ON PLANNING SMOOTH PATHS FOR MARINE VEHICLES

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Abstract: The issue of designing smooth planar paths with maximum curvature and curvature derivative constraints is addressed. The proposed solution can be used to build C^1 class reference paths for marine vehicles linking a set of via points while guaranteeing bounded curvature and curvature derivative.

Keywords: Path planning, bounded curvature.

1. INTRODUCTION

The problem of path following is of great interest in marine applications. Given a curve $y \in R^p$ parametrized by some scalar $s \in R$ (by example the curvilinear abscissa), denoting with $r \in R^q$ the pose (position and orientation) of the vehicle being

$$\dot{r} = f(r, \xi, u) \quad : \quad u \text{ control input} \tag{1}$$

(ξ non manipulable input) and with $d(r, y) \in R$ a suitable distance between the vehicle and the path, the path following control law design problem consists in finding a function $u = u^*(r, y)$ (u^* may eventually depend by ξ too) such that

$$\lim_{t \to \infty} d(r|_{u=u^*}, y) = 0. \tag{2}$$

In most marine applications p is either 2 or 3 and q is either 3 (2D position plus the yaw angle in the planar $p = 2$ case) or 5 (3D position plus pitch and yaw angles in the 3D $p = 3$ case). As opposed to the trajectory tracking problem, where a vehicle is asked to converge on a time parametrized desired pose $r_d(t)$, path following is time invariant in nature: the issue is to converge and stay on a given path without any specific time reference. Many papers have addressed the path following control problem for single marine vehicles and, more recently, for groups of coordinated marine vehicles. As for the single vehicle case, recent results include Indiveri et al. (2000) Encarnacao et al. (2000)Aicardi et al. (2001)Fossen et al. (2003) Breivik and Fossen (2004) Bakaric et al. (2004) where several nonlinear control techniques are employed to address the problem. Generally, performances of these controllers in terms of precision are related to the structure of the reference path. This should be compatible with the kinematics of the vehicle and, in particular, it should satisfy curvature and spatial curvature derivative bounds. The problem of designing suitable paths has received relatively minor attention: in the late eighties Kanayama and Hartman (1989) suggested to approach the planar path design problem starting from the paths curvature, rather than in Cartesian space, to better handle vehicle kinematics and dynamic constraints. Their solution is obtained minimizing the integral over length of the square curvature and of the square

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curvature derivative. The resulting curves are arcs of curves called cubic spirals by the authors. As shown in (Kanayama and Hartman (1989)), this approach allows to compute paths joining an arbitrary pair of poses in a relatively simple way, yet the resulting curve is not guaranteed to have bounded curvature or curvature derivative by desired thresholds. In other approaches, paths are generated joining straight lines and arcs of circles in order to link a given set of via points. By choosing the circle radius su–ciently large, these paths can always satisfy upper bounds on the curvature, but not on its derivative that will be discontinuous. In the planar case these kinds of paths, if suitably designed, can be length optimal with an upper bound on the curvature. This is a classical result by Dubins (1957) who showed that for a forward (only) moving vehicle, the length optimal planar curve joining two assigned poses with bounded curvature $|\kappa| \leq \bar{\kappa}$ is made of segments and circles of radius $1/\bar{\kappa}$. This result was extended in Reeds and Shepp (1990) to the case of a both forwards and backwards moving vehicle showing that, once again, the length optimal bounded curvature paths are made of segments and maximum curvature circular arcs. The resulting curvature proflle with respect to the driven length is a piecewise constant curve taking only null and $\pm\bar{\kappa}$ values. Due to the curvature discontinuity, such curve can be only approximately driven by a vehicle moving with speed $v(t) \neq 0 \ \forall t$. In order to make the curvature versus length proflle $\kappa(l)$ continuous while approximating the length optimal curves, Fraichard and Scheuer (2004) have proposed to link the piecewise constant parts with linear functions $\kappa = \alpha l + \beta$ corresponding to arcs of clothoids (also known as Cornu spirals). This methods allows to set an upper bound on the curvature derivative, i.e. $|\alpha|$. Inspired by this work, the present paper describes a method to generate planar paths with a class C^1 curvature proflle with respect to length and satisfying upper bounds on the curvature and its derivative with respect to length. The basic idea is to build a curvature proflle with respect to length using a C^1 class function to join constant curvature segments over a flnite length. A pictorial representation of the approach as compared to Dubins (1957) and Fraichard and Scheuer (2004) solutions is given in flgure (2). As opposed to alternative solutions, the proposed method can be used to generate globally smooth paths joining assigned via points (ore poses) with upper bounds on the curvature and its derivative. Notice that solutions as the one of Lamiraux and Laumond (2001), while being perhaps better suited to cope with complex environments cluttered with obstacles, generate only locally smooth paths, i.e. piecewise smooth paths among consecutive pairs of poses, without guaranteeing curvature and curvature derivative bounds along the path.

2. BASIC IDEA

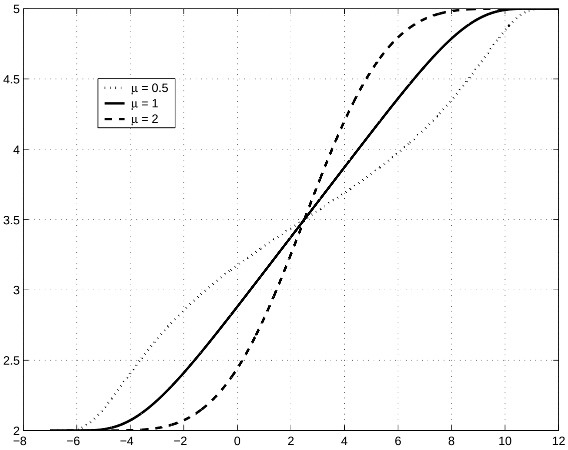

Fig. 1. Examples of $f(x)$ curves for $(x_i, x_f, y_i, y_f) = (-7, 12, 2, 5)$ and $\mu = 0.5, 1, 2$.

Consider the function

$$f_s(\theta) = \begin{cases} \tanh(\tan(\theta)) & \text{if } \theta \in (-\pi/2, \pi/2) \\ 1 & \text{if } \theta \geq \pi/2 \\ -1 & \text{if } \theta \leq -\pi/2. \end{cases} \qquad (3)$$

This function belongs to class C^1 and can be used to smoothly link the lines $y = -1$ and $y = 1$ between the abscissa values $\theta_i = -\pi/2$ and $\theta_f = \pi/2$. Given the basic property

$$\frac{d}{dx} \tanh(x) = 1 - \tanh^2(x), \qquad (4)$$

it can be shown by direct calculation that

$$\begin{aligned} & 0 \leq \frac{d}{d\theta} f_s(\theta) \leq 1 \quad \forall \theta \\ & \frac{d}{d\theta} f_s(\theta) = 1 \quad \text{iff} \quad \theta = 0 \\ & \frac{d^n}{d\theta^n} f_s(\theta) = 0 : n \geq 1 \quad \text{iff} \quad \theta \in I_s \\ & I_s := \{\theta : \theta \leq -\pi/2 \ \cup \ \theta \geq \pi/2\}, \end{aligned}$$

namely that the maximum slope of the curve is 1. In order to generalize such curve to the case where two arbitrary parallel lines $y = y_i$ and $y = y_f$ need to be joint in the distinct x values x_i and x_f ($x_i \neq x_f$) with an upper bound on the maximum slope, consider the following:

$$m := \frac{\pi}{x_f - x_i} \qquad (5)$$

$$p := -\frac{\pi}{2} \left(\frac{x_f + x_i}{x_f - x_i} \right) \qquad (6)$$

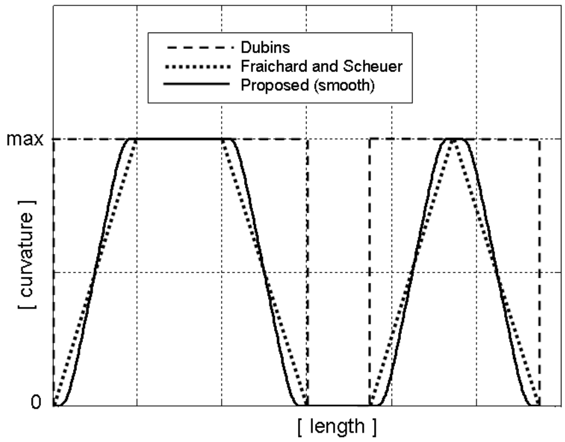

Fig. 2. Comparison of the proposed solution

$$C := \frac{y_f - y_i}{2} \tag{7}$$

$$D := \frac{y_f + y_i}{2} \tag{8}$$

$$\mu > 0 \tag{9}$$

$$f(x) = \begin{cases} C \tanh\left(\mu \tan(mx + p)\right) + D & \text{if } x \in (x_i, x_f) \\ y_f & \text{if } x \geq x_f \\ y_i & \text{if } x \leq x_i \end{cases} \tag{10}$$

that is of class C^1. Notice that with the above defined parameter the linear function $mx + p$ simply maps $[x_i, x_f]$ in $[-\pi/2, \pi/2]$. If μ should satisfy

$$\mu \geq 1 \tag{11}$$

and $C \neq 0$ then the above defined C^1 class function can be proven to satisfy:

$$0 \leq \frac{d}{dx} f(x) \leq \mu m C \quad \forall x \tag{12}$$

$$\frac{d}{dx} f(x) = \mu m C \quad \text{iff} \quad x = x' \tag{13}$$

$$x' := \frac{x_f + x_i}{2} \tag{14}$$

$$\frac{d^n}{dx^n} f(x) = 0 : n \geq 1 \quad \text{iff} \quad x \in I \tag{15}$$

$$I := \{x : x \leq x_i \cup x \geq x_f\}. \tag{16}$$

Notice that x' as defined in equation (14) is simply the mid point between x_f and x_i. Moreover, if condition (11) should be violated, namely if $0 < \mu < 1$, function $f(x)$ would still belong to C^1, but it would be no longer true that its derivative would be always bounded by $\mu m C$ although

$$\left.\frac{d}{dx} f(x)\right|_{x = x^*} = \mu m C$$

would still hold true (see figure 1 for an example).

In the limit $\mu \to \infty$ the $f(x)$ function would tend to a step function. Function (10) can be used to define a class C^1 bounded curvature profile with respect to path length having bounded curvature derivative. In particular letting x be the arc length, y_i and y_f will be the initial (typically zero) and final curvature ($y_f \leq \kappa_{max}$ being κ_{max} the maximum absolute value of the allowed path curvature) and $\sigma \geq \mu m C$ will be the absolute value of the maximum curvature derivative with respect to length. A pictorial example of a similar curvature profile as compared with alternative solutions available in the literature is reported in figure (2). Notice that such profile would correspond to a set of two right hand turns (the curvature is positive) starting and ending on straight lines. Within this setting, each turn starting on a straight line at l_j and ending on a straight line at l_{j+3} would be given by the following class C^1 curvature profile:

$$m_j := \frac{\pi}{l_{j+1} - l_j} \tag{17}$$

$$p_j := -\frac{\pi}{2} \left(\frac{l_{j+1} + l_j}{l_{j+1} - l_j} \right) \tag{18}$$

$$\kappa(l) = \begin{cases} 0 & \text{if } l \leq l_j \\ \frac{\kappa'}{2} \left(\tanh\left(\mu \tan(m_j l + p_j)\right) + 1\right) & \\ & \text{if } l \in (l_j, l_{j+1}) \\ \kappa' & \text{if } l_{j+1} \leq l \leq l_{j+2} \\ \frac{\kappa'}{2} \left(-\tanh\left(\mu \tan(m_{j+2} l + p_{j+2})\right) + 1\right) & \\ & \text{if } l \in (l_{j+2}, l_{j+3}) \\ 0 & \text{if } l \geq l_{j+3} \end{cases} \tag{19}$$

being $l_j : j = 1, 2, \ldots$ a sequence of increasing path length values and κ' such that $|\kappa'| \leq \kappa_{max}$, $\kappa' > 0$ for right hand side turns and $\kappa' < 0$ for left hand side turns.

According to the previous analysis, given an upper bound σ on the maximum absolute value of the curvature derivative with respect to length, the curvature profile parameters of a single turn linking two straight lines would need to satisfy

$$\begin{cases} \mu \leq \frac{\sigma}{mC} \\ \mu \geq 1 \end{cases} \Rightarrow \quad 1 \leq \mu \leq \frac{2\sigma}{\pi |\kappa'|} \Delta l \tag{20}$$

being Δl the length of the nonconstant curvature arcs ($(l_{j+1} - l_j)$ or $(l_{j+3} - l_{j+2})$ in equation (19)). Consequently, the sharpest turns, i.e. the turns corresponding to the maximum heading change in the least travelled distance, with bounded curvature and curvature derivative will correspond to

$$\mu = 1 \tag{21}$$

$$\Delta l = \frac{\pi |\kappa'|}{2\sigma} \tag{22}$$

$$|\kappa(l)| \leq |\kappa'| \leq \kappa_{max} \tag{23}$$

$$\left|\frac{d\kappa}{dl}\right| \le \sigma \tag{24}$$

3. PLANNING SMOOTH PLANAR TURNS

The above results may be employed to plan globally smooth bounded curvature and bounded curvature derivative planar turns. In the sequel the notion of simple turn as opposed to an S shaped turn will be employed: a simple turn corresponds to the case where the curvature along the turn does not change sign, whereas an S shaped turn corresponds to the case where the curvature along the turn takes both positive and negative values. S shaped turns are typically needed to link parallel straight lines to be driven in the same direction, whereas simple turns are usually needed to change heading. In our everyday car driving experience an S shaped turn would be, by example, a change of lane while a U turn would be a simple turn. The parameters relative to a length optimal simple turn of $\Delta\theta$[rad] linking two straight lines may be obtained noticing that

$$\Delta\theta = \kappa' \int_{0}^{\frac{\pi|\kappa^*|}{2\sigma}} \left(\tanh\left(\tan\left(\frac{2\sigma}{|\kappa'|} l - \pi/2\right)\right) + 1\right) dl + \kappa' L_{circ} \tag{25}$$

being L_{circ} the length of the nonnull constant curvature arc ($(l_{j+2} - l_{j+1})$ in equation (19)), κ' such that $|\kappa'| \le \kappa_{max}$ and σ the upper bound of the absolute value of the curvature derivative. The integral in equation (25) can be computed numerically for given sets of σ and κ' yielding the results reported in figure (3).

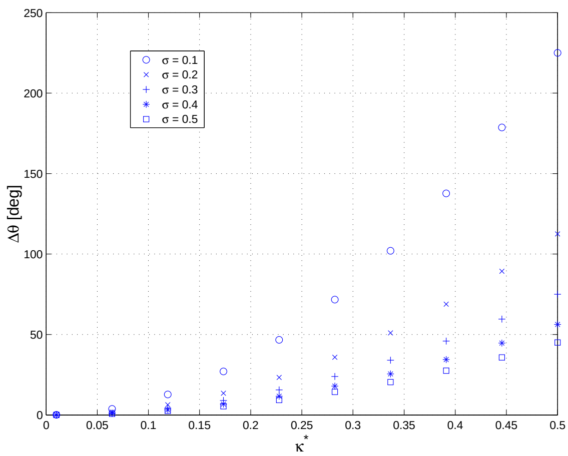

Fig. 3. Length optimal heading variations for different values of σ and κ' with $L_{circ} = 0$ (refer to equation (25)).

Each data point in the plot of figure (3) corresponds to a curve having length $\pi|\kappa'|/\sigma$. The provided data may be used to compute the parameters of the desired smooth turns while satisfying curvature and curvature derivative bounds. If, by example, an 100[deg] turn would be needed with $\sigma = 0.2$ and $\kappa_{max} = 0.4$, it can be concluded from figure (3) that a nonnull circular arc of $\kappa = \kappa'$ curvature will be needed. In particular equation (25) reveals that for $\sigma = 0.2$ and $\kappa' = 0.4$, $\Delta\theta = 72.00 + 0.4\,\frac{180}{\pi} L_{circ}$. Equating $\Delta\theta$ to 100[deg] and solving for L_{circ} it follows that $L_{circ} = 1.2217$[LU][2]. Knowing L_{circ}, σ and κ' the corresponding path may be generated on the basis of equation (19). The resulting path and its curvature profile are depicted in figure (4) where the thick solid parts of the plots correspond to the circular arc of length $L_{circ} = 1.2217$ and radius $R = 1/\kappa' = 2.5$ that is reached and left smoothly.

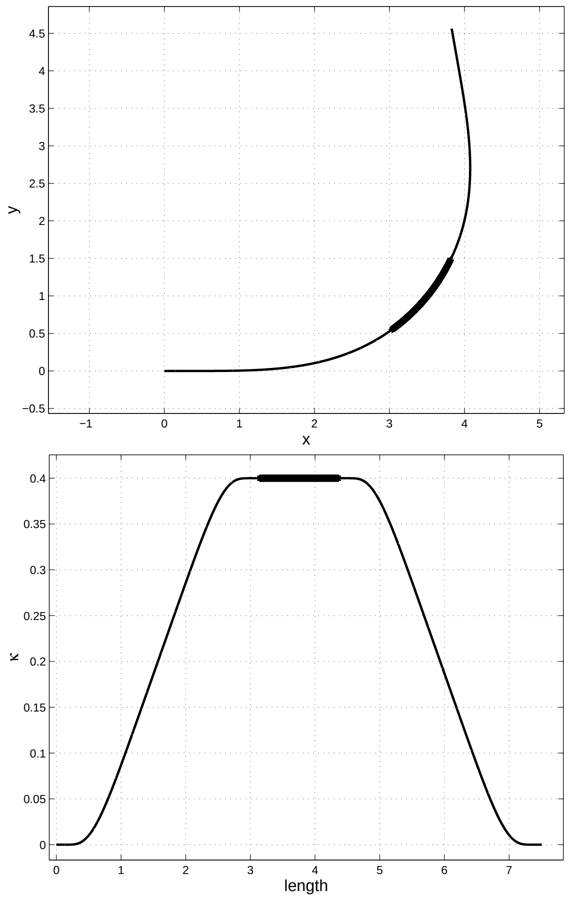

Fig. 4. Smooth path and curvature profile of an 100[deg] turn with curvature bounded by $\kappa_{max} = 0.4$ and curvature derivative bounded by $\sigma = 0.2$. The thick parts of the plots correspond to a circular arc of κ_{max} curvature and L_{circ} length.

[2] [LU] stands for an arbitrary length unit. Notice that $[\kappa]$=[rad][LU]$^{-1}$