i2MAP Concept Review

April 24, 2014

Agenda

- Functional requirements – *what does it have to do.*
- Design requirements – *how do we build it to do the job.*
- Dorado AUV interface & Power Budget
- Overall Design approach
  - Science sensors (CTDO+)
  - Lighting
  - Cameras & lens selection
  - Recording
  - Camera and CODEC testing
  - Overall workflow
  - Power budget update
- AUV operation and navigation issues
- Areas of higher risk

Investigations of Imaging for Midwater Autonomous Platforms (i2MAP)

MBARI's Midwater time-series data provide our researchers with a frame of reference to investigate the response of the midwater community to long-term cycles and climate change.

The goal of the i2MAP project is to develop an autonomous imaging system that will allow us to automate the data collection process; to shift from ROVs to AUVs. The first system to be developed towards reaching this goal is a Midwater Imaging Module for the Dorado AUV.

- Ship time costs will be reduced.
- Data collection dives with AUVs will be less obtrusive and will be conducted at higher speeds, which will result in a more complete assessment of the community.
- The imaging components chosen for the MTS Imaging Module will be capable of higher image resolution (1080p or greater) and higher frame rates (60 -120 fps to enable faster transit). Both features will enhance animal identification, while enabling the AUV to transit at higher speeds, which will reduce avoidance of the vehicle.

Example of a Typical ROV Midwater Transect

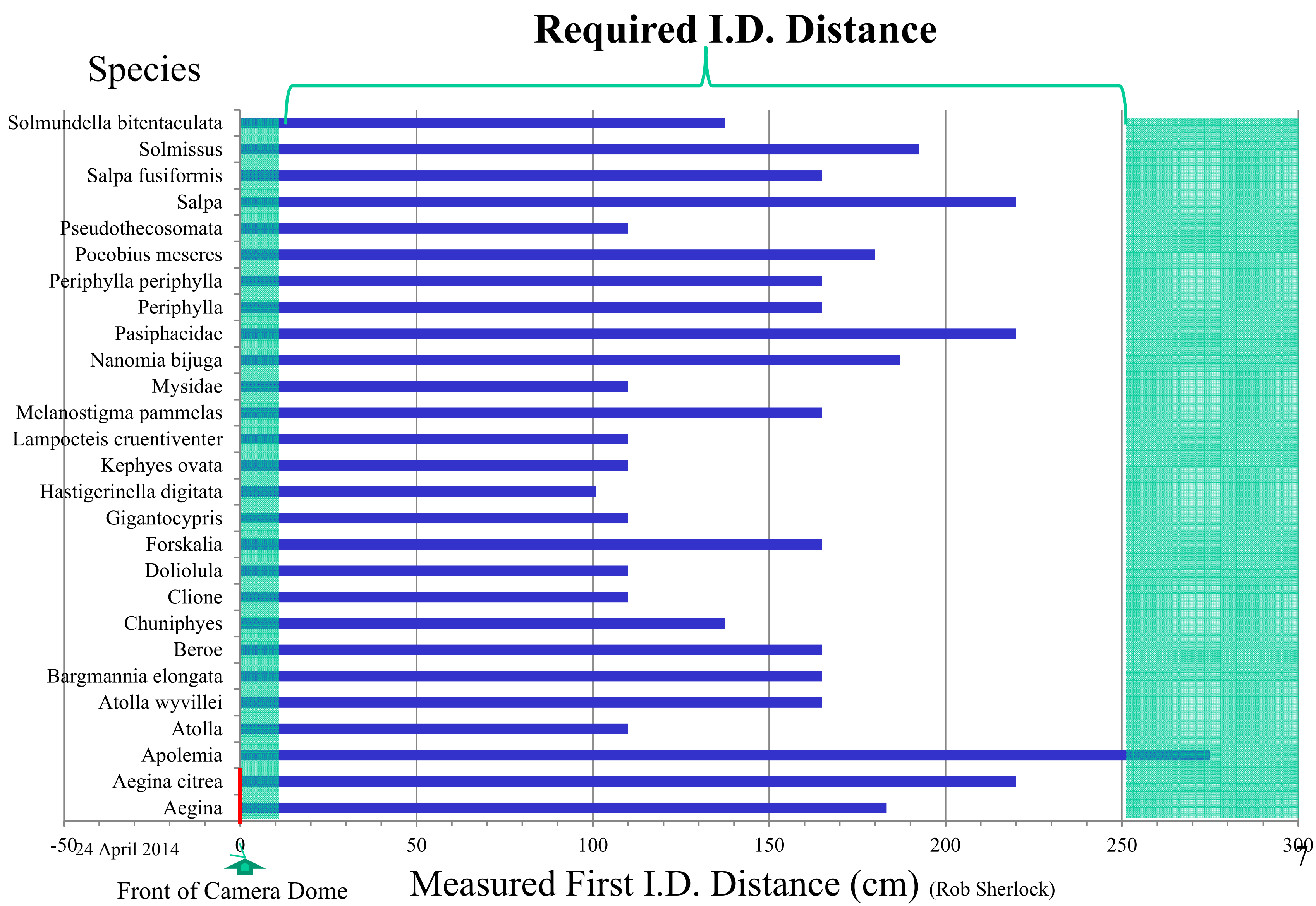
Functional Requirements *(see full text in project folder)*

- The AUV transecting imaging system must replicate the existing ROV based quantitative mid-water transects such that the results of the transects can be compared in a scientifically valid way.
- Maintain the current image data processing and video annotation chain OR create a new processing chain that can be executed with a similar degree of complexity and effort.
- Provide for simultaneous operation with the CTD, oxygen, and fluorometer instruments of the CTD AUV.
- Meet the weight, balance, interface and configuration requirements of a Dorado AUV payload such that extraordinary measures are not required for routine AUV operation.
- Be installed and removed from the AUV Dorado with a minimum of effort, consistent with similar nose cone payloads.

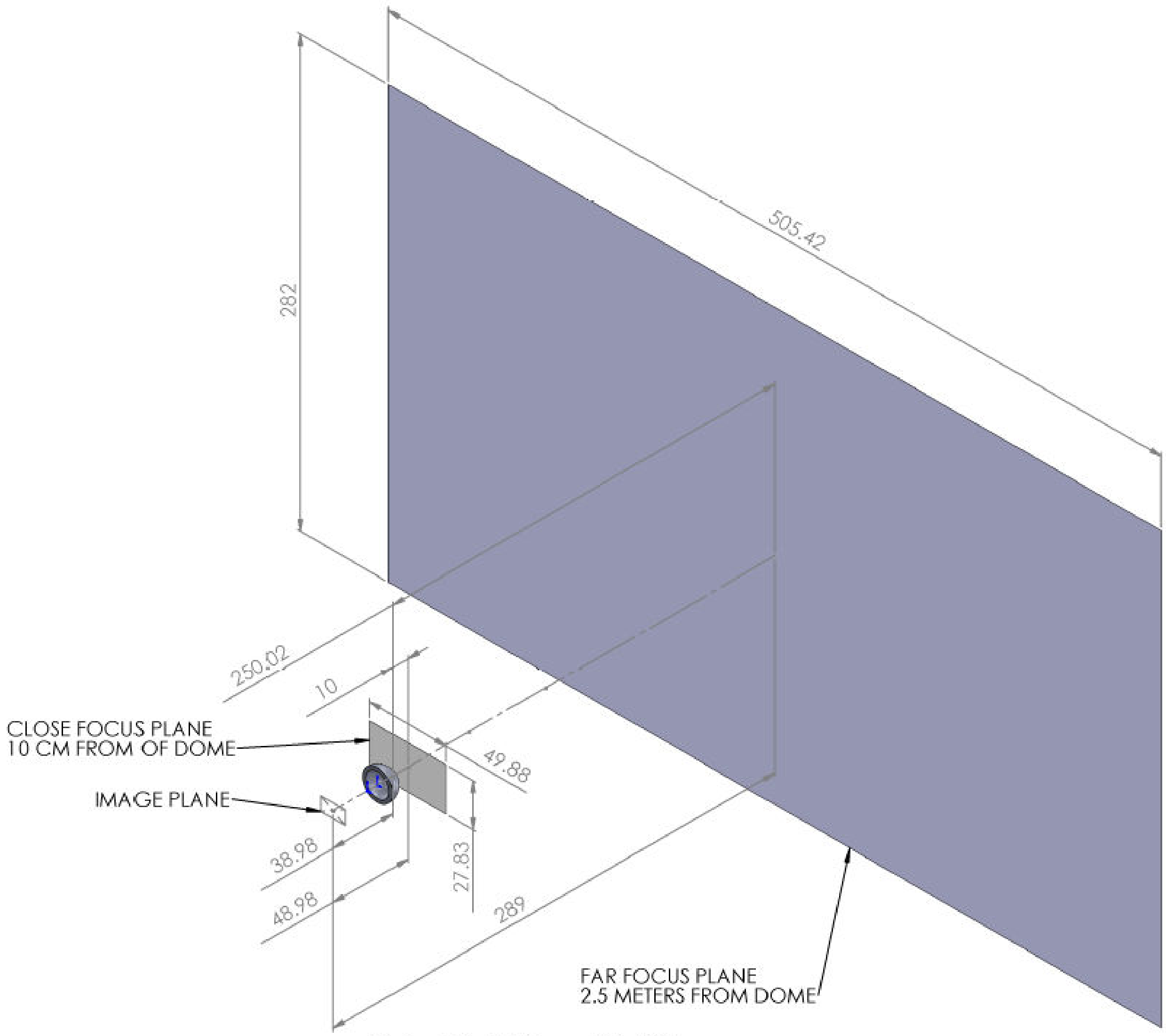
Design Requirements *(see full text in project folder)*

- Capture format. The imaging system must capture in a progressive scan format with a minimum resolution of 1920x1080 4:2:2 format at 10 bits at 60 frames per second or greater (1080 60P) for 120 minutes.
- The video records must include SMPTE timecode data that is synchronous with the environmental instrumentation data.
- Depth of field. The imaging system shall allow the identification of small plankton at a range from 0.1 to 2.5 meters from the camera dome with the camera lens in a fixed focus position. Note: it is desirable but not required that the camera system be able to image objects further from the camera than 2 meters, ideally out to 10 meters distance.
- Width of field. The lens system shall have at least an 80 degree horizontal viewing angle.
- The light bars shall hold 4 DSPL Sealights in a position that minimizes backscatter and glaring.
- The Dorado camera payload shall provide for simultaneous operation with the standard CTD, oxygen (x2), 25 cm transmissometer and fluorometer instruments of the CTD AUV OR duplicate these instruments on the camera payload. The data must be available as part of the standard AUV data load path and be accessible to VARS.
- Operate at specific depths from 50 meters to **1500 meters** performing 11 individual transects of 330 meters length.
  - Dorado shall maintain a constant depth during a commanded transect to within +/- 1 meter.
  - Dorado shall maintain a constant pitch angle during a commanded transect of +/- 10 degrees.
  - Dorado shall maintain a constant roll angle during a commanded transect of +/- 20 degrees.
- The electrical interface to the Dorado AUV must be compatible with:
  - 18 to 36 Volts DC, 15 Amperes maximum continuous current. *Note that SeaCon reports 16A*
  - The data interface is 10/100BaseT Ethernet.

Distance Requirements for the Identification of Representative Organisms in the Camera Field-of View



Depth of Field Test Requirement (all lens and camera combinations)

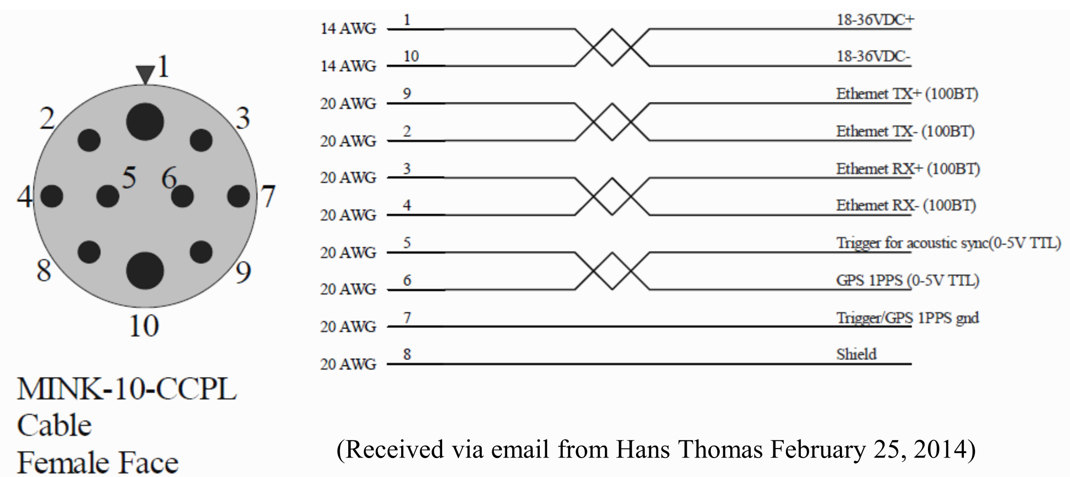


Zeiss Digi Prime T1,9/5 mm

For 2/3" 3CCD 8.8 x 6.6 mm (B4 Mount)

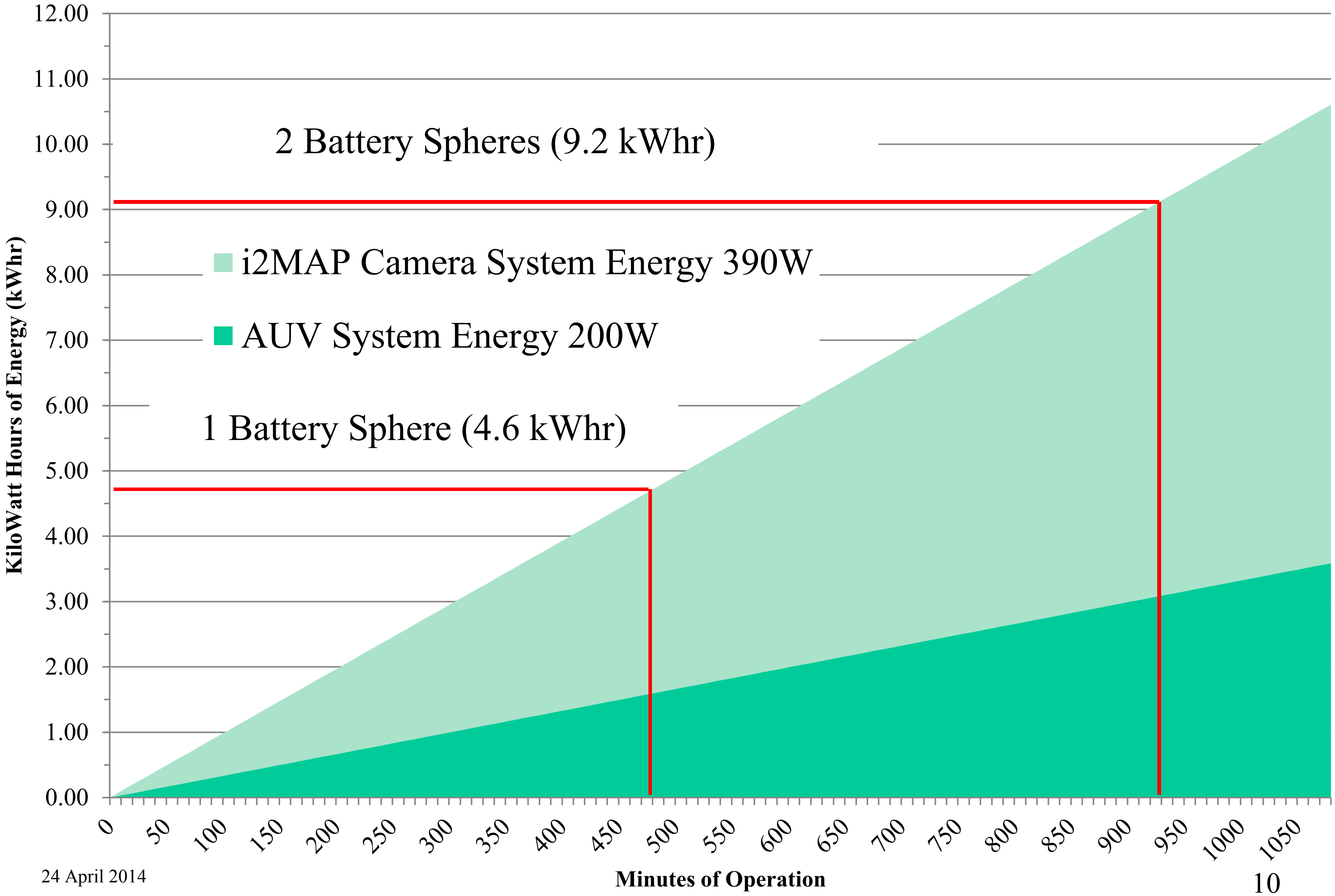
Dorado AUV interface & Power Budget

- **Hardware Interface.**
  - The electrical interface to the Dorado AUV must be compatible with:
    - 18 to 36 Volts DC. (new battery will be 20-36V)
    - 15 Amperes maximum continuous current . (SeaCon reports 16A)
  - NOTE: this is 270 Watts at low battery and 540 Watts at high battery. We need about 390 Watts to run 4 lights full power (20A @ 18VDC).
  - The data interface is 10/100BaseT Ethernet.



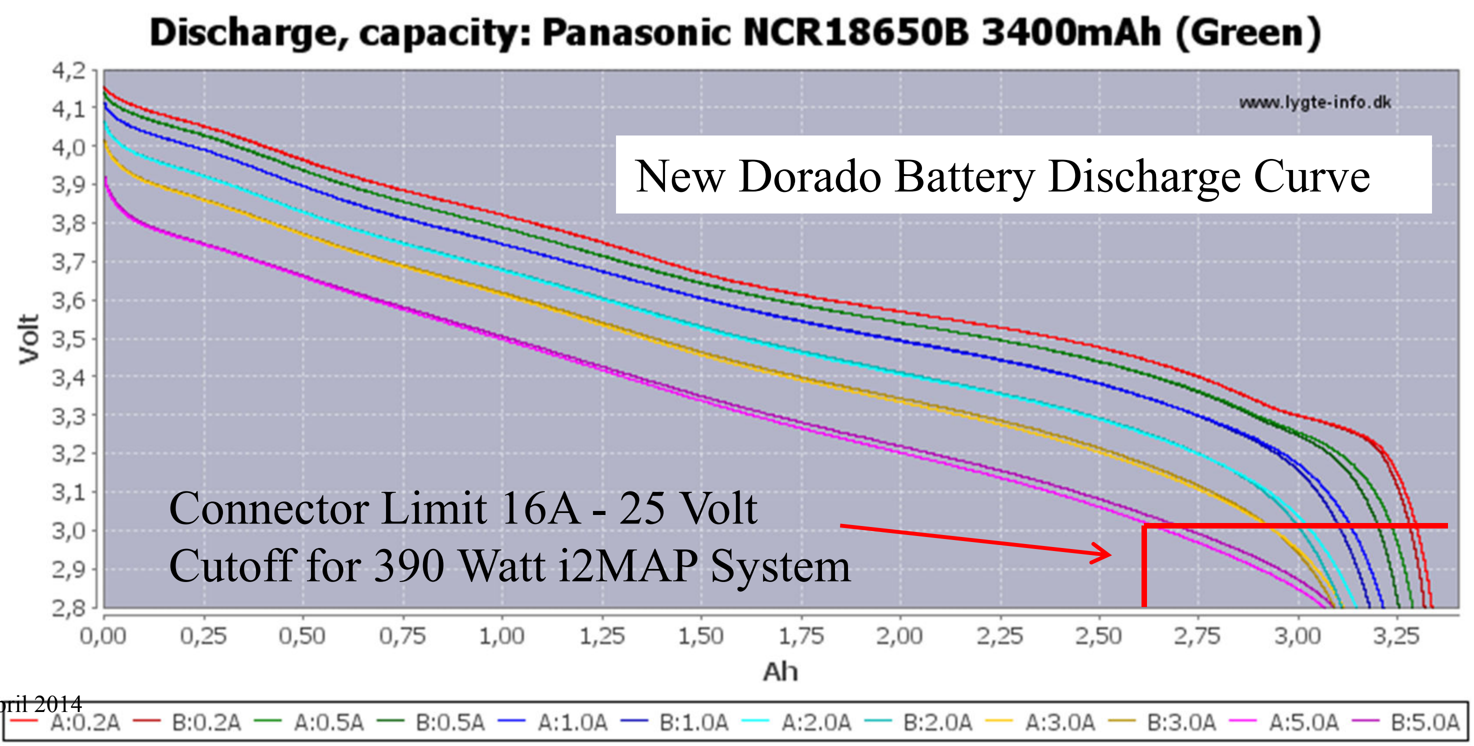
(Received via email from Hans Thomas February 25, 2014)

Total Energy Use by Dorado (1 m/sec.)



i2MAP Limited by the AUV Connector Maximum Current

- With 4 x 80 Watt lights we are limited by the Dorado interface connector 16 Ampere maximum at low battery voltages (<25 Volts).
- Required solution:
  – Cut off i2MAP lights or reduce output when AUV battery voltage drops below 25 Volts.

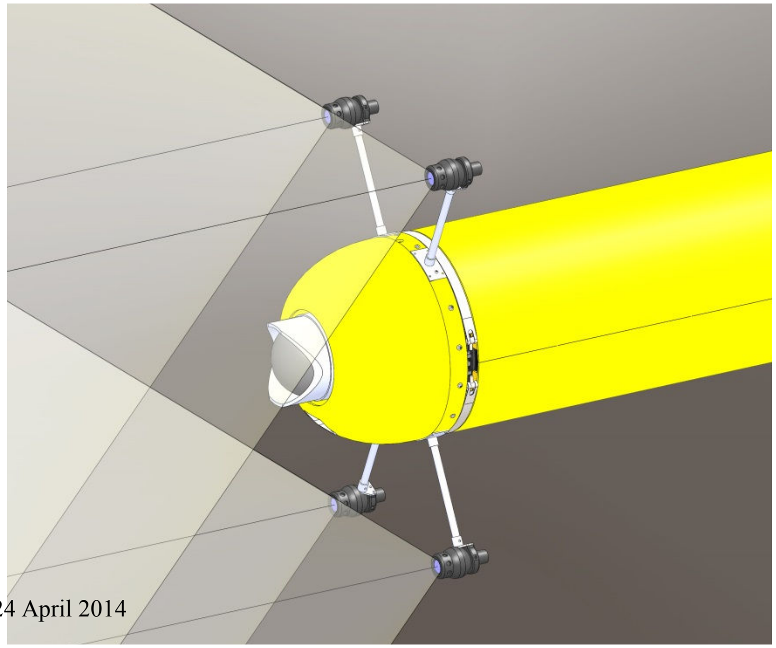


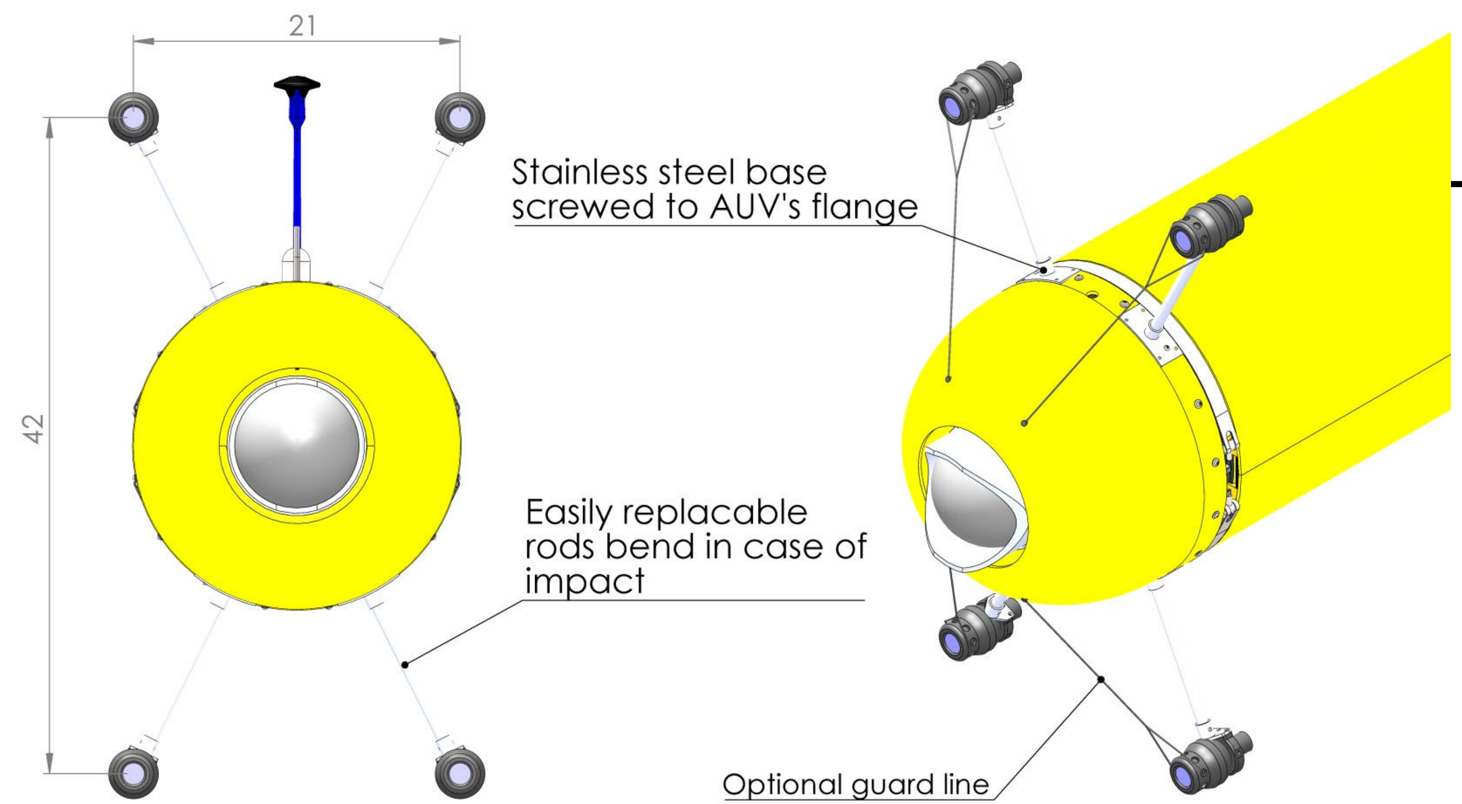
Science Sensors

- SBE 25plus Sealogger CTD Board Set
- SBE4C conductivity sensor
- SBE 5 pump and TC duct
- 2x SBE 43 polarographic oxygen sensors and ducts
- Transmissometer (25 cm)
- Wet Labs ECOPuck backscatter sensor.

Lighting

- 4 DSPL Sealight spheres.
- 21” x 42” rectangular position (same as ROV transects), 1’ back from camera dome.





Problem	Solution
Lights are vulnerable during deployment and recovery	Rods are compliant to avoid damage to lights and fairing. Damaged rods can be replaced immediately on the ship.
Lines or other objects could get caught in the light bars	Risk is minimal according to AUV team. A guard line can be used when operating in high risk areas.
Drag is increased	See Rob's analysis

Camera and Lens Selection

- Video Acquisition format
- Selected list of cameras and lens combinations
- System block diagrams
- Camera testing and evaluation plan
  - Temporal resolution testing
  - Resolution testing
  - Color resolution testing
  - Depth of field (DOF) testing

Video Acquisition Format

- We have set minimum standard of 1920x1080 progressively scanned video at a 60 frames per second rate.
  - 10 Bit color depth in a 4:2:2 format.
    - *Progressive scanning has higher temporal resolution and eliminates interlace artifacts.*
    - *Higher frame rates improve temporal resolution and may allow higher AUV transect speeds.*
  - We can test higher resolutions and frame rates (camera dependent)
    - 4:4:4 color space for example.

- Camera (and lens combinations) come in two groups

  - 2/3” 3-CCD (or CMOS) imager optical prism cameras (B4 lens mount)

  - “Cine” or large format single CMOS imager with a Bayer array (PL or custom lens mount).

2/3” 3-chip Sensor Cameras

- Use f 1.4 optical prism and three imager arrays – need special lens design.
- Use B4 lens mount.
- Lenses are compatible across same type of camera.
- Known for good sensitivity and resolution.
- Fastest frame rate is 60P.
- Small array has large depth of field (DOF).

Sony HDCP1

- Max frame rate: 1920 x 1080 at 60P
- Internal recorder: None
- Recorder interface: 1 x 14-bit HD-SDI 4:2:2
- Lens: Sony 2/3-type bayonet mount (B4).
- Control interface : LAN based Sony system (CCU compatible).

Camera	**Sony HDCP1**	**Recorder**	**AJA Ki Pro Quad recorder**
Format	3 chip 2/3" sensors	CODEC Format	Apple ProRes 422 (HQ) 10 bit 60P
Lens	Zeiss Digi Prime T1,9/5 mm	Recording Time 1080 60P 10 bit 4:2:2 (min.)	**310**
F - Stop	2.5	Recording Media	2 X AJA's Pak512 512GB SSD
DOF (meters)	4.1	**Cost Estimate**	
Horizontal Field of View (deg.)	82.7	Camera	$29,000
Vertical Field of View (deg.)	66.8	Lens	$9,100
Close Focus (from Dome)	0.1	Recorder	$3,995
Far Focus	4.2	Total	**$42,095**

Budget ~ $76k

Ikegami HDL-50/HS

- Max frame rate: 1920 x 1080 at 60P
- Internal recorder: None
- Recorder interface: 2 x HD-SDI 4:2:2 (dual link)
- Lens: B4 2/3-type bayonet mount.
- Control interface : RM-50, RM-51, RM-11, OCP-200 & MCP-200
- Field of view 87 degrees with Zeiss Digi Prime T1,9/5 mm lens.

+ Side

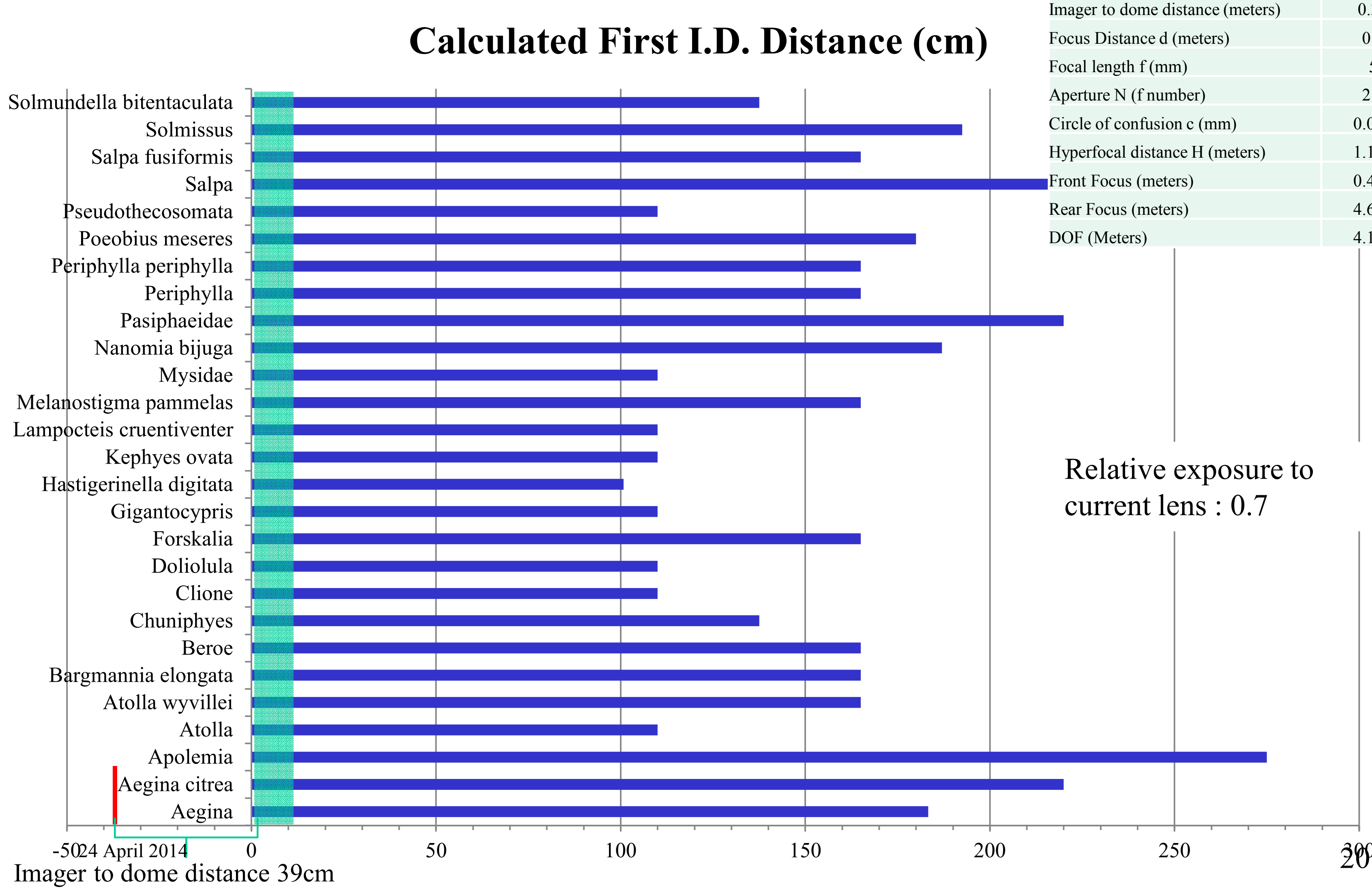
- Very good picture (3CCD).
- Good form factor for housing.
- No extra peripherals.
- Mechanical shutter.
- High quality lens available.
- Good DOF

- Side

- Same $ as Sony P1.
- Ikegami does not make their own imagers – always behind the curve.
- Poor sensitivity.

2/3" Prism Camera with Zeiss Digi Prime T1,9/5 mm @ *f* 2.5 Calculated Depth of Field Focused at 0.9 meter

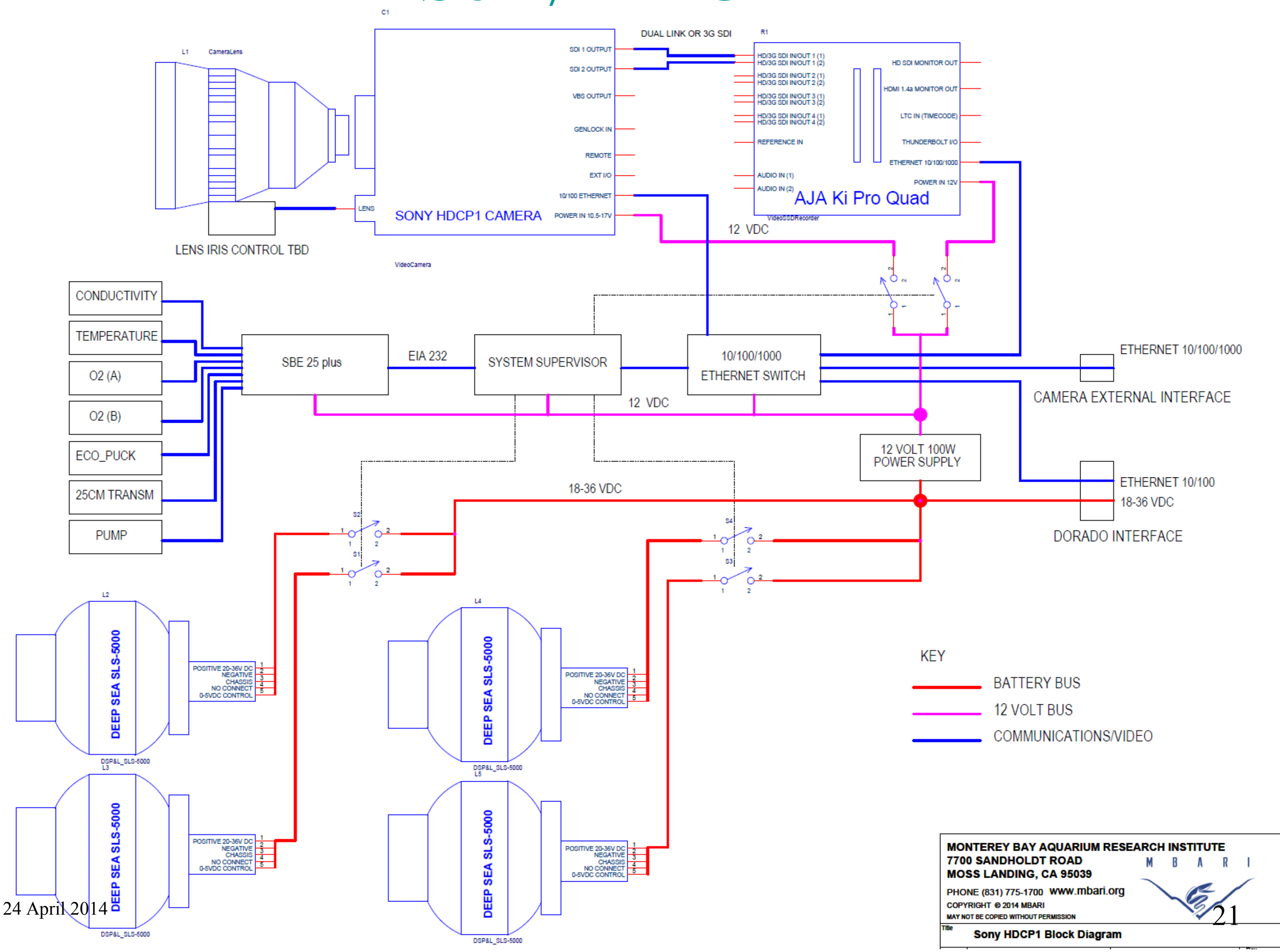
Measurement point	Imager	Dome ~
Imager to dome distance (meters)	0.39	
Focus Distance d (meters)	0.9	
Focal length f (mm)	5	
Aperture N (f number)	2.5	
Circle of confusion c (mm)	0.009	
Hyperfocal distance H (meters)	1.116	0.726
Front Focus (meters)	0.497	0.107
Rear Focus (meters)	4.627	4.237
DOF (Meters)	4.130	



Relative exposure to current lens : 0.7

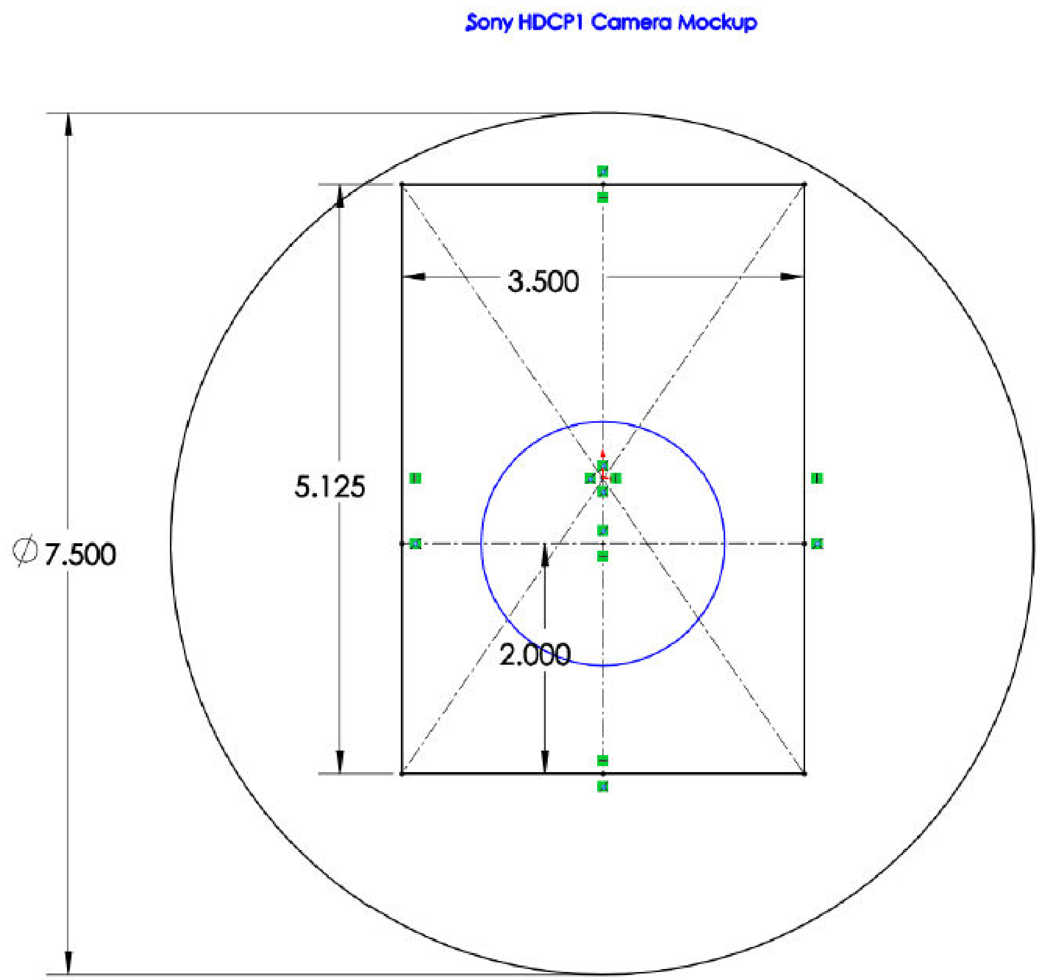
24 April 2014

Sony HDCP1



DUAL LINK OR 3G SDI
CameraLens
SDI 1 OUTPUT
SDI 2 OUTPUT
VBS OUTPUT
GENLOCK IN
REMOTE
EXT I/O
10/100 ETHERNET
POWER IN 10.5-17V
LENS
SONY HDCP1 CAMERA
VideoCamera
LENS IRIS CONTROL TBD
HD/3G SDI IN/OUT 1 (1)
HD/3G SDI IN/OUT 1 (2)
HD/3G SDI IN/OUT 2 (1)
HD/3G SDI IN/OUT 2 (2)
HD/3G SDI IN/OUT 3 (1)
HD/3G SDI IN/OUT 3 (2)
HD/3G SDI IN/OUT 4 (1)
HD/3G SDI IN/OUT 4 (2)
REFERENCE IN
AUDIO IN (1)
AUDIO IN (2)
HD SDI MONITOR OUT
HDMI 1.4a MONITOR OUT
LTC IN (TIMECODE)
THUNDERBOLT I/O
ETHERNET 10/100/1000
POWER IN 12V
AJA Ki Pro Quad
VideoSSDRecorder
12 VDC
CONDUCTIVITY
TEMPERATURE
O2 (A)
O2 (B)
ECO_PUCK
25CM TRANSM
PUMP
SBE 25 plus
EIA 232
SYSTEM SUPERVISOR
10/100/1000
ETHERNET SWITCH
ETHERNET 10/100/1000
CAMERA EXTERNAL INTERFACE
12 VDC
12 VOLT 100W
POWER SUPPLY
ETHERNET 10/100
18-36 VDC
DORADO INTERFACE
18-36 VDC
DEEP SEA SLS-5000
POSITIVE 20-36V DC
NEGATIVE
CHASSIS
NO CONNECT
0-5VDC CONTROL
DSP&L_SLS-5000
KEY
BATTERY BUS
12 VOLT BUS
COMMUNICATIONS/VIDEO
MONTEREY BAY AQUARIUM RESEARCH INSTITUTE
7700 SANDHOLDT ROAD
MOSS LANDING, CA 95039
PHONE (831) 775-1700 www.mbari.org
COPYRIGHT © 2014 MBARI
MAY NOT BE COPIED WITHOUT PERMISSION
M B A R I
Title
Sony HDCP1 Block Diagram

Sony HDCP1 Housing Dia. Mockup



Large Sensor Cameras

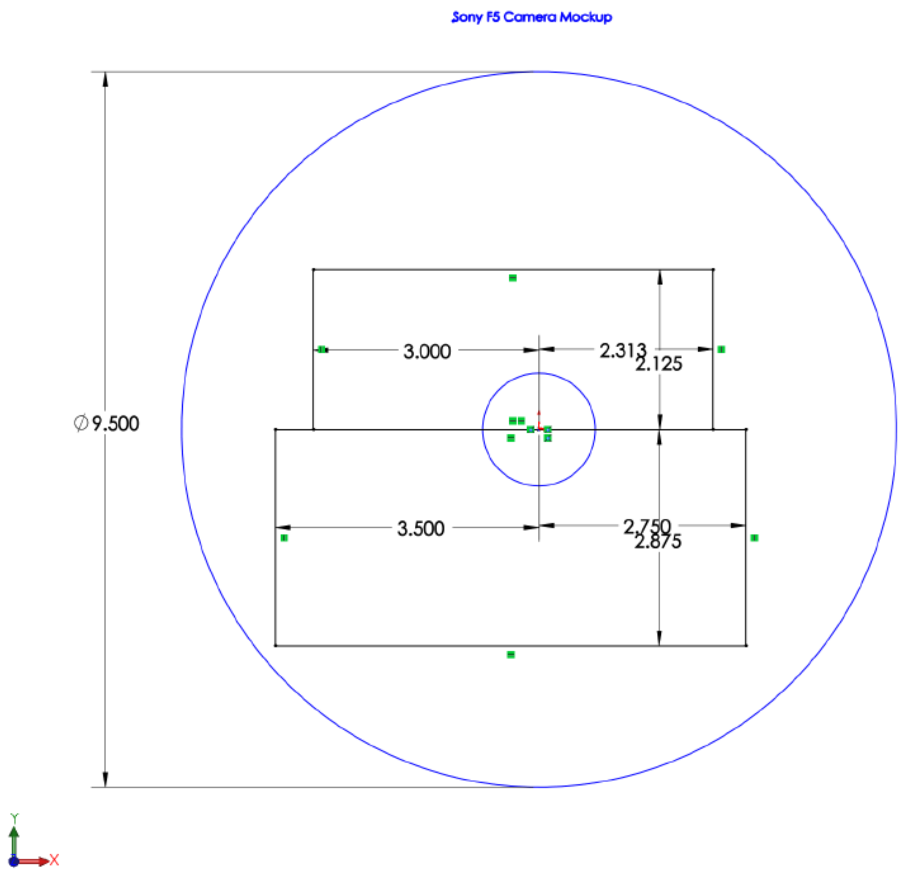
- Known as “cine type” or Super 35 or DIN Super 35
- Single array with Bayer pattern has about 70% of the resolution as 3-chip array.
- Single array creates film-like optical path, can use almost any large format lens, i.e. PL mount, Canon EF mount, Nikon, etc.
- New digital format and cameras often have internal storage and the best compression engines (CODECS).
- Large sensor increases the light sensitivity but reduces the depth of field for a given focal length.

Sony PMW-F5

- Max frame rate: 1920 x 1080 at 60P with XAVC codec, 240 FPS 2K RAW (full res. 60P max.)
- Internal recorder: XAVC codec
- Recorder interface: 3G-SDI with 2K and 4K capability or HDMI 1.4
- Lens: Sony FZ-mount with PL adapter.
- Control interface : LANC ??
- Field of view 80 degrees with 15mm Zeiss CP.2 15mm/T2.9 E-mount prime lens.

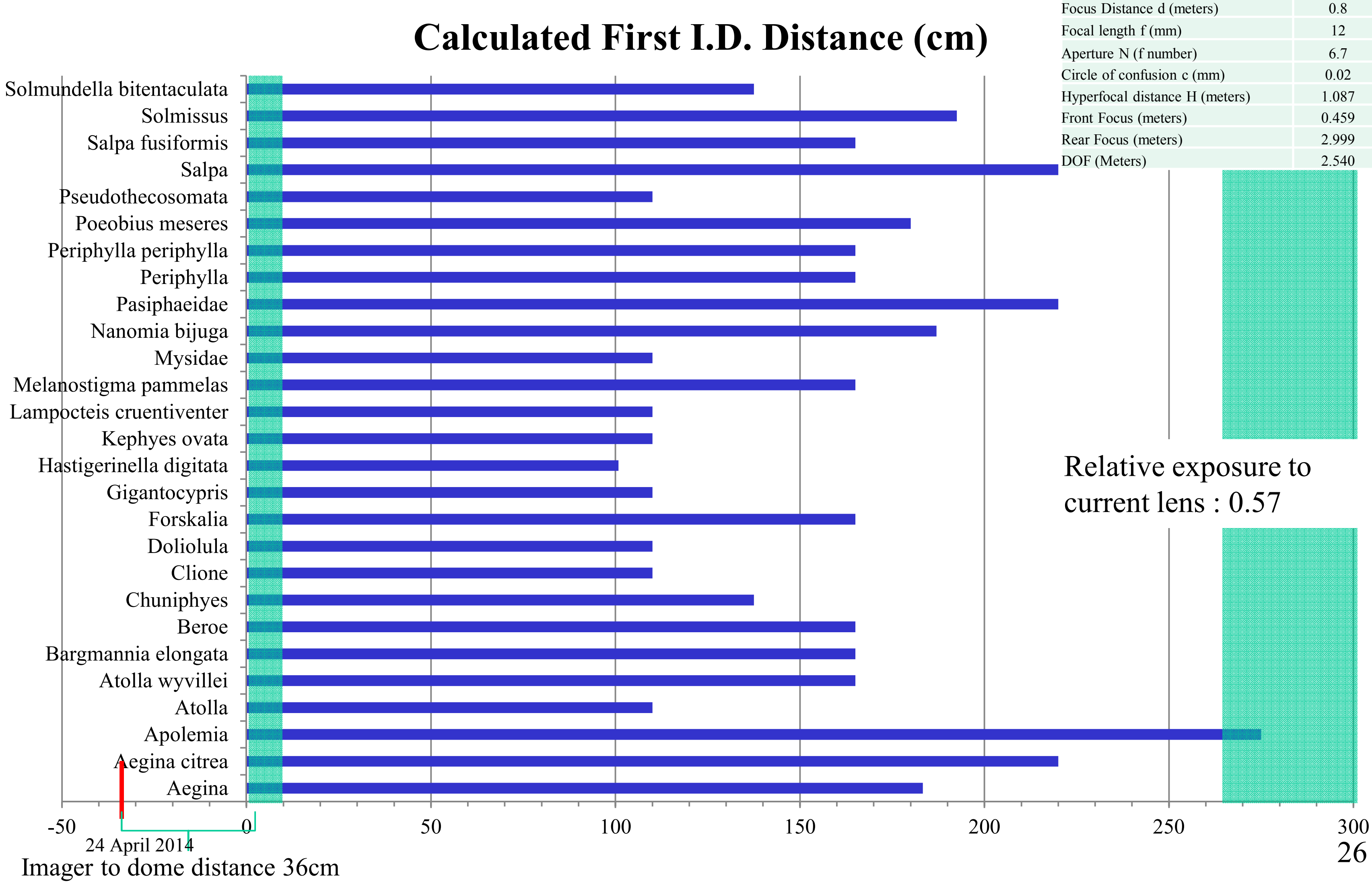
Camera	**Sony PMW-F5**	**Recorder**	**Sony PMW-F5 Internal Storage**
Format	Large Formay Bayer Array Sensors	CODEC Format	1920 X 1080 @ 60P as XVAC codec which is 10bit and 422 sampling
Lens	Arri Ultra Prime UP 10 mm/T2.1	Recording Time 1080 60P 10 bit 4:2:2 (min.)	**120**
F - Stop	4.5	Recording Media	2 X Sony 128GB SxS PRO+ Memory Card
DOF (meters)	3.0	**Cost Estimate**	
Horizontal Field of View (deg.)	100.4	Camera	$17,000
Vertical Field of View (deg.)	64.8	Lens	$13,500
Close Focus (from Dome)	0.1	Recorder	$0
Far Focus	3.5	Total	**$30,500**

Sony PMW-F5 Housing Dia. Mockup



DIN Super 35 Camera with Arri/Zeiss Ultra Prime UP 12 mm/T2 @ ƒ 6.7
Calculated Depth of Field Focused at 0.8 meter

Measurement point	Imager	Dome ~
Imager to dome distance (meters)	0.36	
Focus Distance d (meters)	0.8	
Focal length f (mm)	12	
Aperture N (f number)	6.7	
Circle of confusion c (mm)	0.02	
Hyperfocal distance H (meters)	1.087	0.727
Front Focus (meters)	0.459	0.099
Rear Focus (meters)	2.999	2.639
DOF (Meters)	2.540	



Relative exposure to current lens : 0.57

Sony NEX-FS700UK

- Max frame rate: 1920 x 1080 at 60P with AVCHD codec, 240 FPS 2K RAW (full res. 60P max.)
- Internal recorder: AVCHD codec
- Recorder interface: 3G-SDI with 2K and 4K capability or HDMI 1.4
- Lens: Sony E-mount hot shoe.
- Control interface : LANC
- Field of view 80 degrees with 15mm Zeiss CP.2 15mm/T2.9 E-mount prime lens.

+ Side

- High frame rate.
- High sensitivity.
- Can change lens.
- Low cost ($7700).
- Low cost lens ($5700)

- Side

- Unknown picture quality in our application. (inferior CODEC)
- Not needed features, LCD screen, etc.

Canon EOS C500 4K

- Max frame rate: 1920 x 1080 at 60P with MPG 4 codec, 120 FPS 2K RAW (full res. 60P max.)
- Internal recorder: AVCHD codec
- Recorder interface: 3G-SDI with 2K and 4K capability
- Lens: EF or PL mount.
- Control interface : LANC Remote control through WFT-E6A accessory is also possible
- Field of view 85+? degrees with 14mm Canon CN-E14mm T3.1 L F EF-mount prime lens.

+ Side

- High frame rate.
- High sensitivity.
- Can change lens.
- Low cost lens ($5500)
- Better 4k performance.

- Side

- Not needed features, viewfinder, etc.
- Wrong shape for housing.

Canon EOS-1D C

- Max frame rate: 1920 x 1080 at 60P
- Internal recorder: MOV, Motion JPEG, MPEG-4 AVC (H.264)
- Recorder interface: Does not appear to be available except 1080 60i or 1080 24p.
- Lens: EF mount.
- Control interface : LANC Remote control through WFT-E6A accessory is also possible
- Field of view 85+? degrees with 14mm Canon CN-E14mm T3.1 L F EF-mount prime lens.

+ Side

- High resolution.
- Can change lens.
- Low cost lens ($5500)

- Side

- Not needed features, viewfinder, etc.
- Not optimal for housing.
- Max. rec. file length is 29 minutes.
- External recording resolution poor.
- Poor depth of field (full size 35mm sensor).

Video Recorder Options

- Recording format and recording time
  - Some form of bit reduction “compression” is needed to be able to meet recording time requirement (120 minutes @ 55cm/sec. AUV speed).
  - Chosen minimum standard is ProRes 10 bit 4:2:2 in a 1080 60P format.

Video Format (may not all be supported)	Data Rate in mega bits per second (Mbps)	Ratio N:1
1080p 30 native (4:2:2)	1485	
1080p 60 native (4:2:2)	2970	
D-5 4:1 compression 25P	269	
Apple ProRes 422 10 bit 30P	147	10
Apple ProRes 422 (HQ) 10 bit 30P	220	7
XAVC HD codec 10 bit 4:2:2 60P	223	13
Apple ProRes 422 10 bit 60P	293	10
Apple ProRes 4444 30P	330	
Apple ProRes 422 (HQ) 10 bit 60P	440	7
HDCAM SR 10 bit 4:2:2 30P	440	3
DPX image sequences, SMPTE 268M-2003 60P	440	
Apple ProRes 4444 60P	660	

Video Recorder Options

- Data downloading with out opening housing (over copper).
  - Only one external solid state drive (SSD) recorder will allow downloading over copper – the AJA Ki Pro Quad.
  - Data download via Gigabit Ethernet time as about 1:1 for 1080 60P 4:2:2 video, i.e. 1 hour of recording is 1 hour of download.

- The Sony F-5 camera internal recording option will also support downloading over USB 2.0.
  - (120 minutes maximum recording time 1080 60P). Also 1:1 download time.

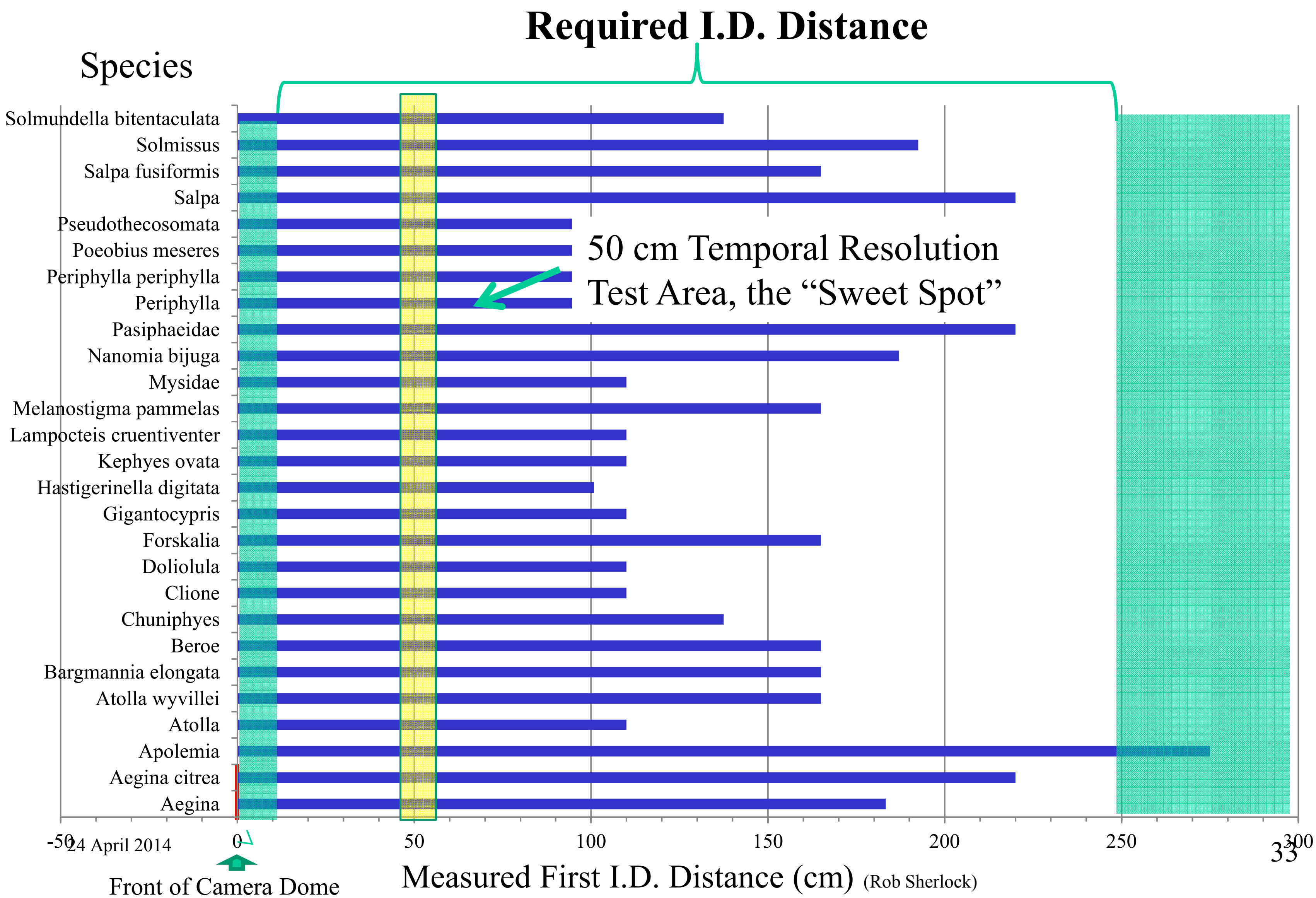
Imaging System Test Problem

- Need to test imaging *systems* ability to resolve motion.
  - To predict performance and optimize AUV transect speeds.
  - To measure the performance of recording CODECs.
  - To measure the tradeoff between shutter speed and camera gain noise.

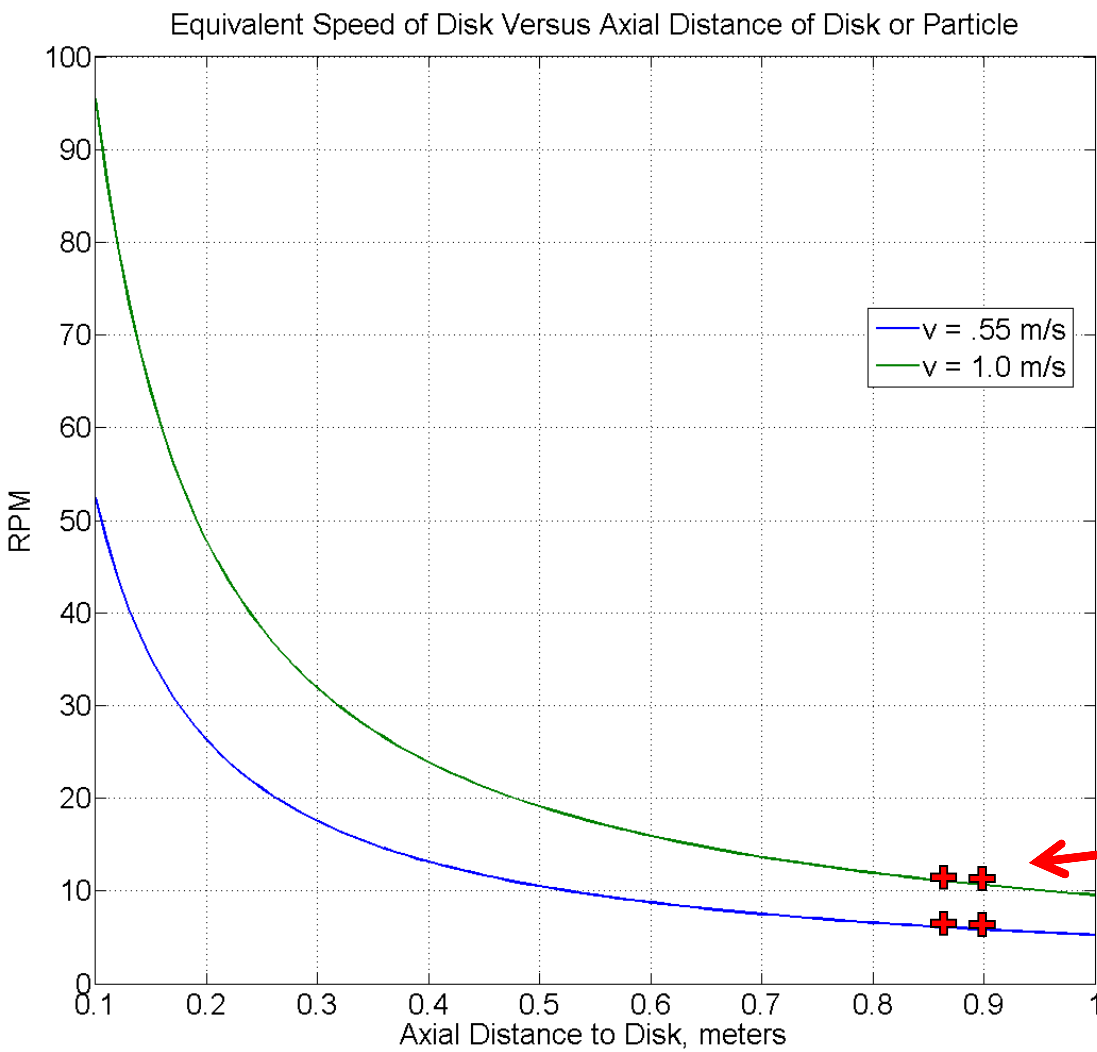
- How can we simulate the motion of small animals towards the camera?

- Use a spinning disk!

Distance Requirements for the Identification of Representative Organisms in the Camera Field-of View



Translation of Vehicle Motion to Focal-Plane



$$v^P = v^T \quad (1)$$

$$-f\,\frac{rv}{x_{3p}^2} = -f\,\frac{r}{x_{3t}}\omega \quad (2)$$

$$\omega = \frac{v x_{3t}}{x_{3p}^2} \quad (3)$$

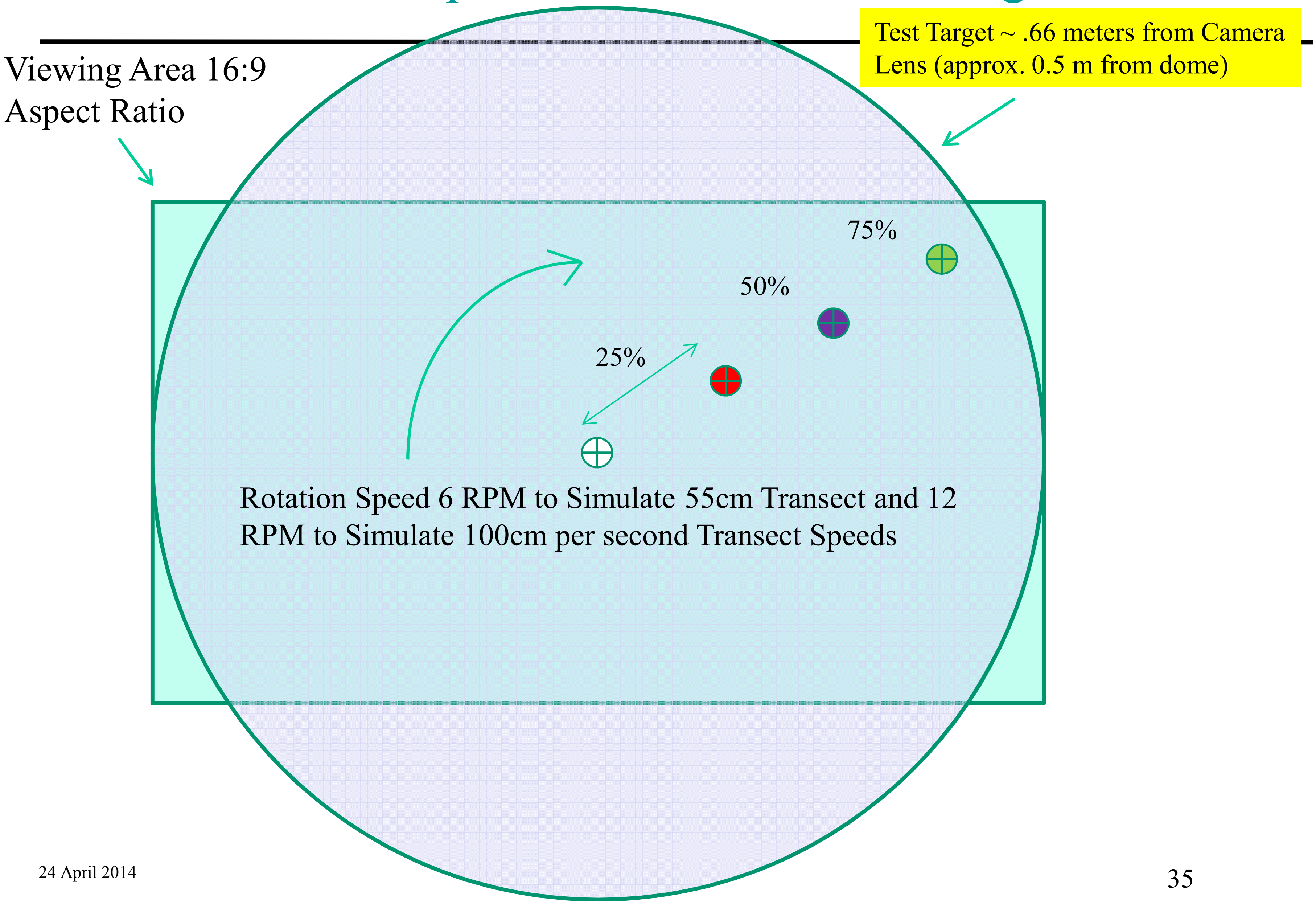
$$\omega = \frac{v}{x_3} \quad (4)$$

x_3p – focal plane to particle
x_3t – focal plane to disk
Eqn. 3 => 4 if they're equal.

Imaging System
Test Points

24 April 2014

Camera Temporal Resolution Test Target



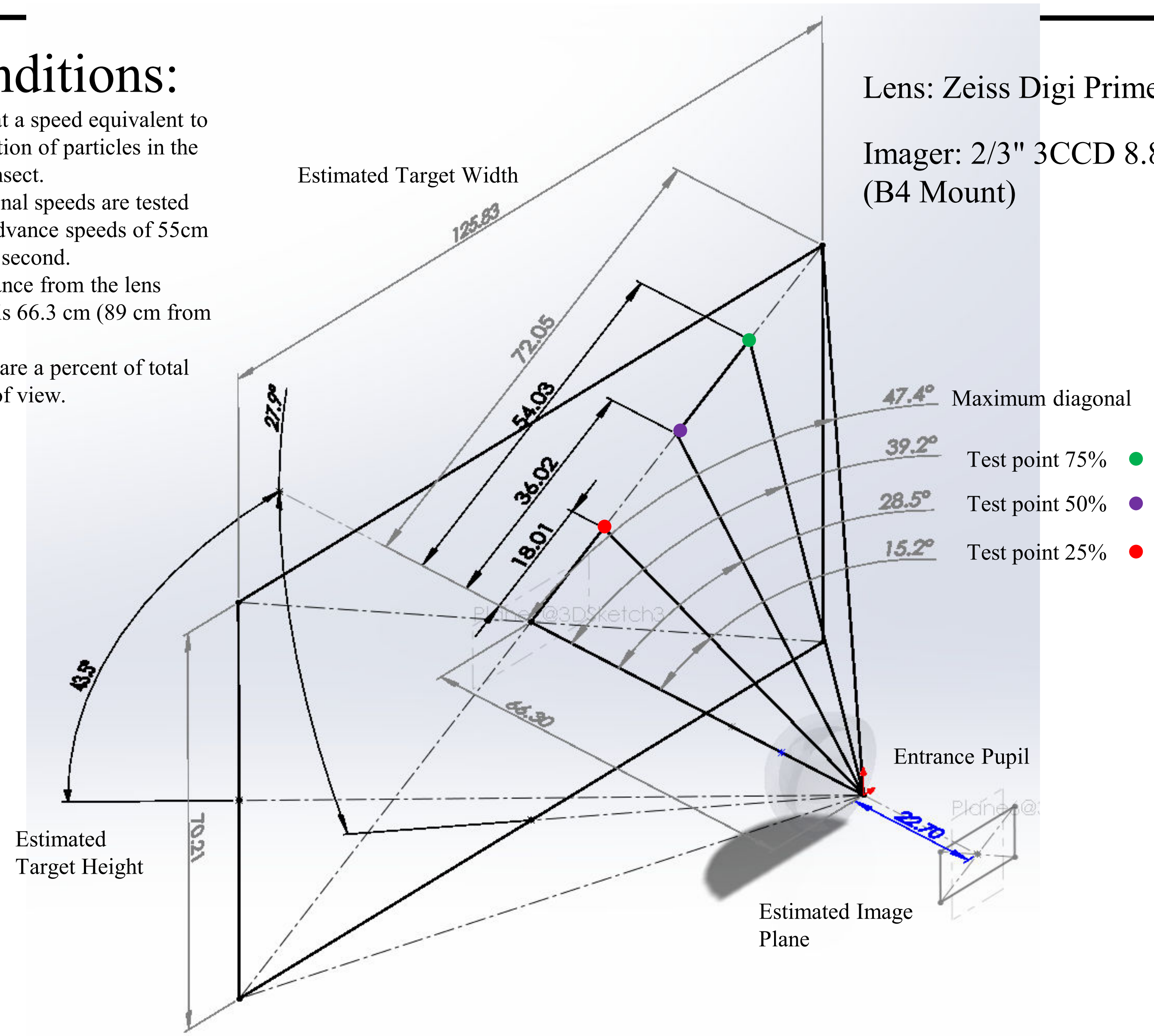
Temporal Resolution Test – 2/3” imager

Test Conditions:

1. Target rotates at a speed equivalent to the angular motion of particles in the actual field transect.
2. The two rotational speeds are tested equivalent to advance speeds of 55cm and 100cm per second.
3. The target distance from the lens entrance pupil is 66.3 cm (89 cm from imager plane).
4. The test points are a percent of total diagonal field of view.

Lens: Zeiss Digi Prime T1,9/5 mm

Imager: 2/3" 3CCD 8.8 x 6.6 mm (B4 Mount)



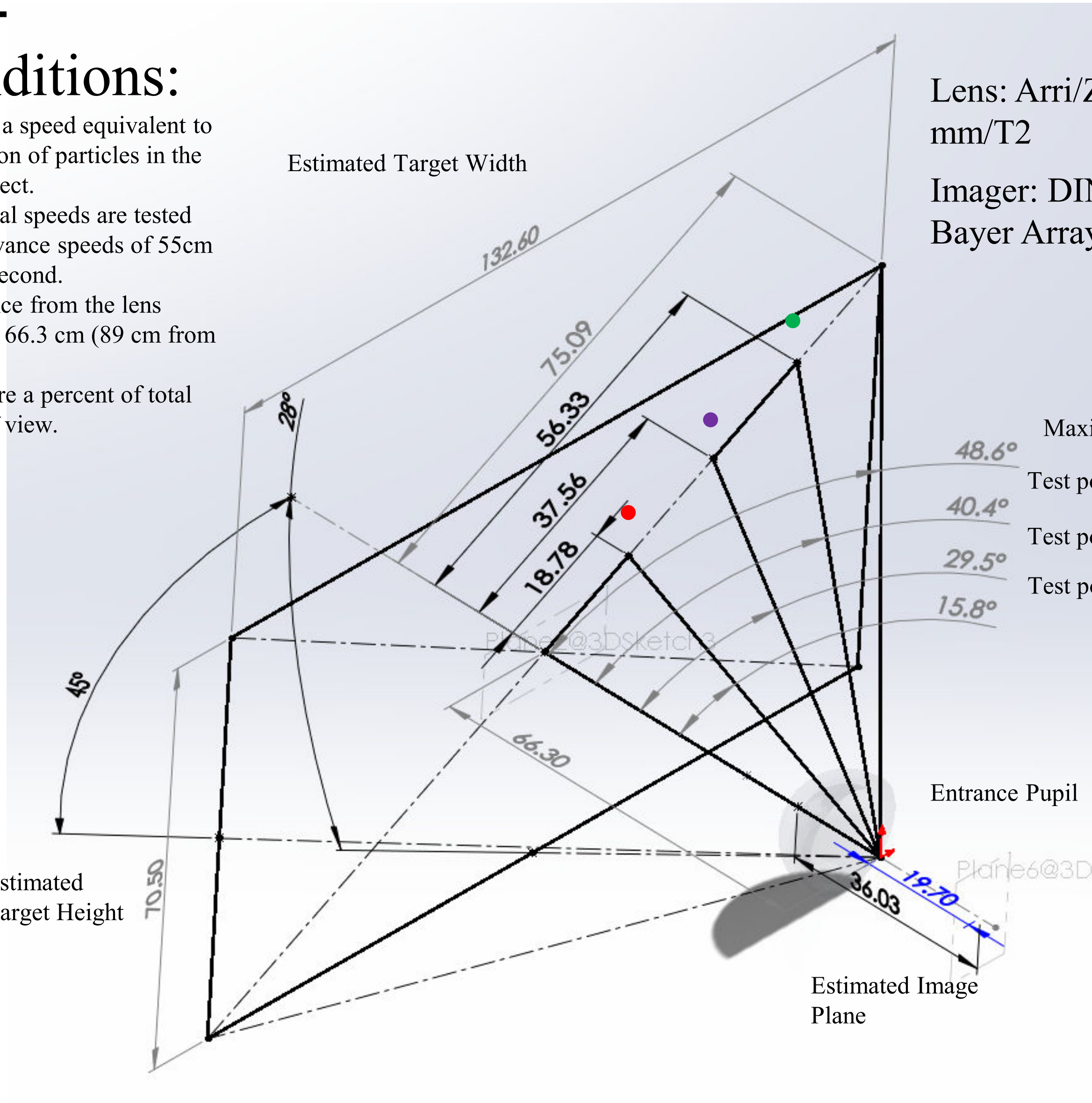
Temporal Resolution Test – super 35 imager

Test Conditions:

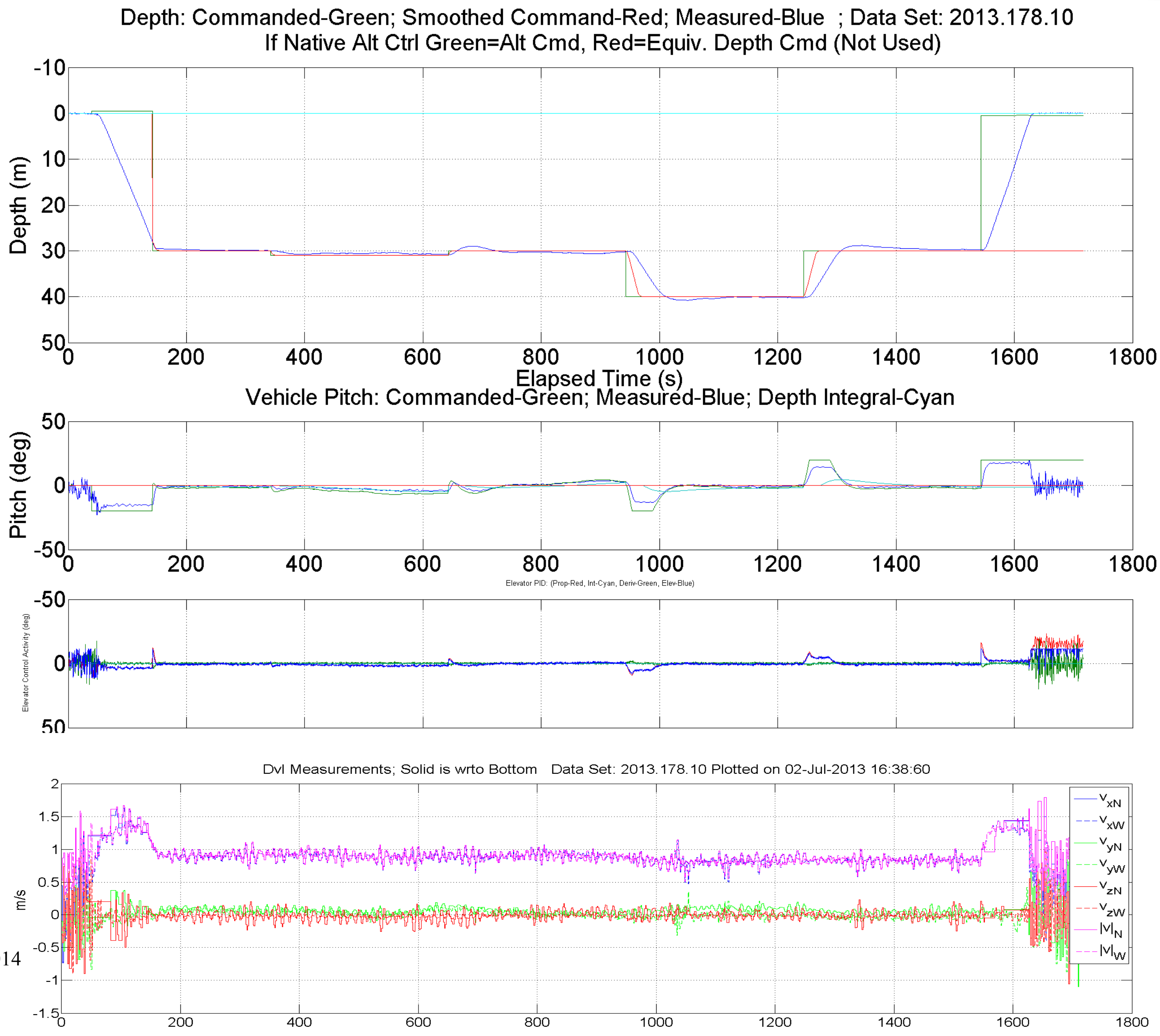
1. Target rotates at a speed equivalent to the angular motion of particles in the actual field transect.
2. The two rotational speeds are tested equivalent to advance speeds of 55cm and 100cm per second.
3. The target distance from the lens entrance pupil is 66.3 cm (89 cm from imager plane).
4. The test points are a percent of total diagonal field of view.

Lens: Arri/Zeiss Ultra Prime UP 12 mm/T2

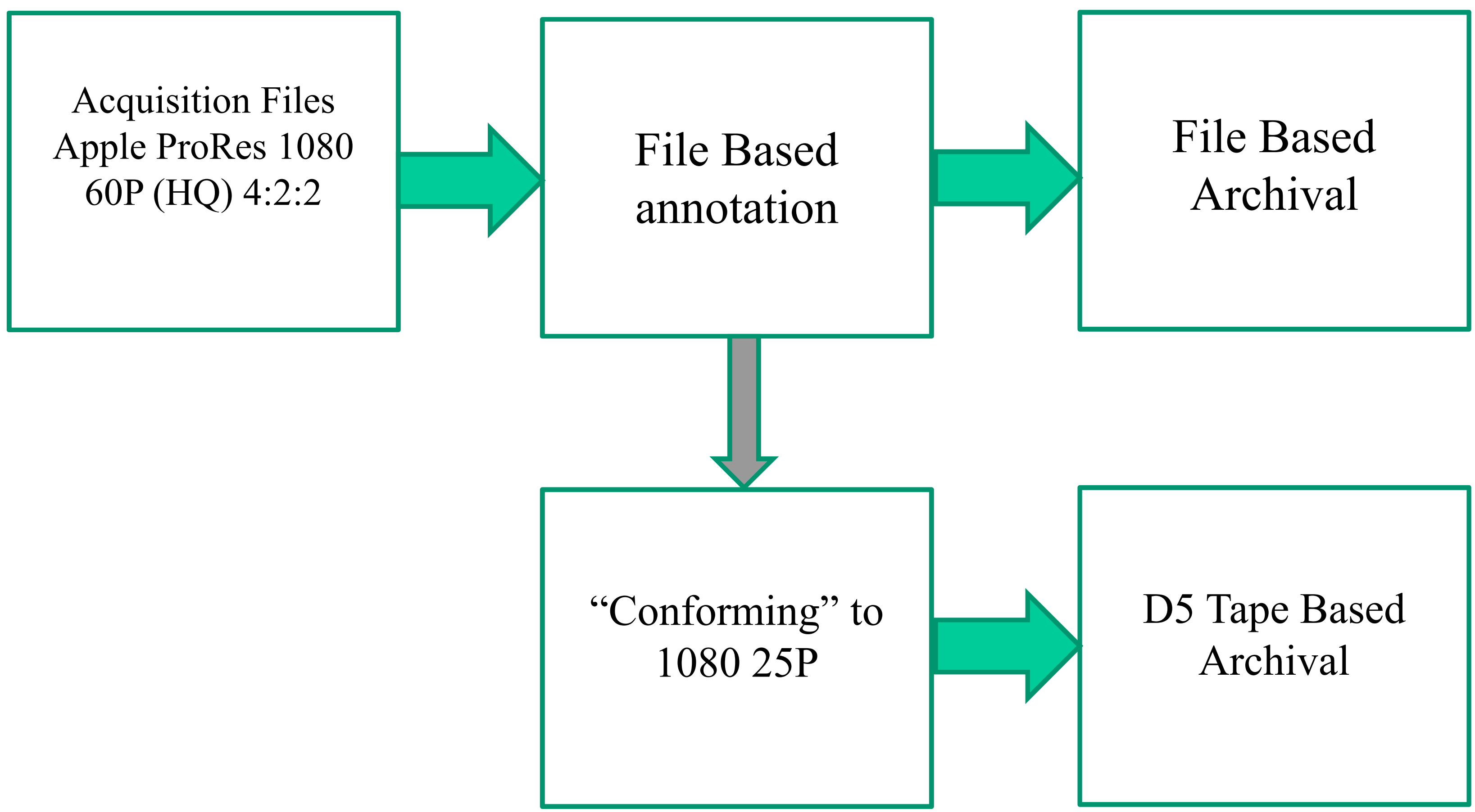
Imager: DIN Super 35 24 x 12.7 mm Bayer Array (PL mount)



Video from AUV



Annotation and Archival Workflow



Vehicle Control And Stability - Two Mostly Independent Problems

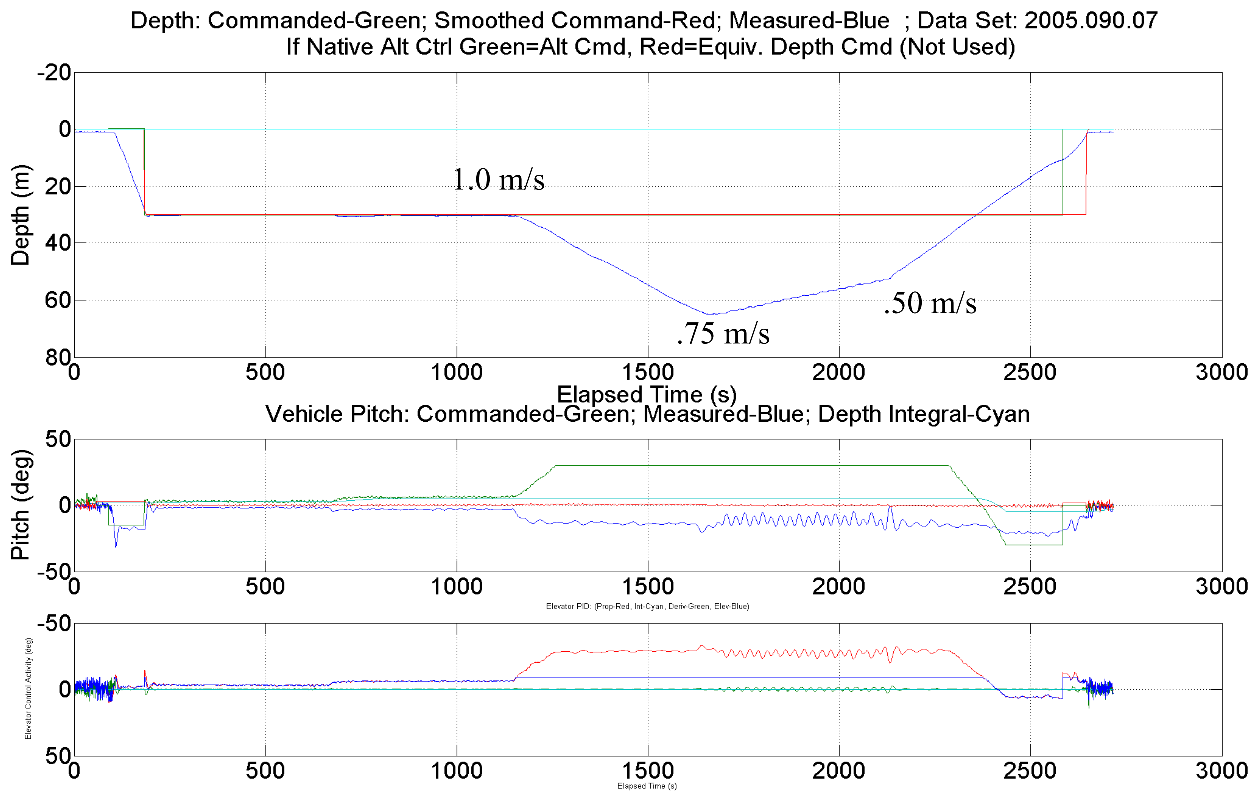
1. Low speed versus buoyancy.
2. Lights and bars on the bow destabilize the vehicle.

Problem #1
Speed Versus Buoyancy

Speed Versus Buoyancy

- AUV typically runs at 1.5 m/s (3 kts).
- Video historically taken at .55 m/s (1.1 kts).
- Buoyancy will overcome lift at .55 m/s:

Mapping AUV
10 lbf Buoyant



Speed versus Video Tradeoff

Vehicle control and stability deteriorate below 1.0 m/s, but video improves.

- Static buoyancy limits slowest speed. Rob asks Mark how fast!
- Camera and lights limit max speed. Mark asks Rob how slow!
- Either body wings or precision ballasting probably necessary below 1.0 m/s.
- Depth control requirement +/- 1 meter.
- Steady-state analysis & sea trials determine if vehicle can hold depth for a given speed and buoyancy.
  - Assuming vehicle is stable.
  - Stall not modeled.
- Sea trials were promising.

Wings 14” Behind Eye Center, With Bars

Problem #2
Stability

Stability

Adding drag or lift or both to the bow is destabilizing.

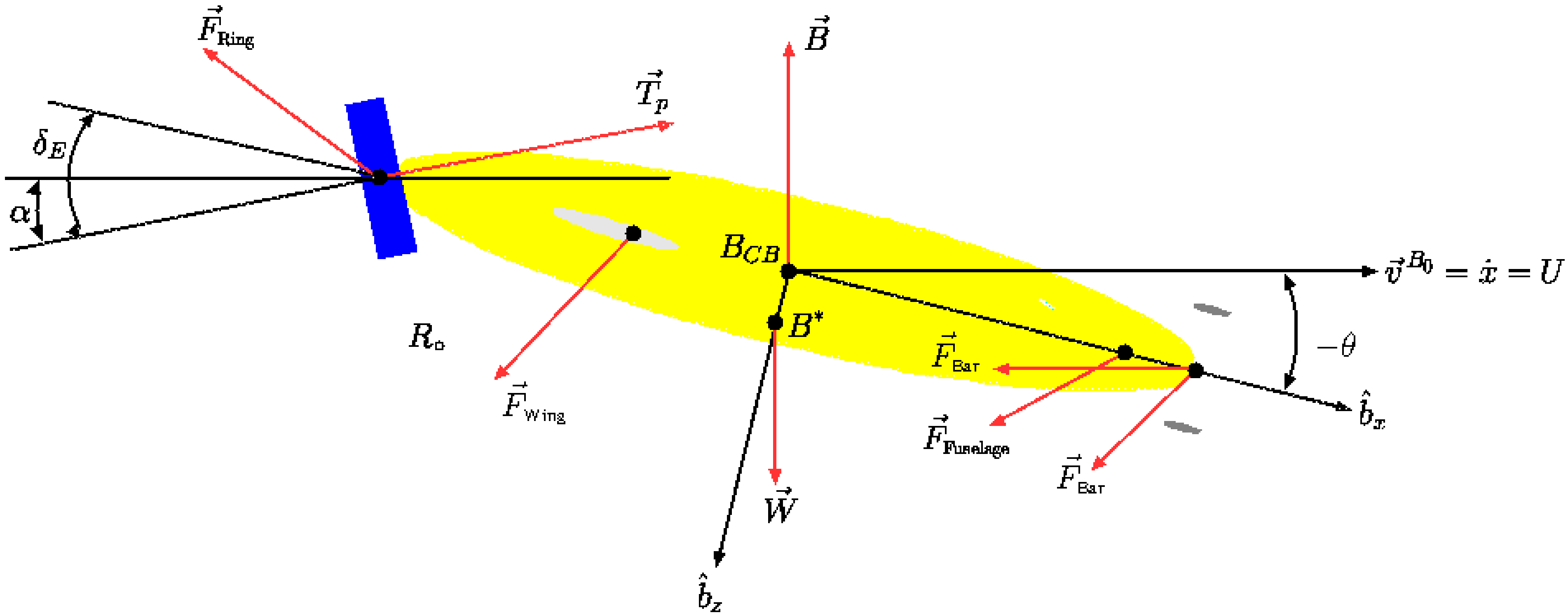
Ctd vehicle.
Streamlined bars.
With lights.
No Wings.
4.5 lbf buoyant.

Bars and Lights

- Shape effects lift, drag and stability.
- Round bar adds drag but not lift:
  - Not as destabilizing (less tendency to oscillate).
  - Affects both vertical and horizontal plane equally.
    - Assuming symmetry.
  - Will consume more propulsion power.
  - May increase the minimum speed due to absence of lift.

Free-Body Diagram

Steady-State Force and Moment Balance in Heave/Pitch (Vertical Plane).



Stability

General Principle: *More drag at the bow tends to destabilize the vehicle.*

- Reduction of stability causes oscillation in depth and heading.
  - Beware that vehicle rate limit will be imposed by camera.
- Winglets on duct may be necessary at lower speeds.
- Body wings mounted aft of the C.M. also increase stability.
- Analysis of natural frequencies + sea trials predict stability.

Conclusions So Far

- Sea-trial data and analysis to this point indicate both problems are solvable.
- Sea trials were performed with streamlined bars:
  - Speeds below 1 m/s at typical ballasting (4 lbf) may require body wings to hold depth.
  - Shifting wings aft increased stability.
  - Significant loss of control with unmodified duct near .5 m/s.
  - Winglets on duct improved control. Further improvements possible with algorithm adjustments.
- Further sea trials and analysis required for round bar.
- More accurate ballasting possible with 1-atm battery spheres versus pressure-tolerant batteries.

Navigation Problem

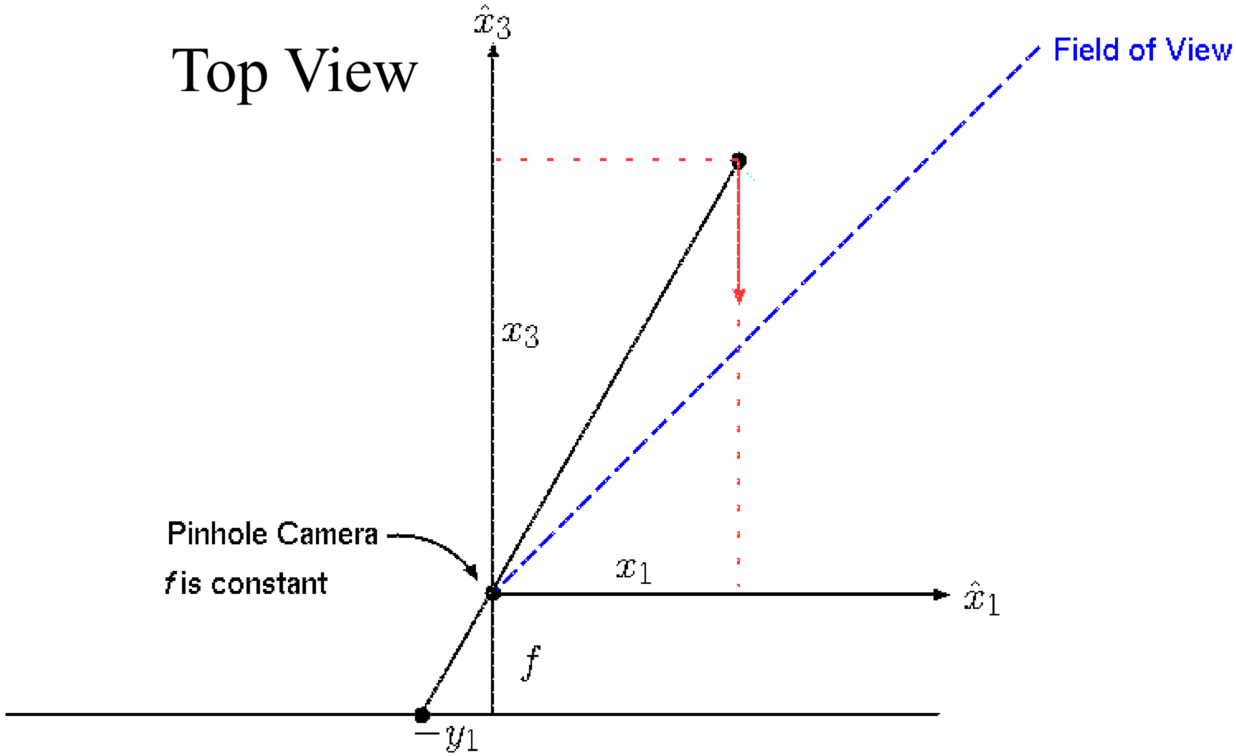
- Navigation accuracy not better than water currents when DVL can't see bottom.
- Will the vehicle get too close to a wall below 600m?
- Assume:
  - 330m transects
  - 5 below canyon rim at 600, 700, 800, 900, 1km.
  - Canyon 3.4 km wide at 1000m.
  - Can dive at .5 m/s vertical rate.
  - Transecting at .5 m/s horizontal speed.
- Then
  - 600/.5 + 5*360/.5 + 4*100/.5 = 5600 seconds = 1hr 33 min.
  - Assume 25 cm/s lateral current => vehicle displaced 1.4 km
  - 300 m margin.

Remaining Issues and Risk Areas

- Resolving optimal transect speed.
- Verification that system is working before launch (monitoring).
- Time code synchronization with science sensor data
- Camera lens control
  - Need iris for ambient light environments (near surface) *not strictly in requirements*.
  - May be desirable to move lens to hyper focus distance (focus to infinity) in some applications *not strictly in requirements* .
- Heat dissipation from camera housing

End

Translation of Vehicle Motion to Focal-Plane



$$\frac{x_1}{x_3} = \frac{-y_1}{f}$$

$$y_1 = -\frac{f}{x_3} x_1$$

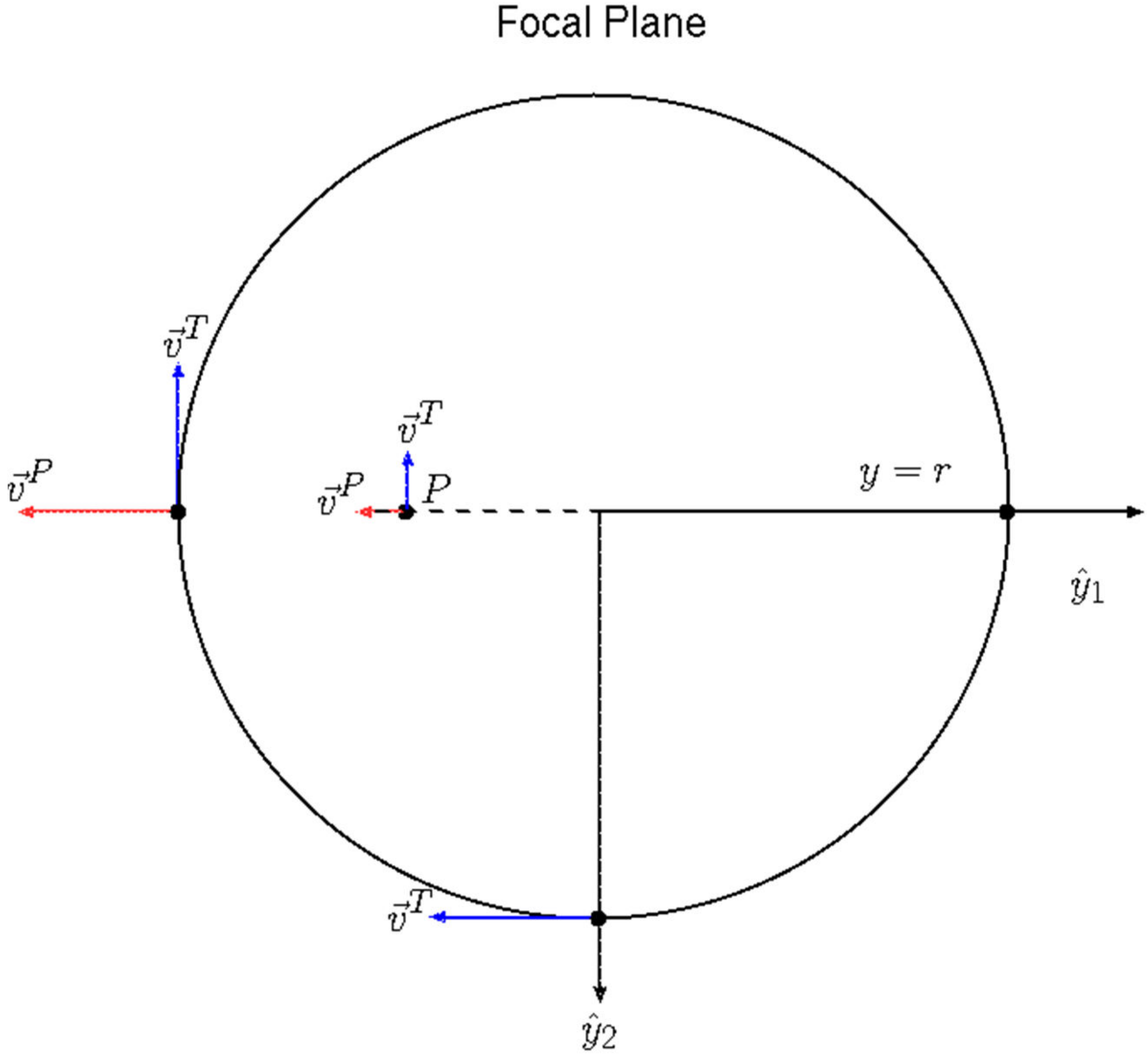
$$\dot{y}_1 = -f\,\frac{x_3\dot{x}_1 - x_1\dot{x}_3}{x_3^2} \quad (1) \quad \text{If } x_1 = r \text{ is constant:}$$

$$= f\,\frac{r\dot{x}_3}{x_3^2} \quad (2) \quad \text{If } \dot{x}_3 = -v\text{:}$$

$$= -f\,\frac{rv}{x_3^2} \quad (3) \quad \text{Let } \dot{y}_1 = v^P\text{:}$$

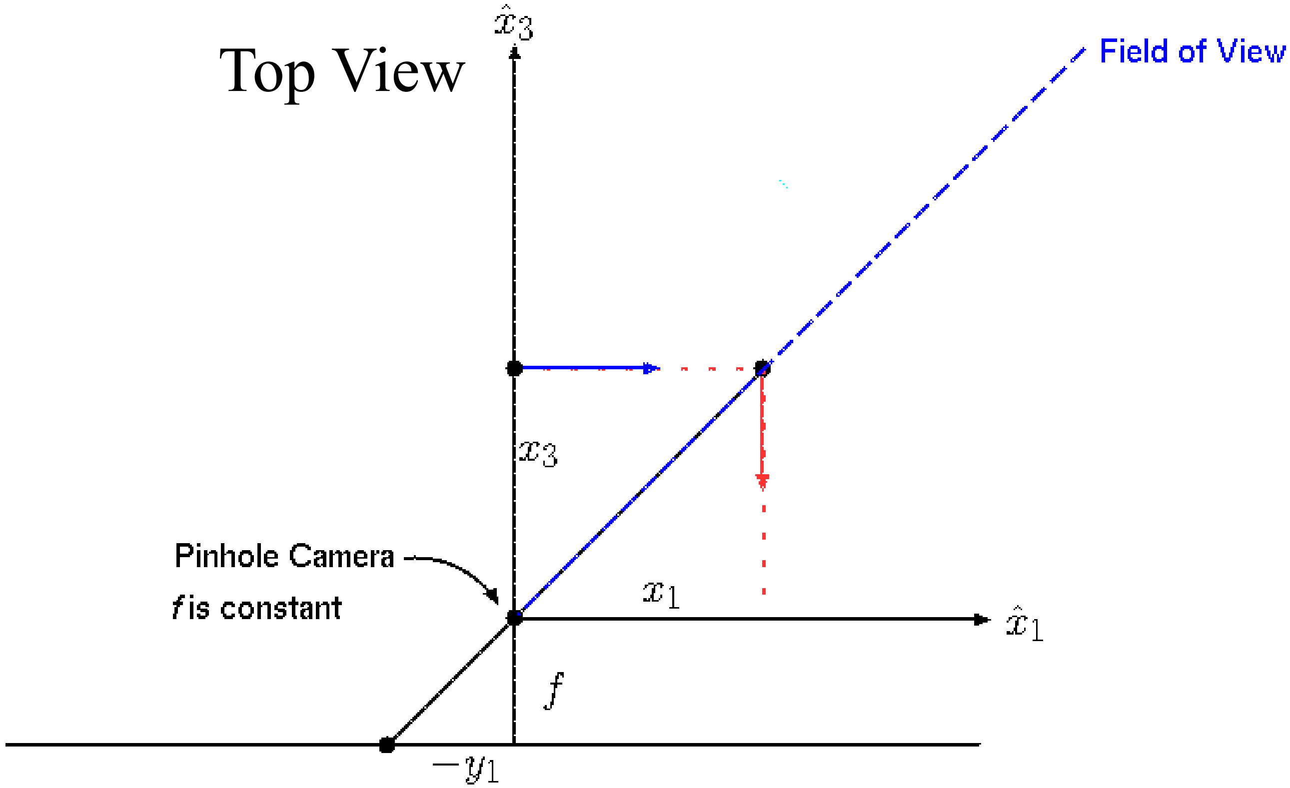
$$v^P = -f\,\frac{rv}{x_3^2} \quad (4)$$

Circular Motion on Focal-Plane



$\vec{v}^P$ – Velocity of point P in water.
$\vec{v}^T$ – Velocity of test point T on disk.

Translation of Circular Motion to Focal-Plane



$$\frac{x_1}{x_3} = \frac{-y_1}{f}$$

$$y_1 = -\frac{f}{x_3}x_1$$

$$\begin{aligned} \dot{y}_1 &= -f\,\frac{x_3\dot{x}_1 - x_1\dot{x}_3}{x_3^2} && (1) \quad \text{If } x_3 \text{ is constant:} \\ &= -f\,\frac{\dot{x}_1}{x_3} && (2) \quad \text{If } \dot{x}_1 = r\omega\text{:} \\ &= -f\,\frac{r}{x_3}\omega && (3) \quad \text{Let } \dot{y}_1 = v_T\text{:} \\ v^T &= -f\,\frac{r}{x_3}\omega && (4) \end{aligned}$$