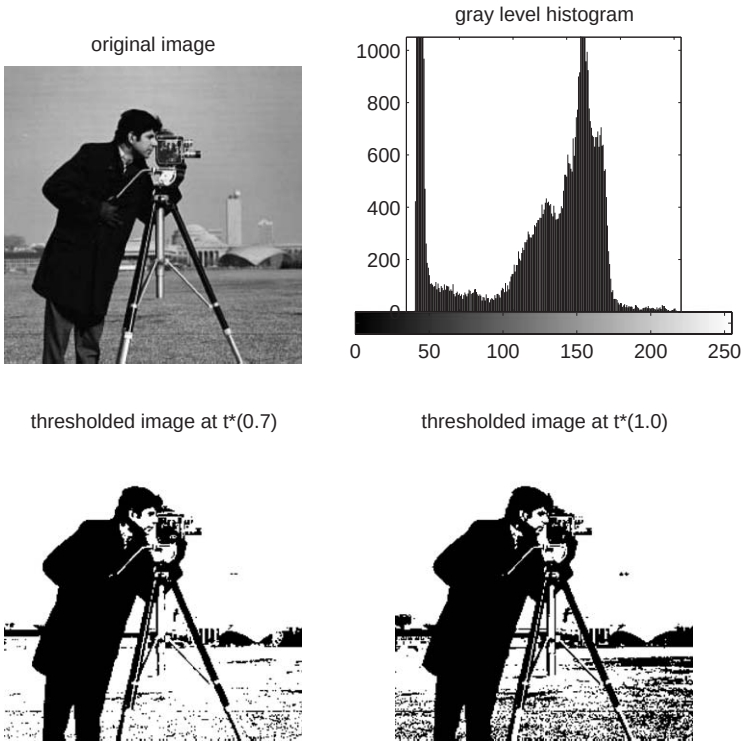

Fig. 2. Cameraman image, its gray level histogram and the thresholded images.

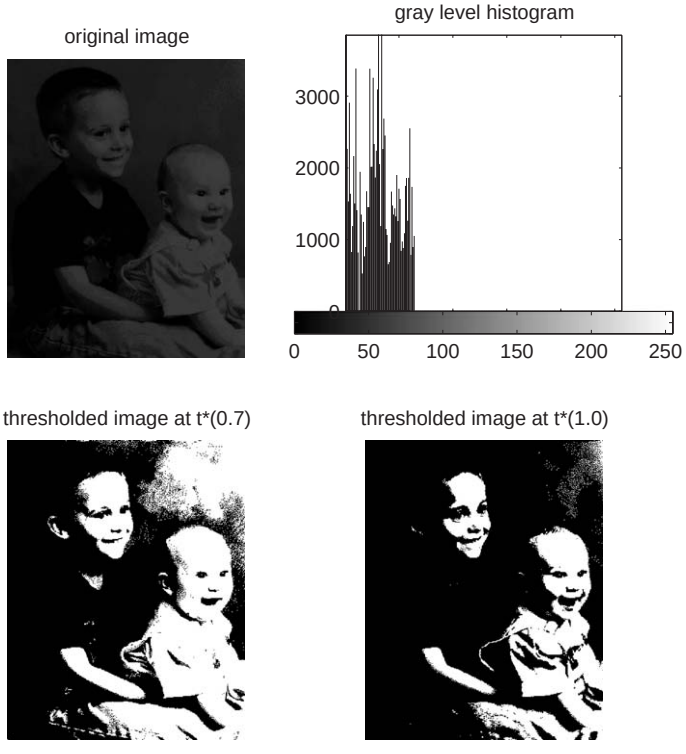

Fig. 3. Kids image, its gray level histogram and the thresholded images.

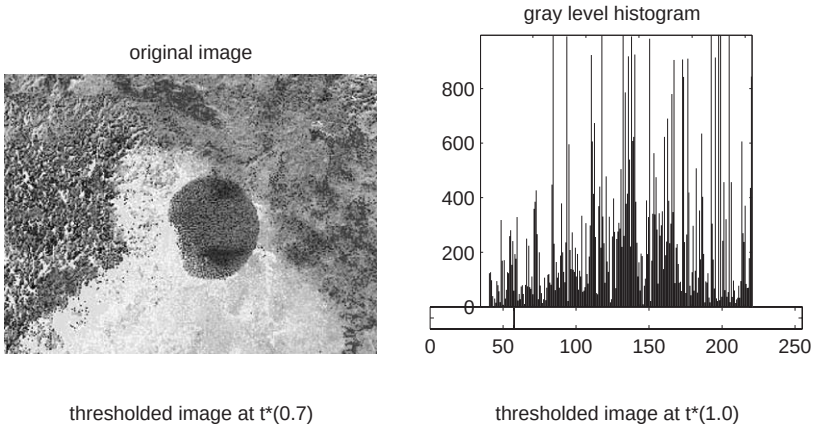

Fig. 4. Shadow image, its gray level histogram and the thresholded images.

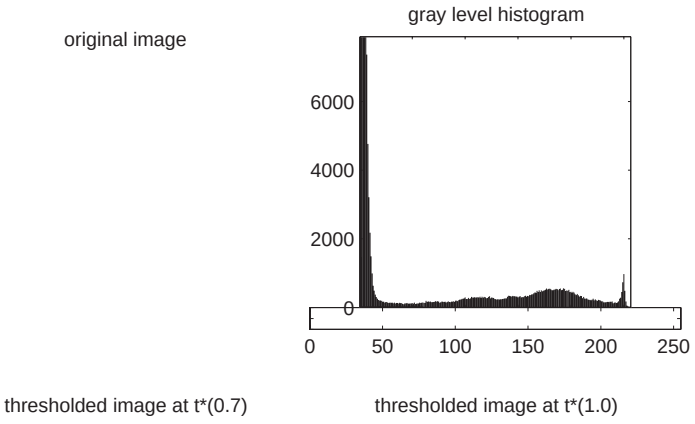

Fig. 5. Moon image, its gray level histogram and the thresholded images.

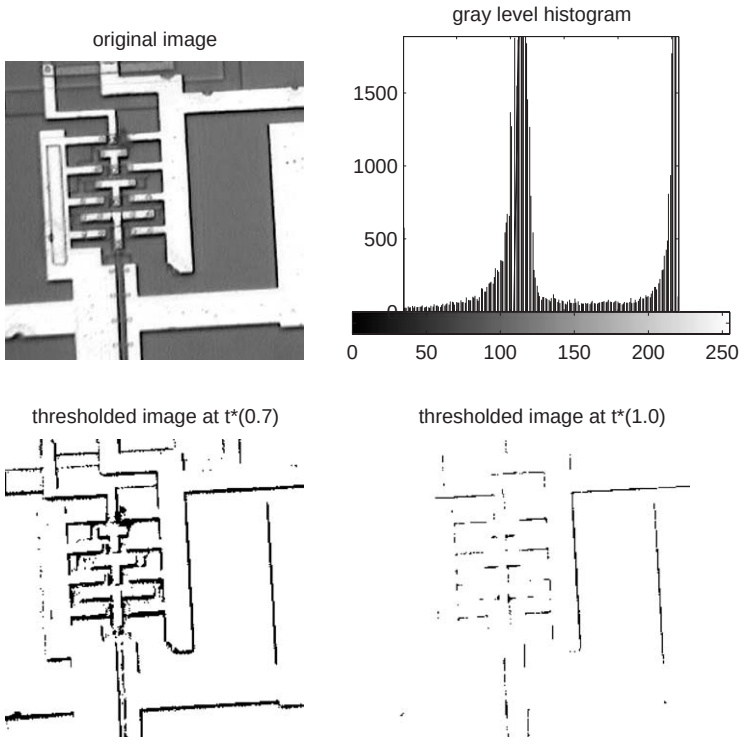

Fig. 10. Ic image, its gray level histogram and the thresholded images.

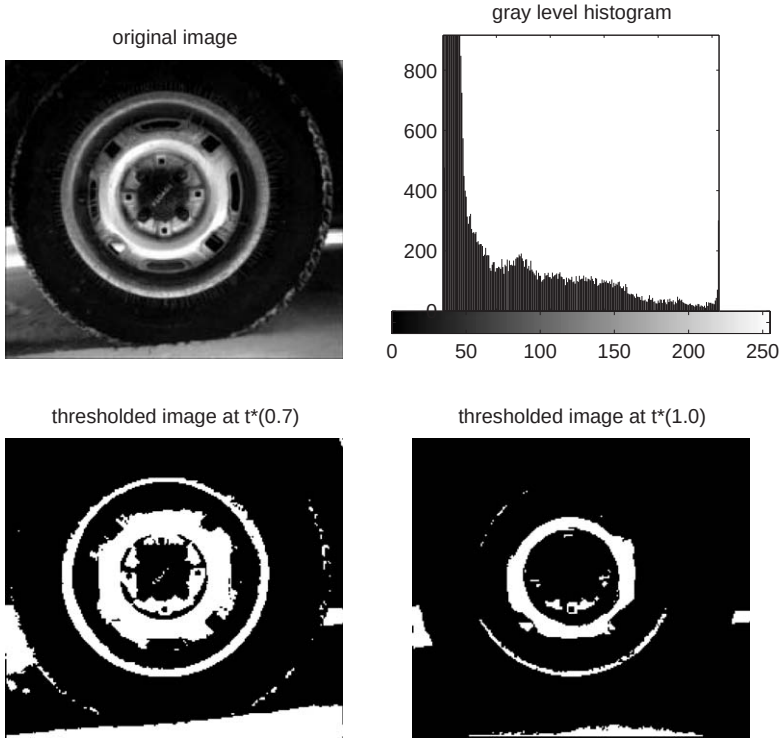

Fig. 11. Tire image, its gray level histogram and the thresholded images.

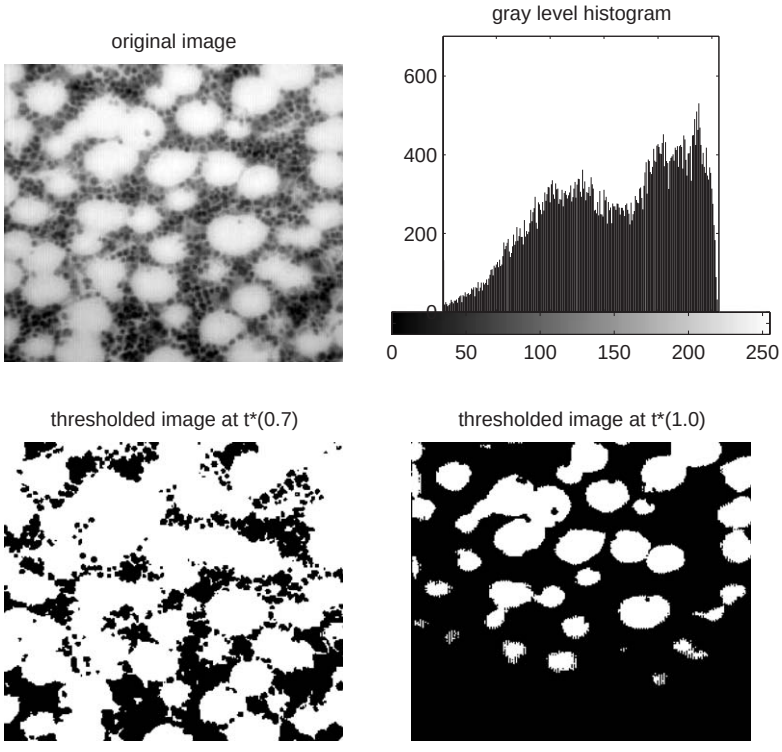

Fig. 12. Bonemarr image, its gray level histogram and the thresholded images.

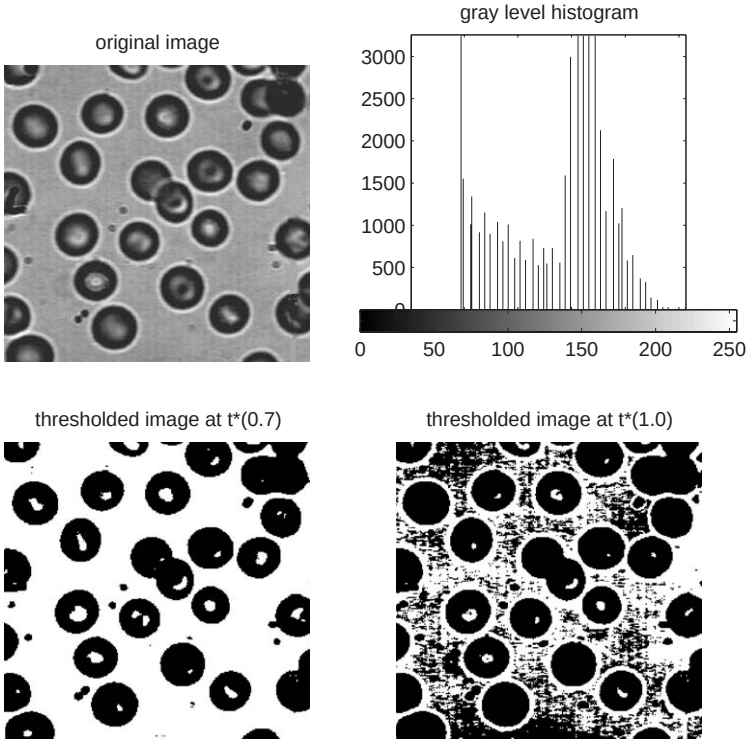

Fig. 13. Blood1 image, its gray level histogram and the thresholded images.

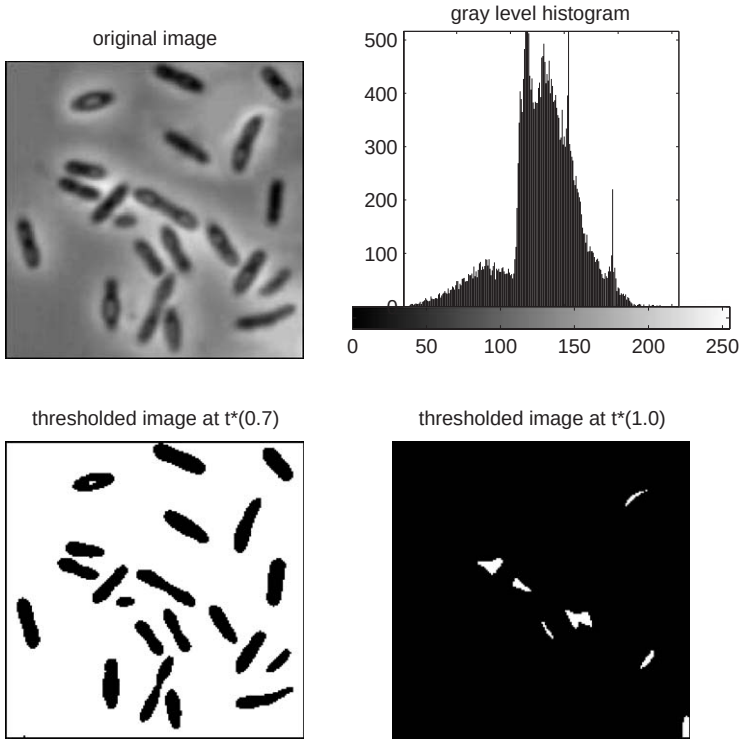

Fig. 14. Bacteria image, its gray level histogram and the thresholded images.

threshold value $t^{\star}(1.0)$ is same as the one obtained by Abutaleb's method [1]. Our analysis is based on how much information is lost due to thresholding. In this analysis, given two thresholded images of a same original image, we prefer the one which lost the least amount of information. Using the above thirteen images and also some other images, we conclude that when α value lies between 0 and 1, our proposed method produced good optimal threshold values. Moreover, the optimal threshold value does not change very much when the fractional α value changes a little. However, when α was greater than 1, this proposed method did not produce good threshold values. In fact the threshold values produced were unacceptable (see the last column of Table 1). When the value of α was one, the threshold value produced was not always a good threshold value (see Figs. 2–14). This new method performs better than Abutaleb's method for fractional values of α.

In this method of thresholding, we have used in addition to the original gray level function $f(x, y)$, a function $g(x, y)$ that is the average gray level value in a 3×3 neighborhood around the pixel (x, y). This approach can be extended to an image pyramid, where an image on the next higher level is composed of average gray level values computed for disjoint 3×3 squares. In order to find out which pyramid level will produce an optimal threshold value, we have tested all the test images with different neighborhood sizes and different α values. In particular, we have considered neighborhood sizes of 2×2, 3×3, 4×4, 5×5, 8×8 and 16×16, respectively. Neighborhood sizes of 3×3, 4×4, 5×5 and 8×8 all produced similar optimal threshold values giving acceptable binary images. From the point of view of computational time and image quality, a neighborhood size of 3×3 or 5×5 with α value around 0.5 or 0.7 would be ideal for thresholding with this proposed method.

4. Conclusion

We developed a method of image segmentation that employs Renyi's entropy of order α and a two-dimensional histogram. The two-dimensional histogram was constructed using the gray value and local average gray value to choose an optimal threshold value. This method of thresholding includes Abutaleb's method [1], and the two-dimensional version of the entropic correlation method [7]. In almost every image used, the proposed method yielded a good threshold value for fractional α, that is, when $0 < \alpha < 1$. For $\alpha > 1$, the proposed method did not yield a good threshold value. When $\alpha = 1$, the method yielded, in some cases, good optimal threshold values and, in some other cases, unacceptable threshold values. This new bilevel method of thresholding can also be extended to multilevel thresholding with some appropriate modifications and will be considered elsewhere. It is already pointed out in the introduction that the

two-dimensional extension gives rise to the exponential increment of computational time. However, we have used the commercial software MATLAB to implement the proposed method and thus we have taken the advantage of the vector computation that MATLAB offers. Because of this our method takes less than a minute on a Macintosh computer with dual 1 GHz PowerPC G4 processors.

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