PRESSURE VESSEL CALCULATIONS

PROJECT: WISSL-2G **DATE: 04/18/2024** **ANALYST: J.T.**

Revisions: (last rev date 4/18/2024)

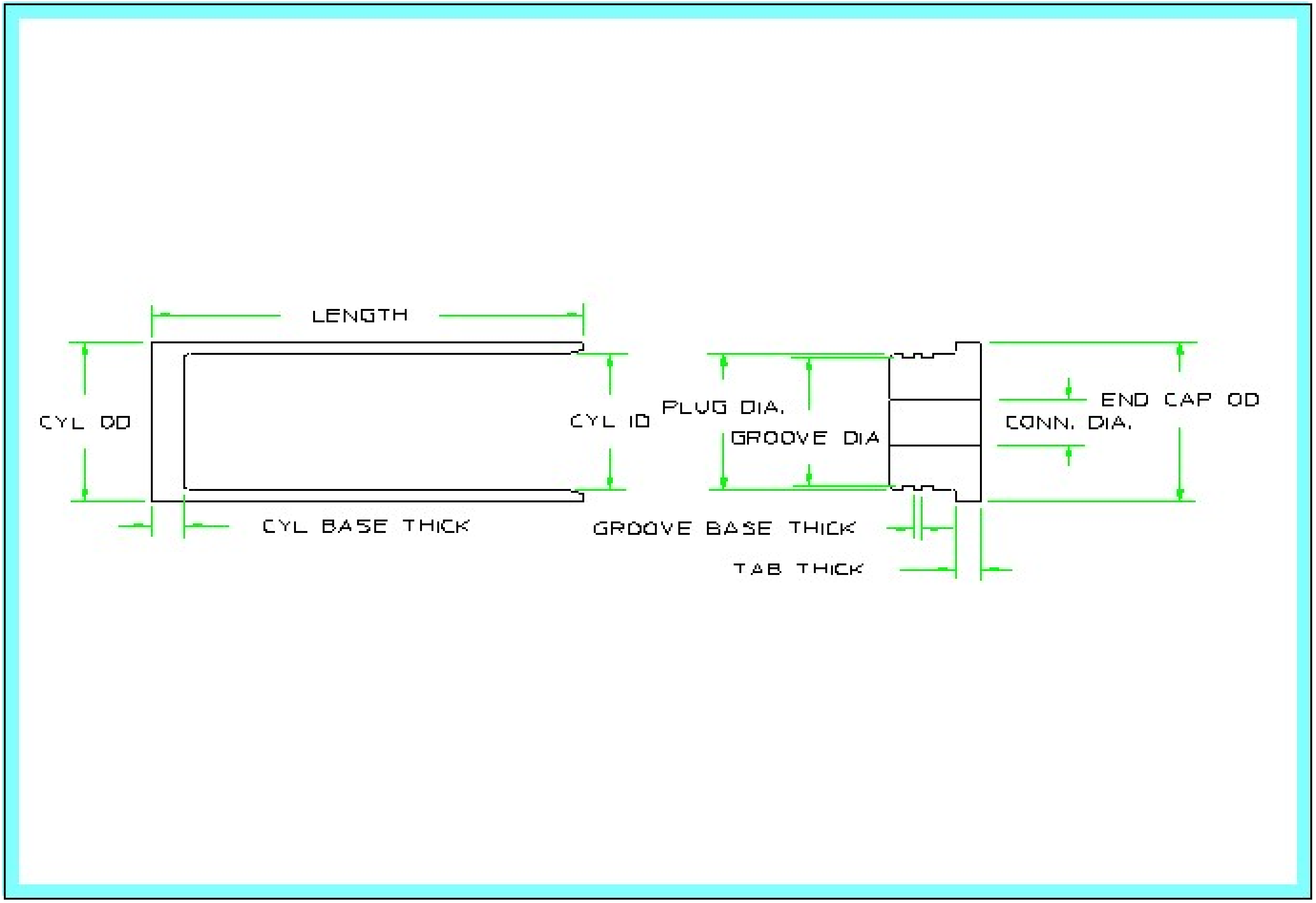
0.1) Direct Conversion Update from J.E. PV6.0 to Mathcad Prime 9 formatting

The following calculations are for determining the margin of safety for a pressure vessel at a given depth. The equations are mostly from Roark's Formulas for Stress and Strain and are referenced when used. The user must input the dimensions, material properties, and design depth. A factor of safety will be included in the calculations to determine the minimum margin of safety. The minimum margin of safety must be greater than zero for the design to be suitable for the depth desired.

The format for this analysis package will be as follows:

A) Input of design parameters including dimensions, material properties, and design depth
B) Calculations for deformation and stresses in cylinders (check for thin wall or thick wall condition)
C) Buckling calculations for cylinders
D) Calculations for deformations and stresses in end plates,
E) Weight calculation
F) Summary of input and list of results including diametral gap, stresses, margins of safety and weight.

Note: The minimum margin of safety will be determined based upon the lowest margin calculated in the sections above. Thus, for a give pressure vessel, the maximum working pressure may be limited by the end plate thickness while for another it may be limited by the length of the cylindrical section. Therefor, if a desired allowable working depth is not reached, the limiting factor needs to be determined in order to effectively iterate the design to meet the requirements. Also, the diametral gap must be checked for the different pressures to assure that it falls within the maximum allowable from the o-ring vendor.


LENGTH
CYL OD
CYL ID
PLUG DIA.
GROOVE DIA
CONN. DIA.
END CAP OD
CYL BASE THICK
GROOVE BASE THICK
TAB THICK

A) Design parameters:

1) Dimensional input:

Enter cylinder dimensions:

Inside Diameter : $ID_{cyl} \equiv 5.6 \cdot in$

Outside Diameter: $OD_{cyl} \equiv 6.375 \cdot in$

O-ring surface diameter: $ID_{cyl\_oring} \equiv 5.6 \cdot in$

Case Length: $L_{cyl} \equiv 22 \cdot in$

Cylinder Base Thickness: $T_{cyl\_base} \equiv 0.5 \cdot in$

Wall thickness:

$$T_{cyl\_wall} \equiv \frac{OD_{cyl} - ID_{cyl}}{2}$$

$$T_{cyl\_wall} = 0.3875 \ in$$

Enter end cap and gland dimensions (O-Ring #2-xxx):

Plug Diameter: $OD_{plug} \equiv 5.596 \cdot in$

Groove Width: $W_{groove} \equiv .188 \cdot in$

Groove Diameter: $ID_{groove} \equiv 5.438 \cdot in$

Groove Base Thickness: $T_{oring\_tab} \equiv 0.147 \cdot in$

End Cap OD: $OD_{cap} \equiv 6.375 \cdot in$

End Cap Thickness: $T_{cap} \equiv 1.450 \cdot in$

Tab Thickness: $T_{endcap\_tab} \equiv 0.7 \cdot in$

Connector Diameter: $D_{conn} \equiv 0.60 \cdot in$

2) Material properties:

Enter material properties: Material: **Titanium Ti-6Al-4V (Grade 5), Annealed**

Modulus of elasticity: $E \equiv 16.5 \cdot 10^3 \cdot ksi$

Material density: $\rho_m \equiv .16 \cdot \frac{lbf}{in^3}$

Poisson's ratio: $\nu \equiv 0.342$

Yield Stress: $S_y \equiv 128 \cdot ksi$

Shear stress: $S_s \equiv 79.8 \cdot ksi$

3) Design depth and resulting pressure:

$depth \equiv 6000 \cdot m$

Water density: $\rho_{sw} \equiv 1023 \ \frac{kgf}{m^3}$

Pressure: $q \equiv depth \cdot \rho_{sw} = \left(8.73 \cdot 10^3\right) \ psi$

n pressure steps: $N_{pressure} \equiv 10$

4) Factors of safety to be used:

for buckling: $FS_{buckling} \equiv 2$

for stress: $FS \equiv 1.25$

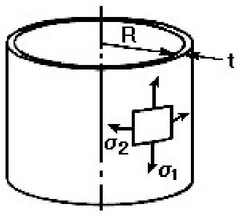
B) Formulas for deformation and stresses in cylinders:

1) Thin walled cylinder calculation:

Reference: Table 28 Formulas for membrane stresses and deformations in thin-walled pressure vessels

Case 1c **Cylindrical shell; uniform internal or external pressure (ends capped)**

Cylindrical shell



Calculated dimensions:

Mean radius:

$$R_{thin} \equiv \left(\frac{ID_{cyl}}{2} + \frac{T_{cyl\_wall}}{2}\right)$$

$$R_{thin} = 2.994\ in$$

Cylinder Height:

$$y \equiv L_{cyl}$$

$$y = 22\ in$$

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$$thinWall \equiv \left(\frac{R_{thin}}{T_{cyl\_wall}} > 10\right) \qquad \frac{R_{thin}}{T_{cyl\_wall}} = 7.726$$

$$thinWall = 0$$

Thin wall stresses:

Meridional (axial) stress:

$$\sigma_{1thin} \equiv \frac{q \cdot R_{thin}}{2 \cdot T_{cyl\_wall}} \qquad \sigma_{1thin} = 33.724\ ksi$$

Circumferential (hoop) stress:

$$\sigma_{2thin} \equiv \frac{q \cdot R_{thin}}{T_{cyl\_wall}} \qquad \sigma_{2thin} = 67.449\ ksi$$

Von Mises Stress calculation:

$$\sigma_{vmthin} \equiv \sqrt{\sigma_{1thin}^{\ 2} - \sigma_{1thin} \cdot \sigma_{2thin} + \sigma_{2thin}^{\ 2}}$$

$$\sigma_{vmthin} = 58.412\ ksi$$

Corresponding margins of safety:

for yield strength:

$$MS_{cythin} \equiv \left(\frac{S_y}{\sigma_{vmthin} \cdot FS} - 1\right) \qquad MS_{cythin} = 0.753$$

THIN WALL SHELL FAILURE AT: $q \cdot \left(1 + MS_{cythin}\right) \cdot FS = \left(1.913 \cdot 10^4\right)\ psi$

Change in height: $\Delta y_{thin} \equiv \dfrac{q \cdot R_{thin} \cdot y}{E \cdot T_{cyl\_wall}} \cdot (0.5 - v)$

$\Delta y_{thin} = 0.014\ in$

Radial displacement of circumference: $\Delta R_{max} \equiv \dfrac{q \cdot R_{thin}^{\ 2}}{E \cdot T_{cyl\_wall}} \cdot \left(1 - \dfrac{v}{2}\right)$

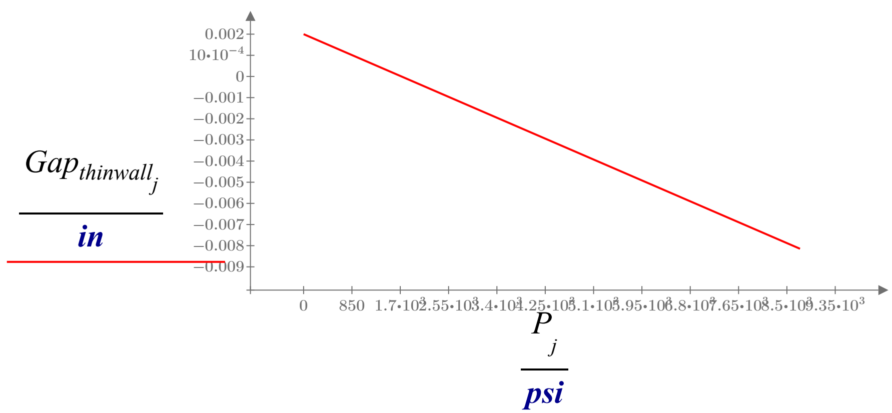
$\Delta R_{max} = 0.01\ in$

Diametral clearance (extrusion gap):

Pressure step: $P_{step} := \dfrac{q}{N_{pressure}}$ $\quad P_{step} = 873.029\ psi \quad j := 0 .. 10 \quad P_j := j \cdot P_{step}$

Extrusion gap (radial):

$$\Delta R_{thinwall_j} := \left(\frac{P_j \cdot R_{thin}^{\ 2}}{E \cdot T_{cyl\_wall}} \cdot \left(1 - \frac{v}{2}\right)\right) \qquad Gap_{thinwall_j} := \frac{ID_{cyl\_oring} - OD_{plug}}{2} - \Delta R_{thinwall_j}$$



$$\frac{Gap_{thinwall_j}}{in} = \begin{bmatrix} 0.002 \\ 9.855 \cdot 10^{-4} \\ -2.903 \cdot 10^{-5} \\ -0.001 \\ -0.002 \\ -0.003 \\ -0.004 \\ -0.005 \\ -0.006 \\ -0.007 \\ -0.008 \end{bmatrix}$$

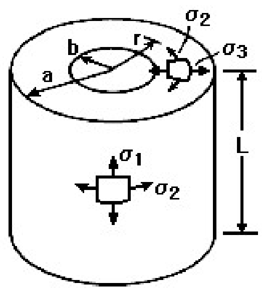
The pressure at which the extrusion gap (GAPj) is equal to the change in interior radius (ΔRj)

$$P_{\Delta 0} := \left(\frac{ID_{cyl\_oring} - OD_{plug}}{2}\right) \cdot \frac{E \cdot T_{cyl\_wall}}{R_{thin}^{\ 2} \cdot \left(1 - \dfrac{v}{2}\right)} \qquad P_{\Delta 0} = \left(1.721 \cdot 10^{3}\right)\ psi$$

2) Thick walled cylinder calculations:

Table 32 Formulas for thick-walled vessels under internal and external loading

Case 1d Cylindrical disk or shell; uniform external pressure in all directions, ends capped



Cylindrical disk or shell

Calculated dimensions:

$a_{thick} \equiv 0.5 \cdot OD_{cyl}$ $b_{thick} \equiv 0.5 \cdot ID_{cyl}$ $L \equiv L_{cyl}$

$a_{thick} = 3.188\ in$ $b_{thick} = 2.8\ in$ $L = 22\ in$

Formulas for change in outer and inner radii, and change in length:

Outer radius:

$$\Delta a_{thick} \equiv \frac{-q \cdot a_{thick}}{E} \cdot \frac{{a_{thick}}^2 \cdot (1 - 2 \cdot \nu) + {b_{thick}}^2 \cdot (1 + \nu)}{{a_{thick}}^2 - {b_{thick}}^2}$$

$\Delta a_{thick} = -0.01\ in$

Inner radius:

$$\Delta b_{thick} \equiv \frac{-q \cdot b_{thick}}{E} \cdot \frac{{a_{thick}}^2 \cdot (2 - \nu)}{{a_{thick}}^2 - {b_{thick}}^2}$$

$\Delta b_{thick} = -0.011\ in$

Length:

$$\Delta L_{thick} \equiv \frac{-q \cdot L}{E} \cdot \frac{{a_{thick}}^2 \cdot (1 - 2 \cdot \nu)}{{a_{thick}}^2 - {b_{thick}}^2}$$

$\Delta L_{thick} = -0.01611\ in$

Formulas for normal stresses as a function of radial thickness, r:

radius range variable

$$step := \frac{a_{thick} - b_{thick}}{10}$$

$$r := b_{thick}, b_{thick} + step \,..\, a_{thick}$$

Longitudinal stress:

$$\sigma_{1thick} \equiv \frac{-q \cdot a_{thick}^{2}}{a_{thick}^{2} - b_{thick}^{2}}$$

$$\sigma_{1thick} = -3.823 \cdot 10^{4}\ psi$$

Circumferential stress:

$$\sigma_{2thick}(r) \equiv \frac{-q \cdot a_{thick}^{2} \cdot \left(b_{thick}^{2} + r^{2}\right)}{r^{2} \cdot \left(a_{thick}^{2} - b_{thick}^{2}\right)}$$

$$\sigma_{2thick}\left(b_{thick}\right) = -7.646 \cdot 10^{4}\ psi$$

Radial stress:

$$\sigma_{3thick}(r) \equiv \frac{-q \cdot a_{thick}^{2} \cdot \left(r^{2} - b_{thick}^{2}\right)}{r^{2} \cdot \left(a_{thick}^{2} - b_{thick}^{2}\right)}$$

$$\sigma_{3thick}\left(b_{thick}\right) = 0\ psi$$

Shear stress @ b:

$$\tau_{thickb} \equiv \frac{q \cdot a_{thick}^{2}}{a_{thick}^{2} - b_{thick}^{2}}$$

$$\tau_{thickb} = \left(3.823 \cdot 10^{4}\right)\ psi$$

Von Mises Stress calculation:

$$\sigma a_{vmthick} \equiv \sqrt{\frac{\left(\sigma_{1thick} - \sigma_{2thick}\left(a_{thick}\right)\right)^{2} + \left(\sigma_{2thick}\left(a_{thick}\right) - \sigma_{3thick}\left(a_{thick}\right)\right)^{2} + \left(\sigma_{3thick}\left(a_{thick}\right) - \sigma_{1thick}\right)^{2}}{2}}$$

$$\sigma a_{vmthick} = 51.096\ ksi$$

$$\sigma b_{vmthick} \equiv \sqrt{\frac{\left(\sigma_{1thick} - \sigma_{2thick}\left(b_{thick}\right)\right)^{2} + \left(\sigma_{2thick}\left(b_{thick}\right) - \sigma_{3thick}\left(b_{thick}\right)\right)^{2} + \left(\sigma_{3thick}\left(b_{thick}\right) - \sigma_{1thick}\right)^{2}}{2}}$$

$$\sigma b_{vmthick} = 66.217\ ksi$$

$$\sigma_{vmthick} = \left(6.622 \cdot 10^{4}\right)\ psi$$

$$\sigma_{vmthick} \equiv \max\left(\sigma a_{vmthick}, \sigma b_{vmthick}\right)$$

Corresponding margins of safety:

for yield strength:

$$MSyield_{cythick} \equiv \left(\frac{S_{y}}{\sigma_{vmthick} \cdot FS} - 1\right)$$

$$MSyield_{cythick} = 0.546$$

Based on Von Mises stress

for shear strength:

$$MSshear_{cythick} \equiv \left(\frac{0.5 \cdot S_{y}}{\tau_{thickb} \cdot FS} - 1\right)$$

$$MSshear_{cythick} = 0.339$$

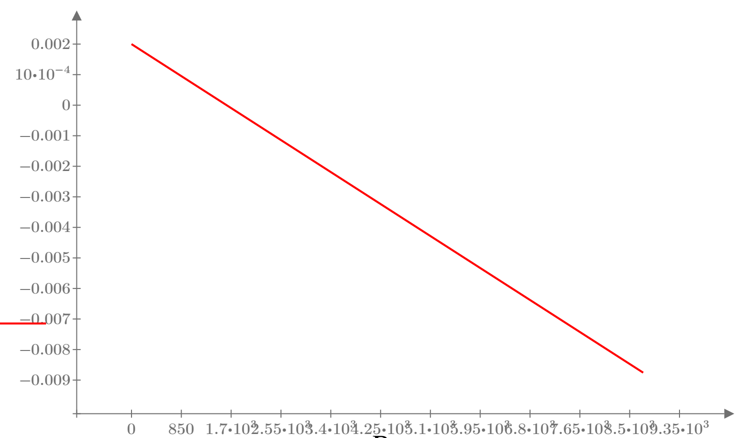
Based on Max. shear stress theory 0.5*Sy

Diametral clearance (extrusion gap):

Pressure step: $P_{step} := \dfrac{q}{N_{pressure}}$ $\qquad P_{step} = 873.029\ psi \qquad j := 0..10 \qquad P_j := j \cdot P_{step}$

Extrusion gap (radial): $\Delta b_{thickwall_j} := \dfrac{P_j \cdot b_{thick}}{E} \cdot \dfrac{a_{thick}^{\ 2} \cdot (2 - \nu)}{a_{thick}^{\ 2} - b_{thick}^{\ 2}}$ $\qquad Gap_{thickwall_j} := \dfrac{ID_{cyl\_oring} - OD_{plug}}{2} - \Delta b_{thickwall_j}$

$\dfrac{Gap_{thickwall_j}}{in}$



$\dfrac{P_j}{psi}$

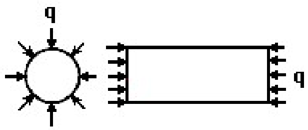
$$\frac{Gap_{thickwall_j}}{in} = \begin{bmatrix} 0.002 \\ 9.244 \cdot 10^{-4} \\ -1.513 \cdot 10^{-4} \\ -0.001 \\ -0.002 \\ -0.003 \\ -0.004 \\ -0.006 \\ -0.007 \\ -0.008 \\ -0.009 \end{bmatrix}$$

C) Buckling calculation for cylinders:

Table 35 Formulas for elastic stability of plates and shells

Case 20a — **Thin tube with closed ends under uniform external pressure, lateral and longitudinal; ends held circular**

Thin tube with closed ends under uniform external pressure, lateral and longitudinal



Calculated dimensions:

Tube dimensions:

radius: $r_{buckling} \equiv \left(\frac{ID_{cyl}}{2} + \frac{T_{cyl\_wall}}{2} \right)$ length: $L = 22\ in$

$r_{buckling} = 2.994\ in$

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$condition := \frac{r_{buckling}}{T_{cyl\_wall}} > 10$ $condition = 0$

Critical pressure:

The critical pressure for the initial buckling of a cylinder is:

number of lobes: $n \equiv 2..10$

$$p_n \equiv \frac{E \cdot \frac{T_{cyl\_wall}}{r_{buckling}}}{1 + \frac{1}{2} \cdot \left(\frac{\pi \cdot r_{buckling}}{n \cdot L} \right)^2} \cdot \left(\frac{1}{n^2 \cdot \left(1 + \left(\frac{n \cdot L}{\pi \cdot r_{buckling}} \right)^2 \right)^2} + \frac{n^2 \cdot T_{cyl\_wall}^2}{12 \cdot r_{buckling}^2 \cdot \left(1 - \nu^2 \right)} \cdot \left(1 + \left(\frac{\pi \cdot r_{buckling}}{n \cdot L} \right)^2 \right)^2 \right)$$

$p_{cropped} \equiv \text{submatrix}\left(p, 2, 10, 0, 0 \right)$ note: must trim the full p matrix to just the index range defined by n so that min doesn't pull empty 0's

$p_{critical} \equiv 0.8 \cdot min\left(p_{cropped} \right)$ $p_{critical} = \left(1.235 \cdot 10^4 \right)\ psi$

Where experimental values of pcritical are +/- 20% of pmin !!!

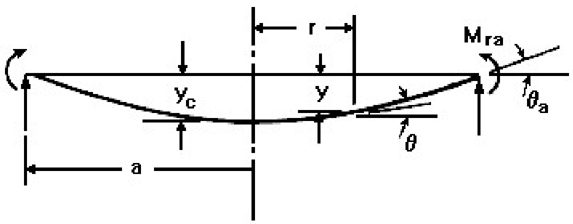
Margin of safety on buckling:

$MS_{buckling} \equiv \left(\frac{p_{critical}}{q \cdot FS_{buckling}} - 1 \right)$ $MS_{buckling} = -0.293$

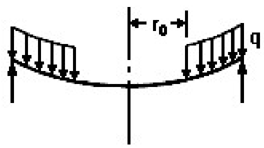
D) Solid circular plate simply supported; uniformly distributed pressure from ro to a

Table 24 Case 10a

Solid circular plate



Uniformly distributed pressure from ro to a



/?????????????

CLOSED END CYLINDER BASE CALCS:

Check stress in bottom:

Cylinder Base Thickness:

$t_{base} \equiv T_{cyl\_base}$

$a_{base} \equiv 0.5 \cdot ID_{cyl}$

If assumed simply supported:

$t_{base} = 0.5\ in$

$a_{base} = 2.8\ in$

$r_{base0} \equiv 0$

Plate Constant:

$$D_{base} \equiv \frac{E \cdot t_{base}^{\ 3}}{12 \cdot \left(1 - \nu^2\right)}$$

$D_{base} = \left(1.946 \cdot 10^5\right)\ lbf \cdot in$

$$G_{17} \equiv \frac{3 + \nu}{16}$$

$G_{17} = 0.209$

$$M_{cbase} \equiv \frac{q \cdot a_{base}^{\ 2} \cdot (3 + \nu)}{16}$$

$M_{cbase} = \left(1.43 \cdot 10^4\right)\ lbf \cdot \frac{in}{in}$

$$\sigma_{base} \equiv 6 \cdot \frac{M_{cbase}}{t_{base}^{\ 2}}$$

$\sigma_{base} = 343.117\ ksi$

$$Q_{abase} \equiv \frac{-q \cdot a_{base}}{2}$$

$Q_{abase} = -1.222 \cdot 10^4\ \frac{lbf}{in}$

$$ycenter_{base} \equiv \frac{-q \cdot a_{base}^{\ 4}}{64 \cdot D_{base}} \cdot \frac{5 + \nu}{1 + \nu}$$

$ycenter_{base} = -0.171\ in$

Margin: $MS_{base} \equiv \left(\frac{S_y}{\sigma_{base} \cdot FS} - 1\right)$

$MS_{base} = -0.702$

check small deflection assumption (deflection < t/2)

$$\frac{ycenter_{base}}{t_{base}} = -0.343$$

END CAP DEFLECTION AND STRESSES:

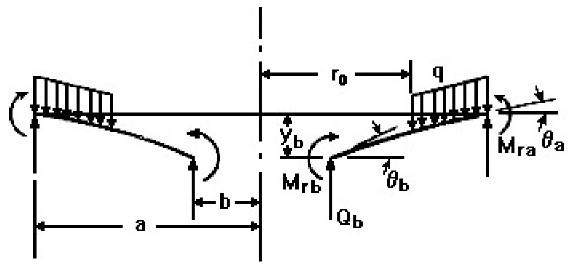
Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Annular plate with uniformly distributed pressure q over the portion from ro to a; outer edge simply supported

This file corresponds to Cases 2a - 2d in *Roark's Formulas for Stress and Strain.*

Annular plate with a uniformly distributed pressure q over the portion from ro to a

Outer edge simply supported, inner edge free



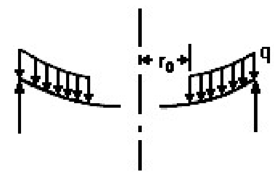


Plate dimensions:

thickness: $t_{endcap} \equiv T_{cap}$

outer radius: $a_{endcap} \equiv \dfrac{ID_{cyl\_oring}}{2}$

inner radius: $b_{endcap} \equiv \dfrac{D_{conn}}{2}$ $\qquad b_{endcap} = 0.3\ in$

Shear modulus: $G \equiv \dfrac{E}{2 \cdot (1+v)}$ $\qquad G = \left(6.148 \cdot 10^{6}\right)\ psi$

$M_{rbendcap} \equiv 0.0 \cdot lbf$ $\qquad Q_{bendcap} \equiv 0 \cdot \dfrac{lbf}{in}$ $\qquad y_{aendcap} \equiv 0 \cdot in$ $\qquad M_{raendcap} \equiv 0 \cdot in \cdot lbf$ $\qquad r_{0endcap} \equiv b_{endcap}$

in*lbf / in $\qquad$ **in*lbf / in**

Plate Constant

$D_{endcap} \equiv \dfrac{E \cdot t_{endcap}{}^{3}}{12 \cdot \left(1-v^{2}\right)}$

$D_{endcap} = \left(4.7471 \cdot 10^{6}\right)\ lbf \cdot in$

in*lbf / in

$$C_{endcap4} \equiv \frac{1}{2} \cdot \left((1+v) \cdot \frac{b_{endcap}}{a_{endcap}} + (1-v) \cdot \frac{a_{endcap}}{b_{endcap}} \right)$$

$$C_{endcap1} \equiv \frac{1+v}{2} \cdot \frac{b_{endcap}}{a_{endcap}} \cdot \ln\left(\frac{a_{endcap}}{b_{endcap}}\right) + \frac{1-v}{4} \cdot \left(\frac{a_{endcap}}{b_{endcap}} - \frac{b_{endcap}}{a_{endcap}}\right)$$

$$C_{endcap7} \equiv \frac{1}{2} \cdot \left(1-v^2\right) \cdot \left(\frac{a_{endcap}}{b_{endcap}} - \frac{b_{endcap}}{a_{endcap}}\right)$$

$$L_{endcap14} \equiv \frac{1}{16} \cdot \left(1 - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^4 - 4 \cdot \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2 \cdot \ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right)$$

$$L_{endcap11} \equiv \frac{1}{64} \cdot \left(1 + 4 \cdot \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2 - 5 \cdot \left(\frac{r_{0endcap}}{a_{endcap}}\right)^4 - 4 \cdot \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2 \cdot \left(2 + \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2\right) \cdot \ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right)$$

$$L_{endcap17} \equiv \frac{1}{4} \cdot \left(1 - \left(\frac{1-v}{4}\right) \cdot \left(1 - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^4\right) - \left(\frac{r_{0endcap}}{a_{endcap}}\right)^2 \cdot \left(1 + (1+v) \cdot \ln\left(\frac{a_{endcap}}{r_{0endcap}}\right)\right)\right)$$

$$y_{endcapb} \equiv \frac{-q \cdot {a_{endcap}}^4}{D_{endcap}} \cdot \left(\frac{C_{endcap1} \cdot L_{endcap17}}{C_{endcap7}} - L_{endcap11}\right)$$

$$y_{endcapb} = -0.008\ in$$

$$Q_a \equiv \frac{-q}{2 \cdot a_{endcap}} \cdot \left({a_{endcap}}^2 - {r_{0endcap}}^2\right)$$

$$\theta_{endcapb} \equiv \frac{q \cdot {a_{endcap}}^3}{D_{endcap} \cdot C_{endcap7}} \cdot L_{endcap17}$$

$$\theta_{endcapb} = 0.112\ deg$$

$$Q_a = -1.208 \cdot 10^4\ \frac{lbf}{in}$$

$$\theta_{endcapa} \equiv \frac{q \cdot {a_{endcap}}^3}{D_{endcap}} \cdot \left(\frac{C_{endcap4} \cdot L_{endcap17}}{C_{endcap7}} - L_{endcap14}\right)$$

$$\theta_{endcapa} = 0.223\ deg$$

Transverse

$$M_{endcapt} \equiv \frac{\theta_{endcapb} \cdot D_{endcap} \cdot \left(1 - v^2\right)}{b_{endcap}}$$ $$M_{endcapt} = \left(2.734 \cdot 10^4\right)\ lbf$$ $$\sigma_{endcap} \equiv \frac{6 \cdot M_{endcapt}}{t_{endcap}{}^2}$$ $$\sigma_{endcap} = \left(7.801 \cdot 10^4\right)\ psi$$

check small deflection
assumption
(deflection < t/2)

$$\frac{y_{endcapb}}{t_{endcap}} = -0.005$$

Should be < 1 ???

$$\frac{y_{endcapb}}{.5 \cdot t_{endcap}} = -0.011$$

Margi
n:

$$MS_{endcap} \equiv \left(\frac{S_y}{\sigma_{endcap} \cdot FS} - 1\right)$$ $$MS_{endcap} = 0.313$$

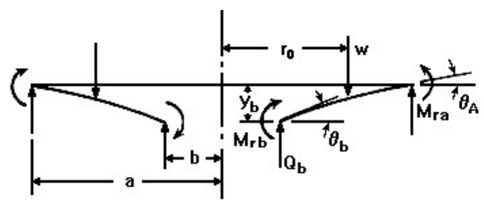
Stresses in tabs:

For a conservative approach, assume the outer edge of the tabs are free and the inner edge fixed

A) Top tab:

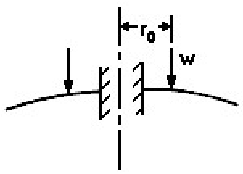
Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Annular plate with uniform annular line load w at radius ro; outer edge free



Case 1l:

Outer edge free, inner edge fixed



Applied Unit Load @ OD of Case ??

$t_{endcap\_tab} \equiv T_{endcap\_tab}$

$D_{endcap\_tab} \equiv \frac{E \cdot t_{endcap\_tab}^{\ 3}}{12 \cdot (1 - \nu^2)}$

$D_{endcap\_tab} = (5.341 \cdot 10^5)\ lbf \cdot in$

Loading on tab:

$W_{endcap\_tab} \equiv q \cdot \pi \cdot \frac{ID_{cyl\_oring}^{\ 2}}{4}$

$W_{endcap\_tab} = (2.15 \cdot 10^5)\ lbf$

$y_{bendcap\_tab} \equiv 0$

$\theta_{bendcap\_tab} \equiv 0$

$M_{aendcap\_tab} \equiv 0$

$Q_{aendcap\_tab} \equiv 0$

$a_{endcap\_tab} \equiv \frac{OD_{cap}}{2}$

$b_{endcap\_tab} \equiv \frac{OD_{plug}}{2}$

$r_{endcap\_tab0} \equiv a_{endcap\_tab}$

$t_{endcap\_tab} = 0.7\ in$

$b_{endcap\_tab} = 2.798\ in$

$w_{endcap\_tab} \equiv \frac{W_{endcap\_tab}}{\pi \cdot OD_{plug}}$

$w_{endcap\_tab} = (1.223 \cdot 10^4)\ \frac{lbf}{in}$

$a_{endcap\_tab} = 3.188\ in$

$$C_{endcap\_tab2} \equiv \frac{1}{4} \cdot \left(1 - \left(\frac{b_{endcap\_tab}}{a_{endcap\_tab}}\right)^2 \cdot \left(1 + 2 \cdot \ln\left(\frac{a_{endcap\_tab}}{b_{endcap\_tab}}\right)\right)\right)$$

$$C_{endcap\_tab3} \equiv \frac{b_{endcap\_tab}}{4 \cdot a_{endcap\_tab}} \cdot \left(\left(\left(\frac{b_{endcap\_tab}}{a_{endcap\_tab}}\right)^2 + 1\right) \cdot \ln\left(\frac{a_{endcap\_tab}}{b_{endcap\_tab}}\right) + \left(\frac{b_{endcap\_tab}}{a_{endcap\_tab}}\right)^2 - 1\right)$$

$$C_{endcap\_tab8} \equiv \frac{1}{2} \cdot \left(1 + v + (1 - v) \cdot \left(\frac{b_{endcap\_tab}}{a_{endcap\_tab}}\right)^2\right)$$

$$C_{endcap\_tab9} \equiv \frac{b_{endcap\_tab}}{a_{endcap\_tab}} \cdot \left(\frac{1+v}{2} \cdot \ln\left(\frac{a_{endcap\_tab}}{b_{endcap\_tab}}\right) + \left(\frac{1-v}{4}\right) \cdot \left(1 - \left(\frac{b_{endcap\_tab}}{a_{endcap\_tab}}\right)^2\right)\right)$$

$$L_{endcap\_tab9} \equiv \frac{r_{endcap\_tab0}}{a_{endcap\_tab}} \cdot \left(\frac{1+v}{2} \cdot \ln\left(\frac{a_{endcap\_tab}}{r_{endcap\_tab0}}\right) + \frac{1-v}{4} \cdot \left(1 - \left(\frac{r_{endcap\_tab0}}{a_{endcap\_tab}}\right)^2\right)\right)$$

$$L_{endcap\_tab6} \equiv \frac{r_{endcap\_tab0}}{4 \cdot a_{endcap\_tab}} \cdot \left(\left(\frac{r_{endcap\_tab0}}{a_{endcap\_tab}}\right)^2 - 1 + 2 \cdot \ln\left(\frac{a_{endcap\_tab}}{r_{endcap\_tab0}}\right)\right)$$

$$L_{endcap\_tab3} \equiv \frac{r_{endcap\_tab0}}{4 \cdot a_{endcap\_tab}} \cdot \left(\left(\left(\frac{r_{endcap\_tab0}}{a_{endcap\_tab}}\right)^2 + 1\right) \cdot \ln\left(\frac{a_{endcap\_tab}}{r_{endcap\_tab0}}\right) + \left(\frac{r_{endcap\_tab0}}{a_{endcap\_tab}}\right)^2 - 1\right)$$

$$M_{endcap\_tabrb} \equiv \frac{-w_{endcap\_tab} \cdot a_{endcap\_tab}}{C_{endcap\_tab8}} \cdot \left(\frac{r_{endcap\_tab0} \cdot C_{endcap\_tab9}}{b_{endcap\_tab}} - L_{endcap\_tab9}\right) \qquad M_{endcap\_tabrb} = -5.28 \cdot 10^3 \ lbf$$

$$y_{endcap\_tabmax} \equiv \frac{-w_{endcap\_tab} \cdot a_{endcap\_tab}{}^3}{D_{endcap\_tab}} \cdot \left(\frac{C_{endcap\_tab2}}{C_{endcap\_tab8}} \cdot \left(\frac{r_{endcap\_tab0} \cdot C_{endcap\_tab9}}{b_{endcap\_tab}} - L_{endcap\_tab9}\right) - \frac{r_{endcap\_tab0} \cdot C_{endcap\_tab3}}{b_{endcap\_tab}} + L_{endcap\_tab3}\right)$$

$$y_{endcap\_tabmax} = -4.777 \cdot 10^{-4} \ in$$

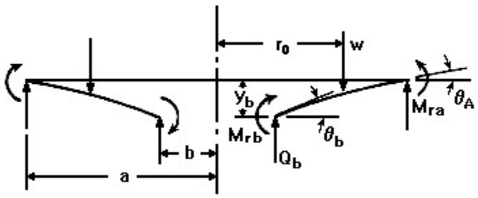
$$\sigma_{endcap\_tab} \equiv \frac{6 \cdot M_{endcap\_tabrb}}{t_{endcap\_tab}{}^2} \qquad \sigma_{endcap\_tab} = -6.465 \cdot 10^4 \ psi \qquad Q_{endcap\_tabb} \equiv \frac{w_{endcap\_tab} \cdot r_{endcap\_tab0}}{b_{endcap\_tab}} \qquad Q_{endcap\_tabb} = \left(1.393 \cdot 10^4\right) \frac{lbf}{in}$$

Margin: $MS_{endcap\_tab} \equiv \left(\frac{S_y}{\left|\sigma_{endcap\_tab}\right| \cdot FS} - 1\right) \quad MS_{endcap\_tab} = 0.584$

B) O-ring groove tab:

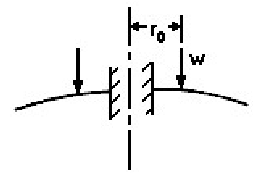
Table 24 Formulas for shear, moment and deflection of flat circular plates of constant thickness

Annular plate with uniform annular line load w at radius ro; outer edge free



Case 1I:

Outer edge free, inner edge fixed



For o-ring tab: $t_{oring} \equiv T_{oring\_tab}$

$$D_{oring} \equiv \frac{E \cdot t_{oring}^{3}}{12 \cdot (1 - \nu^2)} \qquad D_{oring} = (4.946 \cdot 10^3)\ lbf \cdot in$$

Loading on tab:

$$W_{oring} \equiv q \cdot \pi \cdot \frac{OD_{plug}^{2} - ID_{groove}^{2}}{4} \qquad W_{oring} = (1.195 \cdot 10^4)\ lbf \qquad w_{oring} \equiv \frac{W_{oring}}{\pi \cdot ID_{groove}}$$

$y_{oringb} \equiv 0$ $\theta_{oringb} \equiv 0$ $M_{oringra} \equiv 0$ $Q_{oringa} \equiv 0$

$$w_{oring} = 699.712\ \frac{lbf}{in}$$

$a_{oring} \equiv 0.5 \cdot OD_{plug}$ $b_{oring} \equiv 0.5 \cdot ID_{groove}$ $r_{oring0} \equiv a_{oring}$

$$C_{oring2} \equiv \frac{1}{4} \cdot \left(1 - \left(\frac{b_{oring}}{a_{oring}}\right)^2 \cdot \left(1 + 2 \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right)\right)\right)$$

$$C_{oring3} \equiv \frac{b_{oring}}{4 \cdot a_{oring}} \cdot \left(\left(\left(\frac{b_{oring}}{a_{oring}}\right)^2 + 1\right) \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right) + \left(\frac{b_{oring}}{a_{oring}}\right)^2 - 1\right)$$

$$L_3 \equiv \frac{r_{oring0}}{4 \cdot a_{oring}} \cdot \left(\left(\left(\frac{r_{oring0}}{a_{oring}}\right)^2 + 1\right) \cdot \ln\left(\frac{a_{oring}}{r_{oring0}}\right) + \left(\frac{r_{oring0}}{a_{oring}}\right)^2 - 1\right)$$

$$L_{oring9} \equiv \frac{r_{oring0}}{a_{oring}} \cdot \left(\frac{1+v}{2} \cdot \ln\left(\frac{a_{oring}}{r_{oring0}}\right) + \frac{1-v}{4} \cdot \left(1 - \left(\frac{r_{oring0}}{a_{oring}}\right)^2\right)\right)$$

$$y_{oringmax} \equiv \frac{-w_{oring} \cdot a_{oring}{}^4}{b_{oring} \cdot D_{oring}} \cdot \left(\frac{C_{oring2} \cdot C_{oring9}}{C_{oring8}} - C_{oring3}\right)$$

$$y_{oringmax} = -2.357 \cdot 10^{-5}\ in$$

Margin: $MS_{oring} \equiv \left(\frac{S_y}{\left|\sigma_{oring}\right| \cdot FS} - 1\right)$ $MS_{oring} = 5.517$

$$C_{oring8} \equiv \frac{1}{2} \cdot \left(1 + v + (1 - v) \cdot \left(\frac{b_{oring}}{a_{oring}}\right)^2\right)$$

$$C_{oring9} \equiv \frac{b_{oring}}{a_{oring}} \cdot \left(\frac{1+v}{2} \cdot \ln\left(\frac{a_{oring}}{b_{oring}}\right) + \left(\frac{1-v}{4}\right) \cdot \left(1 - \left(\frac{b_{oring}}{a_{oring}}\right)^2\right)\right)$$

$$L_{oring6} \equiv \frac{r_{oring0}}{4 \cdot a_{oring}} \cdot \left(\left(\frac{r_{oring0}}{a_{oring}}\right)^2 - 1 + 2 \cdot \ln\left(\frac{a_{oring}}{r_{oring0}}\right)\right)$$

$$M_{oringrb} \equiv \frac{-w_{oring} \cdot a_{oring}}{C_{oring8}} \cdot \left(\frac{r_{oring0} \cdot C_{oring9}}{b_{oring}} - L_{oring9}\right)$$

$$M_{oringrb} = -56.591\ lbf$$

$\sigma_{oring} \equiv \frac{6 \cdot M_{oringrb}}{t_{oring}{}^2}$ $\sigma_{oring} = -1.571 \cdot 10^4\ psi$

$Q_{oringb} \equiv \frac{w_{oring} \cdot r_{oring0}}{b_{oring}}$ $Q_{oringb} = 720.042\ \frac{lbf}{in}$

E) WEIGHT & DISPLACEMENT CHANGE CALCULATION:

1. Case Weight Calc.:

Cylinder: cross sectional area: $ac_x \equiv \frac{\pi \cdot \left(OD_{cyl}^2 - ID_{cyl}^2\right)}{4}$ $ac_x = 7.289\ in^2$

internal volume: $vc_i \equiv \pi \cdot \frac{ID_{cyl}^2}{4} \cdot \left(L_{cyl} - T_{cyl\_base}\right)$ $vc_i = 529.547\ in^3$

external volume: $vc_o \equiv \frac{\pi \cdot OD_{cyl}^2}{4} \cdot L_{cyl}$ $vc_o = 702.22\ in^3$

weight: $W_{cyl} \equiv ac_x \cdot L_{cyl} \cdot \rho_m$ $W_{cyl} = 25.657\ lbf$

End Cap: volume:

Tab: $v_{tab} \equiv \frac{\pi \cdot OD_{cap}^2}{4} \cdot T_{endcap\_tab}$ $v_{tab} = 22.343\ in^3$

Plug: $v_{plug} \equiv \frac{\pi \cdot OD_{plug}^2}{4} \cdot \left(T_{cap} - T_{endcap\_tab}\right)$ $v_{plug} = 18.446\ in^3$

Cap: $V_{cap} \equiv v_{plug} + v_{tab}$ $V_{cap} = 40.79\ in^3$

Weight: $W_{cap} \equiv V_{cap} \cdot \rho_m$ $W_{cap} = 6.526\ lbf$

Total Weight: $Wp_{dry} \equiv W_{cyl} + 2 \cdot W_{cap}$ $Wp_{dry} = 38.71\ lbf$

$Wp_{wet} \equiv Wp_{dry} - \left(vc_o + 2 \cdot v_{tab}\right) \cdot \rho_{sw}$ $Wp_{wet} = 11.106\ lbf$

2. Volume Change of Case at Depth:

Radius change due to press: $\Delta R \equiv \mathbf{if}\left(thinWall, \Delta R_{max}, \left|\Delta a_{thick}\right|\right)$ $\Delta R = 0.01\ in$

Length change due to press: $\Delta L \equiv \mathbf{if}\left(thinWall, \Delta y_{thin}, \left|\Delta L_{thick}\right|\right)$ $\Delta L = 0.016\ in$

Dished End Cap
Spherical Segment Vol.: $\delta V_{endcap} \equiv \pi \cdot \left|y_{endcapb}\right| \cdot \left(\frac{ID_{cyl\_oring}^{2}}{8} + \frac{y_{endcapb}^{2}}{6}\right)$ $\delta V_{endcap} = 0.095\ in^3$

external volume of cyl. (at depth): $V_{ext\_depth} \equiv \frac{\pi}{4} \cdot \left(OD_{cyl} - 2 \cdot \Delta R\right)^2 \cdot \left(L_{cyl} - \Delta L\right) + 2 \cdot v_{tab} - 2 \cdot \delta V_{endcap}$

$V_{ext\_depth} = 741.8142\ in^3$

external volume of cyl. (initial): $V_{ext} \equiv vc_o + 2 \cdot v_{tab}$ $V_{ext} = 746.9063\ in^3$

Pressure Case Volume Change ΔV: $\Delta V_{case} \equiv V_{ext} - V_{ext\_depth}$ $\Delta V_{case} = 5.0920\ in^3$

Assume Bulk Modulus of CH2O = 325,000 psi. $K \equiv 325000. \cdot psi$ $V_{CH2O} \equiv V_{ext}$

Compressed water vol. at depth: $\Delta V_{CH2O} \equiv \frac{q \cdot V_{CH2O}}{K}$ $\Delta V_{CH2O} = 20.0637\ in^3$

$\Delta\rho_{sw} \equiv \frac{q \cdot \rho_{sw}}{K}$ $\Delta\rho_{sw} = \left(9.928 \cdot 10^{-4}\right)\ \frac{lbf}{in^3}$ $\rho_{sw} + \Delta\rho_{sw} = 0.0380\ \frac{lbf}{in^3}$ $\rho_{sw} = 0.0370\ \frac{lbf}{in^3}$

Wet weight at depth: $Wp_{wet\_depth} \equiv Wp_{dry} - \left(V_{ext\_depth}\right) \cdot \left(\rho_{sw} + \Delta\rho_{sw}\right)$ $Wp_{wet\_depth} = 10.557\ lbf$

F) Summary

Thin Wall Calcs: Is it thin wall (1=yes)? $thinWall = 0$

Meridional (axial) stress: $\sigma_{1thin} = 33.724\ ksi$

Change in height: $\Delta y_{thin} = 0.014\ in$

Circumferential (hoop) stress: $\sigma_{2thin} = 67.449\ ksi$

Radial displacement of circumference: $\Delta R_{max} = 0.01\ in$

Margin of Safety: $MS_{cythin} = 0.753$

Thick Wall Calcs

Axial Stress: $\sigma_{1thick} = -3.823 \cdot 10^4\ psi$

Change in OD (radius): $\Delta a_{thick} = -0.010\ in$

Hoop Stress: $\sigma_{2thick}\left(b_{thick}\right) = -7.646 \cdot 10^4\ psi$

Change in ID (radius): $\Delta b_{thick} = -0.011\ in$

Radial Stress: $\sigma_{3thick}\left(b_{thick}\right) = 0\ psi$

Change in Length: $\Delta L_{thick} = -0.01611\ in$

Shear Stress: $\tau_{thickb} = \left(3.823 \cdot 10^4\right)\ psi$

Von Mises Stress: $\sigma_{vmthick} = 66.217\ ksi$

Margins of Safety: **for yield strength:** $MSyield_{cythick} = 0.546$ **for shear strength:** $MSshear_{cythick} = 0.339$

Closed End Cylinder Base Calcs:

$\sigma_{base} = 343.117\ ksi$ $ycenter_{base} = -0.171\ in$ $Q_{abase} = -1.222 \cdot 10^4\ \frac{lbf}{in}$ **Margin:** $MS_{base} = -0.702$

Buckling Calcs: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$\boxed{condition} := \frac{r_{buckling}}{T_{cyl\_wall}} > 10$ $condition = 0$

$p_{critical} = 12.349\ ksi$ **Margin of safety on buckling:** $MS_{buckling} = -0.293$

END CAP Calcs

END CAP Circular Plate

$y_{endcapb} = -0.008\ in$ $Q_a = -1.208 \cdot 10^4\ \frac{lbf}{in}$ $\sigma_{endcap} = \left(7.801 \cdot 10^4\right)\ psi$

MARGIN OF SAFETY: $MS_{endcap} = 0.313$

END CAP tabs:

$\sigma_{endcap\_tab} = -6.465 \cdot 10^4\ psi$ $Q_{endcap\_tabb} = \left(1.393 \cdot 10^4\right)\ \frac{lbf}{in}$

MARGIN OF SAFETY: $MS_{endcap\_tab} = 0.584$

O-RING tab:

$\sigma_{oring} = -1.571 \cdot 10^4\ psi$ $y_{oringmax} = -2.357 \cdot 10^{-5}\ in$ $Q_{oringb} = 720.042\ \frac{lbf}{in}$

MARGIN OF SAFETY: $MS_{oring} = 5.517$

CASE WEIGHTS

$W_{cyl} = 25.657\ \boldsymbol{lbf}$ $W_{cap} = 6.526\ \boldsymbol{lbf}$

Case + 2ea. Caps =

$Wp_{dry} = 38.71\ \boldsymbol{lbf}$ $Wp_{wet} = 11.106\ \boldsymbol{lbf}$

$Wp_{wet\_depth} = 10.557\ \boldsymbol{lbf}$

CASE VOLUME CHANGE

$V_{ext} = 746.906\ \boldsymbol{in}^3$

$\Delta V_{case} = 5.092\ \boldsymbol{in}^3$

$\Delta V_{CH2O} = 20.064\ in^3$

EndCap Flange/Case Bearing Stress

Bearing Area: $A_b := \frac{\pi}{4} \cdot \left(OD_{cap}^{\ 2} - ID_{cyl\_oring}^{\ 2}\right)$ $A_b = 7.289\ \boldsymbol{in}^2$

End Cap Force: $F_b := q \cdot \frac{\pi}{4} \cdot \left(OD_{cap}^{\ 2}\right)$ $F_b = \left(2.787 \cdot 10^5\right)\ lbf$

Bearing Stress: $\sigma_b := \frac{F_b}{A_b}$ $\sigma_b = \left(3.823 \cdot 10^4\right)\ psi$

Margin of safety: $MS_{bearing} := \left(\frac{S_y}{\sigma_b \cdot FS} - 1\right)$ $MS_{bearing} = 1.68$

EndCap Flange Shear Thru Stress

Flange Shear Area: $A_s := \pi \cdot \left(ID_{cyl\_oring}\right) \cdot T_{endcap\_tab}$ $A_s = 12.315\ \boldsymbol{in}^2$

End Cap Force: $F_s := q \cdot \frac{\pi}{4} \cdot \left(ID_{cyl\_oring}^{\ 2}\right)$ $F_s = \left(2.15 \cdot 10^5\right)\ lbf$

Shear Stress: $\sigma_s := \frac{F_s}{A_s}$ $\sigma_s = \left(1.746 \cdot 10^4\right)\ psi$

Margin of Safety: $MS_{shear} := \left(\frac{S_s}{\sigma_s \cdot FS} - 1\right)$ $MS_{shear} = 2.656$