

Pt i: UNKNOWN POINT (x_i, y_i, z_i)

$(x_i^0, y_i^0, z_i^0) \rightarrow$ APPROX LOCATION OF Pt i

Pt j: KNOWN POINT (x_j, y_j, z_j)

SPEED OF SOUND IN WATER = C

C_0 = APPROX SPEED OF SOUND IN WATER

MEASURED TRAVEL TIME BETWEEN Pt i and j = t_{ij}

ADJUSTED TRAVEL TIME

$$\hat{t}_{ij} = \frac{1}{C} [(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2]^{1/2}$$

LINEARIZATION

$$\hat{t}_{ij} = t_{ij}^0 + \frac{\partial t_{ij}}{\partial x_i} \Delta x_i + \frac{\partial t_{ij}}{\partial y_i} \Delta y_i + \frac{\partial t_{ij}}{\partial z_i} \Delta z_i + \frac{\partial t_{ij}}{\partial C} \Delta C$$

$$\Delta x_i = x_i - x_i^0$$

$$\Delta y_i = y_i - y_i^0$$

$$\Delta C = C - C_0$$

$$\Delta z_i = z_i - z_i^0$$

$$t_{ij}^0 = \frac{1}{C_0} [(x_j - x_i^0)^2 + (y_j - y_i^0)^2 + (z_j - z_i^0)^2]^{1/2}$$

$$b_1 = -\frac{\partial t_{ij}}{\partial x_i} = -\frac{\partial}{\partial x_i} \left[\frac{1}{C} [(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2]^{1/2} \right]$$

$$= \frac{1}{C^2} \frac{(x_j - x_i)}{\hat{t}_{ij}} \approx \frac{1}{C_0^2} \frac{(x_j - x_i^0)}{t_{ij}^0}$$

$$b_2 = -\frac{\partial t_{ij}}{\partial y_i} = \frac{1}{C^2} \frac{(y_j - y_i)}{\hat{t}_{ij}} \approx \frac{1}{C_0^2} \frac{(y_j - y_i^0)}{t_{ij}^0}$$

$$b_3 = -\frac{\partial t_{ij}}{\partial z_i} = \frac{1}{C^2} \frac{(z_j - z_i)}{\hat{t}_{ij}} \approx \frac{1}{C_0^2} \frac{(z_j - z_i^0)}{t_{ij}^0}$$

$$b_4 = -\frac{\partial t_{ij}}{\partial C} = \frac{1}{C^2} \hat{t}_{ij} \approx \frac{1}{C_0} t_{ij}^0$$

$$\hat{t}_{ij} = t_{ij}^0 - b_1 \Delta x_i - b_2 \Delta y_i - b_3 \Delta z_i - b_4 \Delta c$$

$$\hat{t}_{ij} = t_{ij} + v_{ij}$$

t_{ij} = OBSERVED TRAVEL TIMES

v_{ij} = RESIDUAL

$$v_{ij} + b_1 \Delta x_i + b_2 \Delta y_i + b_3 \Delta z_i + b_4 \Delta c = t_{ij}^0 - t_{ij} = f_{ij} \quad \left. \begin{array}{l} 1 \text{ EQN FOR} \\ \text{EACH OBSERVED} \\ \text{TRAVEL TIMES} \end{array} \right\}$$

$N = 4$ OBSERVED TRAVEL TIMES
 $\rightarrow$ DETERMINED SYSTEM

$N > 4$ OBSERVED TRAVEL TIMES
 $\rightarrow$ OVER-DETERMINED SYSTEM
 (USE LEAST SQUARES)

$$\begin{array}{c} v \\ \left[\begin{array}{c} \\ \\ \\ \end{array} \right] \\ N \times 1 \end{array} + \begin{array}{c} B \\ \left[\begin{array}{c} \\ \\ \\ \end{array} \right] \\ N \times 4 \end{array} = \begin{array}{c} \left[\begin{array}{c} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ \Delta c \end{array} \right] \\ 4 \times 1 \end{array} = \begin{array}{c} f \\ \left[\begin{array}{c} \\ \\ \\ \end{array} \right] \\ N \times 1 \end{array} \quad \left. \begin{array}{l} \text{ONE ROW} \\ \text{FOR} \\ \text{EACH} \\ \text{OBSERVED} \\ \text{TRAVEL TIME.} \end{array} \right\}$$