Depth of Field Calculations and Constants for the Cooke Range of S4 Prime Lenses

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(1) Rigorous formulae for the limits of the depth of field are as follows:

L_N = n+ H(L-n) / (H+(L-n-f)) for Infinity > L > n+2f

L_F = n + H(L-n) / (H-(L-n-f)) for n+2f < L < n+f+H

NOTE: If L is greater than n+f+H then, obviously, L_F will try to be greater than infinity and you will get a not very useful negative answer from the L_F formula. Also, even if L = n+f+H, then L_F calculates as infinity and your computer will tell you that.

L_N is the nearest approach of the depth of field and L_F is the farthest limit of the depth of field, both measured from the film plane.

L is the focussing distance measured from the film

f is the focal length of the lens

n is the distance of the front nodal point from the film, *taking account of the forward movement of the lens for focussing.* However, for all practical purposes with non macro photography you can use n_0 for n, where n_0 is the infinity focus position of the front nodal point measured from the film plane (the focal plane in this case). We give a value for n_0 for each of the Cooke S4 prime lenses in the table of constants below, and also a formula for you to calculate a rigorous value for n if you are doing macro photography (that is, not with Cooke S4 prime lenses).

H is the Hyperfocal distance = f^2 / (d x fno), where d is the blur circle diameter (typically 0.025mm) and fno is the stop number. H is the distance of the plane on which you focus if you want the depth of field to stretch to infinity. This distance is actually measured from the *front focal plane* of the lens. The depth of field in this case then stretches from infinity down to H/2 (also measured from the front focal plane of the lens). The front focal plane of the lens is at (n+f) from the film plane, so if you want to use the lens at its hyperfocal setting, focus at L = n+f+H, and your depth of field will then stretch from infinity down to n+f+H/2. You will probably want to use n_0 rather than n in these latter formulae, and that is perfectly acceptable in all practical cases.

(2) These formulae are of course purely geometrical, that is, they do not take account of diffraction. On the other hand, they are geometrically rigorous, that is, they do cope with semi-macro photography situations as long as the object that is focussed on is not closer to the film plane than L = n+2f (when the magnification is 1:1). The formulae may be modified to cope with magnifications greater than 1:1, but that is not relevant for the Cooke range of S4 lenses.

(3)One subtlety that can be important in critical work is the fact that all cine lenses these days are aperture calibrated for Tstops. This means that the they are working at an f-number aperture which is larger (numerically smaller) than the T-stop number setting, so they have a little less depth of field than if the T-number is used in place of the f-number. Thus, to calculate the correct f-number aperture for the calculation of H requires knowledge of the transmission of the lens. The relationship between the precise f-number and the T-stop number in terms of lens transmission is

$$\text{f-number} = \text{T-stop} \times T^{1/2}$$

Where T is the transmission of the lens (eg 0.9, or 90%)

For the Cooke S4 prime lenses we may assume a transmission of better than 0.9 and compute a safe value for $T^{1/2}$ based on T = 0.9 (that is $T^{1/2}$ ~0.95), which will give a slightly conservative depth of field. In this way the value of the f-number that is used to calculate H can be 0.95 x T-stop. For example, at a T-stop aperture of 5.6 , use f/5.3. If you want to be super-careful you might note that T5.6 is an abbreviation of T5.657, and use f/5.37, but frankly, this would be being irrelevantly precise.

(4) With zoom lenses, and prime lenses in which there are floating components for focussing, it is of course necessary to use the correct value fur n_0 (or n) and for f in the above formulae, for each focal length setting. Both f and n_0 are constant for all the Cooke S4 prime lenses.

(3) Using the rigorous value for n (rather than using n_0) is not necessary when using Cooke prime lenses in the normal way, that is, when you are not focussing closer than the engraved focus scale indications. However, if you are using the above formulae to calculate depth of field for semi macro work (for focussing distances between say 10 times the focal length and 4 times the focal length) it is necessary to calculate a rigorous value for n, as follows

$$n = n_0 + f(Q/2 - 1 - (Q^2 / 4 - Q)^{1/2})$$

Where $Q = (L - n_0 + f) / f$

Using n_0 rather than n at close focus distances results in an optimistic value for the depth of field. But as long as L is greater than about 10 times the focal length then this optimistic error will be no more than about 5%, which is negligible. On the other hand, when L is equal to about 5 times the focal length this factor is about 20%, and it rises rapidly as the focus distance approaches the 1:1 position. At the 1:1 position the error is about 200% (that is, the calculated depth of field is three times the rigorously calculated depth of field).

As you approach the 1:1 focus position the depth of field will be small and the calculated depth of field will tend to be used more rigorously than when you doing normal non-macro photography.

(6) Here is the table of the required constants for the Cooke S4 lenses:

f mm	14	16	18	21	25	27	32	35	40	50	65	75	100	135
n_0 mm	118	116	95.5	88.6	82.1	76.9	79.9	71.3	62.1	26.5	28.4	25.2	36.3	69.6