Time and Energy Efficient Trajectory Generator for Autonomous Underwater Vehicle Docking Operations

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***Abstract*—This paper presents a novel real-time quasi-optimal trajectory generator, based on the inverse dynamics in the virtual domain (IDVD) method, to produce a reliable and efficient guidance system for autonomous underwater vehicle (AUV) docking operations. To this end, a challenging docking scenario is defined in a cluttered operating field, encompassing ocean currents and no-fly zones. Using the IDVD method, a trajectory that takes into consideration the hydrodynamic model of the vehicle is generated and the optimality of this trajectory, in regards to mission time and energy expenditure of the vehicle, is considered. Computer simulations demonstrate that the IDVD-based strategy enables the guiding of a vehicle into the dock by satisfying the final boundary conditions of the dock's position and orientation. Generated trajectories are feasible, smooth and realizable using for the vehicle's low-level auto-pilot module. In terms of computation, the solution is suitable for real-time implementation that incorporates uncertainty handling of the operating environment. For further analysis, the generated trajectory is evaluated on a high fidelity AUV simulator. The result of this latter test also demonstrates applicability of the utilized IDVD method for optimization of docking trajectories.**

Keywords—trajectory optimization; autonomous underwater docking; IDVD; AUV simulator; real-time computation

I. INTRODUCTION

As AUV missions grow more comprehensive and are required to spend longer periods of time underwater, the use of docking systems that permit battery recharging and data downloads, are steadily becoming increasingly important for oceanic research and underwater survey operations [1], [2]. Classic guidance laws such as line-of-sight (LOS) [3], linear terminal guidance [4], pure pursuit guidance [5] have been applied for the underwater guidance and docking operation. These guidance laws typically are limited in that they operate based only on the geometry and kinematics of the vehicle but the major advantage of them is their relative simplicity of implementation. Taking into account a series of assumptions and simplifications, these laws can be considered as an optimal guidance approach in a sense of minimizing the drift in the midcourse phase and miss distance of the terminal phase. More recently, AI-based techniques, such as fuzzy logic [6] and evolutionary algorithms [7], [8], have also been used to address the docking and rendezvous problems. It is noteworthy to mention that in all abovementioned studies there has been less attention on providing a universal framework that takes into account minimization of cost functions, like energy expenditure or mission time during docking operations subjected to all vehicular and environmental constraints. In other words, the docking problem has been less a focus of research from optimal control theory perspective.

Being influential for design, synthesis and analysis of complex phenomena, optimal control theory is able to provide a universal framework for developing a reliable and efficient underwater docking trajectory optimization or equivalently underwater docking guidance system. However, obtaining an analytical solution is the major difficulty with this kind of formulation. Numerical approaches are being utilized instead to find out a good approximation of the solution [9]. The well-known Pontryagin's maximum principle (PMP), as a main core of indirect methods, provides the necessary conditions for the optimal solution so that the control Hamiltonian adopts the minimum value over the permissible control interval. Direct methods, on the other hand, transcribe an infinite optimal control problem (OCP) to a finite nonlinear programming problem (NLP) by discretizing all or sub-sets of states and/or controls at first and then parametrizing them based on various interpolating functions to keep the continuous nature of an OCP. As there is no need to use the calculus of variations, direct methods, currently, are very popular and generally tractable. A plethora of the state-of-the-art NLP solvers are available facilitating the use of direct methods. Though the solution obtained by the direct methods are sub-optimal as opposed to that of indirect ones, direct methods have shown a robust performance with a large convergence radius and they are promising methods suitable for real-time implementation [9]. A number of studies have been conducted for underwater trajectory optimization and docking system of an AUV using direct methods [10]-[12].

The approach adopted in this paper is to define the docking problem in a context of trajectory optimization problem. By

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doing this, an OCP is formed in which the states of the system steer between the boundary conditions (initial pose of the vehicle and pose of the docking station (DS)), based on the control inputs while a set of constraints on states and control variables are satisfied and a cost functional like mission time and/or the vehicle's energy expenditure is minimized. Here, the proposed OCP is conceptualized as a simultaneous time-energy trajectory minimization problem resulting in smooth and realizable acceleration commands and state trajectories in a free final time while the physical limitations of the vehicle and environmental constraints are met. To do so, the Bolza cost functional [9] is defined first and then Pontryagin's necessary conditions are developed. Thereafter, a direct optimization method, referred to as the inverse dynamics in the virtual domain (IDVD) [13] is employed to solve the proposed OCP and eventually to provide an efficient set of trajectories for guiding AUV into DS. It is desired to have a set of trajectories which are both realistically tractable and computationally efficient in a cluttered operation field with ocean currents and no-fly zones. Using only a small set of optimization variables, IDVD can provide robust quasi-optimal solutions with a fast computational procedure for trajectory optimization tailored for a real-time implementation.

This paper is organized as follows. Section II presents the AUV modelling and docking problem in a form of a two point boundary value problem (TPBVP). In Section III, the Pontryagin's necessary conditions are developed. Section IV introduces IDVD and its key aspects for underwater docking trajectory optimization. In Section V, simulation results obtained from IDVD are presented and then their properness for a realistic docking scenario is investigated by an AUV simulator. Finally, the paper ends with conclusions in Section VI.

II. AUV Modelling and Underwater Docking Problem

The general 6-DoF nonlinear dynamic model of AUV has been developed [14]. Standard assumptions on neglecting roll and roll rate, relatively small pitch angle and slow-changing pitch rate, and a small side slip angle causing small sway and sway rate [15],[16], results in a 4-DoF AUV model

$$\begin{aligned} \dot{x} &= u\cos(\psi) + c_x \\ \dot{y} &= u\sin(\psi) + c_y \\ \dot{z} &= w \end{aligned} \tag{1}$$

$$\begin{aligned} m\dot{u} - (X_u + X_{u|u|}|u|)u &= T_u \\ m\dot{w} - (Z_w + Z_{w|w|}|w|)w &= T_w \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{\psi} &= r \\ I_z\dot{r} - (N_r + N_{r|r|}|r|)r &= T_r \end{aligned} \tag{3}$$

where x, y, z are the coordinates of the AUV's center of gravity in a local tangent frame $\{n\}$; u, w are surge and heave speed components respectively, in the body coordinate frame $\{b\}$ relative to the water, ψ is the yaw angle and r is the yaw rate, c_x and c_y are the northerly and easterly components of the current velocity, T_u, T_w, T_r are the control inputs in surge, heave and yaw directions. Meanwhile, the AUV is characterized by the mass of the vehicle m and its inertia around the z-axis I_z, components of the linear terms drag X_u, Z_w, N_r, and components of the quadratic terms drag, $X_{u|u|}$, $Z_{w|w|}$, $N_{r|r|}$. The values for these parameters were adopted from [17].

The utilized state and control vectors, considering the vehicle's hydrodynamic model, are introduced by $X=[x\ y\ z\ \psi\ u\ w\ r]^T$ and $U=[T_u\ T_w\ T_r]^T$, respectively. The underwater docking problem, in a context of TPBVP, for driving AUV through the DS, is defined as follow; starting from $X_0=[x_0\ y_0\ z_0\ \psi_0\ u_0\ w_0\ r_0]^T$ with $U_0=[T_{u;0}\ T_{w;0}\ T_{r;0}]^T$, it is desired to bring the AUV to some equilibrium state $X_f=[x_f\ y_f\ z_f\ \psi_f\ u_f\ w_f\ r_f]^T$ with corresponding control $U_f=[T_{u;f}\ T_{w;f}\ T_{r;f}]^T$ while obeying constraints and minimizing a performance index. The constraints include physical limitations on the AUV's states and controls

$$\begin{aligned} &|r| \le r^{\max} \\ &|T_u| \le T_u^{\max}, |T_w| \le T_w^{\max}, |T_r| \le T_r^{\max} \end{aligned} \tag{4}$$

along with multiple path constraints of the form

$$(x - x_{nf})^2 + (y - y_{nf})^2 + (z - z_{nf})^2 \ge r_{nf}^{\ 2} \tag{5}$$

where the vector $[x_{nf}\ y_{nf}\ z_{nf}]^T$ denotes the center of no-fly zone and r_{nf} is its radius. The performance index is represented in a form of Bolza cost functional

$$J = \min_{u(t),t_0,t_f} \Phi(t_0, X(t_0), t_f, X(t_f)) + \int_{t_0}^{t_f} L(t, X(t), U(t))dt \tag{6}$$

and consists of Lagrange term $L(.)$ and Mayer term $\Phi(.)$. In this study, the Lagrange term is a function of control inputs and final mission time and the Mayer term is zero. Ultimately, the normalized Bolza cost function is

$$J = \frac{1}{t_f (T_u^{\max})^2} \int_0^{t_f} (T_u^2 + T_w^2 + T_r^2 + 2t_f (T_u^{\max})^2 T_{\max}^{-2}\, t)dt \tag{7}$$

where T_{max} is a time scaling factor. The proposed cost function (7) allows the AUV to perform docking operations with a smooth trajectory which minimizes the control forces as well as mission duration.

III. Statement of the Pontryagin's Necessary Conditions

The Pontryagin's necessary conditions are established upon the Hamiltonian function. Let us define the Hamiltonian for the unconstrained OCP in this paper as a scalar function $H:[t_0,t_f]\times\Re^{nx}\times\Re^{nu}\times\Re^{nx}\to\Re$ that is

$$H(t, X(t), U(t), \lambda(t)) = L(t, X(t), U(t)) + \lambda^T(t) f(t, X(t), U(t)) \tag{8}$$

where $\lambda(t)$ is a co-state vector and $f(t,X(t), U(t))$ represents the hydrodynamic model of AUV in (1)-(3). The Hamiltonian for the proposed OCP is

$$
\begin{aligned}
H &= -\frac{1}{w_1}(T_u{}^2 + T_w{}^2 + T_r{}^2) + \lambda_1(u\cos(\psi) + c_x) + \lambda_2(u\sin(\psi) + c_y) \\
&+ \lambda_3 w + \frac{\lambda_4}{m}(X_u u + X_{u|u|}|u|u + T_u) + \frac{\lambda_5}{m}(Z_w w + Z_{w|w|}|w|w + T_w) \\
&+ \lambda_6 r + \frac{\lambda_7}{I_z}(N_r r + N_{r|r|}|r|r + T_r).
\end{aligned} \tag{9}
$$

Accordingly, the necessary conditions are

$$
\begin{aligned}
&\dot{X} = \frac{\partial H}{\partial \lambda}, \dot{\lambda} = -\frac{\partial H}{\partial X} \\
&\frac{\partial H}{\partial U} = 0, H|_{t=t_f} = 0.
\end{aligned} \tag{10}
$$

For the constrained OCP, (4)-(5) are taken into account and the augmented Hamiltonian is developed with respect to (9)

$$\overline{H} = H + \mu^T g(X,U,t) \tag{11}$$

where μ is the vector of Lagrangian multipliers and $g(X,U,t)$ is the general representation of constrains on states and controls. The condition for generating optimal control sequences is similar to (10) in which the derivatives of (11) with respect to U are equal to zero and additionally the Karush–Kuhn–Tucker (KKT) conditions for the Lagrangian multipliers to be satisfied

$$
\mu_i \begin{cases} \le 0 & g_i(X,U,t) = g_i^L \\ = 0 & g_i^L \le g_i(X,U,t) \le g_i^U \\ \ge 0 & g_i(X,U,t) = g_i^U \\ \text{unrestricted} & g_i^L = g_i^U \end{cases} \tag{12}
$$

where g^L and g^U represent the lower and upper bounds of constraint functions. The proposed constraints make the process of finding an analytical solution very challenging and may even be impossible to determine; therefore the need for employing a numerical method is vital. In principle, the proposed necessary conditions of optimality construct the core of indirect methods. There exist a series of practical difficulties making the indirect methods unsuitable for highly constrained trajectory optimization problems [18]. For instance, the initial guess on co-state variables must be properly provided to start the iterative methods. With respect to the non-physical nature of co-states, it is indeed a cumbersome procedure to determine suitable initial guess and an improper one can result in a non-converged or non-optimal solution. There is also a possibility that even relying on a reasonable guess, the adjoint equations in (10) turns out to be an ill conditioned case. Therefore, in spite of a small loss of the optimality, the direct methods are the most viable approaches to deal with such problems.

IV. Underwater Docking Using IDVD

The IDVD method is employed to design a robust guidance system allowing autonomous docking with a fixed funnel-shaped DS. The idea of IDVD method is to use the differential flatness property of the system dynamics, for performing the optimization in the output space. This is achieved by expressing the states and controls as functions of the output and its derivatives. Optimization is carried out in the virtual domain as opposed to the time domain enabling decoupling optimization of the 3D path and speed profile along it and thus providing a reliable tool for the fast prototyping of optimal trajectories [13].

To formulate the underwater docking problem using IDVD, the following steps are adopted:

- ***Step 1*** Generate a reference function in the virtual domain (domain) that is independent of time derivative constraints.
- ***Step 2*** Convert the reference trajectory back into the time domain using the speed factor (λ).
- ***Step 3*** Employ inverse dynamics to calculate states and controls.
- ***Step 4*** Execute optimization routine considering boundary condition, constraints and performance index.

For the proposed underwater docking problem, the reference functions are defined for the three spatial coordinates x, y, z using some analytically-defined basis functions of some abstract scaled argument $\bar{\tau} = \tau / \tau_f \in [0;1]$

$$
\begin{aligned}
&x(\bar{\tau}) = P_x(\bar{\tau}) = a_{0x} + a_{1x}\bar{\tau} + a_{2x}\bar{\tau}^2 + a_{3x}\bar{\tau}^3 + a_{4x}\bar{\tau}^4 \\
&+ b_{1x}\sin(\pi\bar{\tau}) + b_{2x}\sin(2\pi\bar{\tau}) \\
&y(\bar{\tau}) = P_y(\bar{\tau}) = a_{0y} + a_{1y}\bar{\tau} + a_{2y}\bar{\tau}^2 + a_{3y}\bar{\tau}^3 + a_{4y}\bar{\tau}^4 \\
&+ b_{1y}\sin(\pi\bar{\tau}) + b_{2y}\sin(2\pi\bar{\tau}) \\
&z(\bar{\tau}) = P_z(\bar{\tau}) = a_{0z} + a_{1z}\bar{\tau} + a_{2z}\bar{\tau}^2 + a_{3z}\bar{\tau}^3 + a_{4z}\bar{\tau}^4 \\
&+ b_{1z}\sin(\pi\bar{\tau}) + b_{2z}\sin(2\pi\bar{\tau})
\end{aligned} \tag{13}
$$

These spatial reference functions provide substantial flexibility for varying the curvature of trajectory using higher-order derivatives at the initial and end points. Differentiating (13) three times with respect to the argument $\bar{\tau}$ and taking into account the initial and final boundary conditions on the states and their derivatives the unknown coefficients in (13) are determined

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & \pi & 2\pi \\
0 & 1 & 2 & 3 & 4 & -\pi & 2\pi \\
0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 6 & 12 & 0 & 0 \\
0 & 0 & 0 & 6 & 0 & -\pi^3 & -8\pi^2
\end{bmatrix}
\begin{bmatrix} a_{0\eta} \\ a_{1\eta} \\ a_{2\eta} \\ a_{3\eta} \\ a_{4\eta} \\ b_{1\eta} \\ b_{2\eta} \end{bmatrix}
=
\begin{bmatrix} x_0 \\ x_f \\ x_0'\tau_f \\ x_f'\tau_f \\ x_0''\tau_f{}^2 \\ x_f''\tau_f{}^2 \\ x_0'''\tau_f{}^3 \end{bmatrix}. \tag{14}
$$

It is important to note that the spatial trajectory formulated in the virtual domain does not define the speed profile along it. It is the mapping between the virtual and physical domains that creates the speed profile. This mapping is done using the so-called speed factor.

$$\lambda(\tau)=\frac{d\tau}{dt} \tag{15}$$

The discrete representation of (15) relating time and virtual space is

$$\lambda_j=\Delta\tau\,\Delta t_{j-1}^{\;-1}\ ,\ \ j=2,\ldots,N \tag{16}$$

where

$$\Delta\tau=\tau_f(N-1)^{-1}$$

and N determines the number of computational nodes along the arc τ_f. Regarding the subscript of the time in (16), it is also necessary to compute the time step as it is not constant. This time step is calculated based on the division of distance between two computational nodes along the arc over the speed

$$\Delta t_{j-1}=\frac{\sqrt{(x_j-x_{j-1})^2+(y_j-y_{j-1})^2+(z_j-z_{j-1})^2}}{\sqrt{u_{j-1}^{\;2}+w_{j-1}^{\;2}+c_x^{\;2}+c_y^{\;2}+2c_x(u_{j-1}\cos(\psi_{j-1}))+c_y\sin(\psi_{j-1})}} \tag{17}$$

Now, the rest of states and controls are computed

$$\begin{aligned}
u_j&=\sqrt{(\lambda_j x'_{\tau;j}-c_x)^2+(\lambda_j y'_{\tau;j}-c_y)^2}\\
\psi_j&=\tan^{-1}\left(\frac{\lambda_j y'_{\tau;j}-c_y}{\lambda_j x'_{\tau;j}-c_x}\right)\\
w_j&=\lambda_j z'_{\tau;j}\\
r_j&=\frac{-(\lambda y'-c_y)(\lambda' x'+\lambda x'')+(\lambda x'-c_x)(\lambda' y'+\lambda y'')}{(\lambda y'-c_y)^2+(\lambda x'-c_x)^2}
\end{aligned} \tag{18}$$

$$\begin{aligned}
T_{u;j}&=m\lambda_j u'_{\tau;j}-(X_u+X_{uu}|u_j|)u_j\\
T_{w;j}&=m\lambda_j w'_{\tau;j}-(Z_w+Z_{ww}|w_j|)w_j\\
T_{r;j}&=I_z\lambda_j r'_{\tau;j}-(N_r+N_{rr}|r_j|)r_j
\end{aligned} \tag{19}$$

When all parameters (states and controls) are computed in each of the N points, the performance index is computed

$$J=k_t\left(\sum_{j=1}^{N-1}\Delta t_j-t_f\right)^2+k_u\sum_{j=1}^{N-1}(T_{u;j}^2+T_{w;j}^2+T_{r;j}^2)\Delta t_j \tag{20}$$

V. Simulation Results

Employing the IDVD method enables the real-time generation of quasi-optimal underwater docking trajectories to guide AUV into a fixed funnel-shaped DS. The proposed DS is represented with a pose vector $\eta_f=[x_f\ y_f\ z_f\ \psi_f]^T$ in which the location of DS in the $\{n\}$ frame is defined by x_f=150m, y_f=75m, z_f=10m, and orientation of DS with respect to the true North equals to ψ_f=225° (the DS heading angle is the angle between the centerline of the funnel entrance and the true North). The AUV's initial pose is defined by a vector of $\eta_0=[x_0\ y_0\ z_0\ \psi_0]^T$ in which the initial location of the vehicle in the $\{n\}$ frame comprises x_0=50m, y_0=50m, z_0=5m and the initial heading angle with respect to the true North is defined as ψ_0=10°. The rest of initial states is defined as follows: u_0=0.3 m/s, w_0=0 m/s, and r_0=0 °/s. A 2D current disturbance of magnitude c_x=0.25 m/s, c_y=0.25 m/s with respect to the North and East respectively. The constraint corresponding to the physical limitations of the vehicle's actuators and yaw rate is shown by (4) in which T_u^{max}= T_w^{max}=20 N, T_r^{max}=20 Nm, and r^{max}=15 °/s. Three no-fly zones, as mentioned in Section II, are considered to make the docking scenario more challenging.

Fig. 1 illustrates the 3D collision-free maneuvering path of AUV that satisfactorily meets the corresponding final boundary value that is the location of DS. In Fig. 2a the trajectories associated with the vehicle's forward and vertical velocities in the $\{b\}$ frame are shown; also time history of the vehicle's heading and heading rate can be found in Fig. 2b. The generated controls by IDVD are illustrated in Fig. 3a. Fig. 3b demonstrates the changes of speed factor and non-linear mapping between the virtual and physical domains.

As seen from Figs. 1-3, the generated trajectories using IDVD satisfy the assigned boundary conditions and produce smooth control time histories meeting all the constraints. This smoothness is due to the fact that IDVD solution enforces the higher-order derivatives to be feasible.

To be more specific, for the considered scenario, IDVD results in the normalized control expenditure per second of J_u=0.4 and normalized time cost J_t=0.7(equivalent to t_f=101.4s). It is noteworthy to mention that IDVD presents a robust compromise in simultaneous minimization of energy expenditure and mission time, which in principle are contradictory. In other words, minimization of time does not result in violation of controls and singular arcs. This result becomes more advantageous for real world implementation, taking into account the efficient computation time of IDVD method. For this scenario, using the MATLAB development environment (and the fmincon function for optimization) on a desktop computer with an Intel i7 3.40 GHz quad-core processor, the results were generated with a computation time t_{CPU}=5.9s. This real-time computation time stems from the nature of IDVD in which the underlying problem is encapsulated with a few number of varied parameters (only, 7 in this study).

To validate generated trajectories in a realistic sense, an AUV simulator is utilized [19], [20]. For this purpose, the generated trajectories are applied to the AUV within the simulator and the fidelity and tractability of trajectories are investigated.

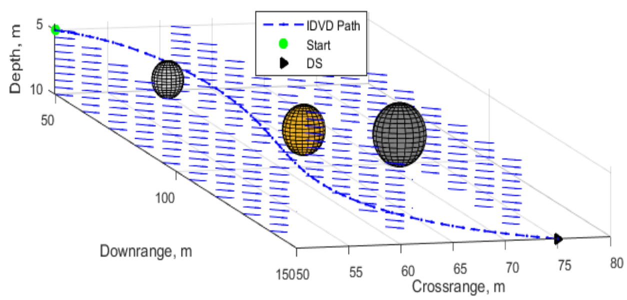

Fig. 1. The 3D path for traveling from the initial to DS position.

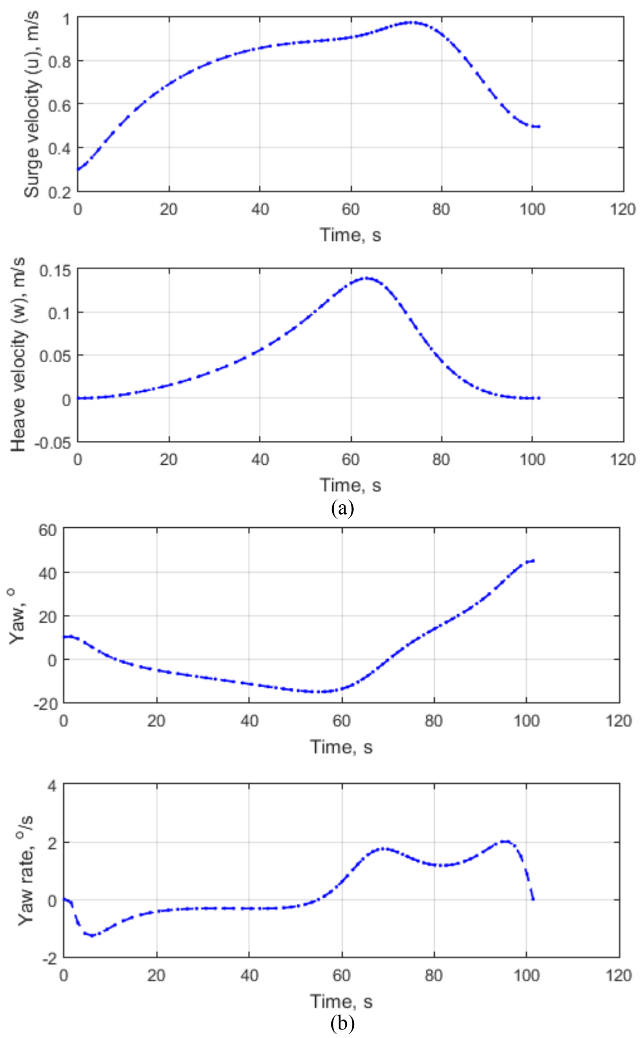

Fig. 2. Time history of surge and heave velocities (a), along with vehicle's heading and heading rate (b).

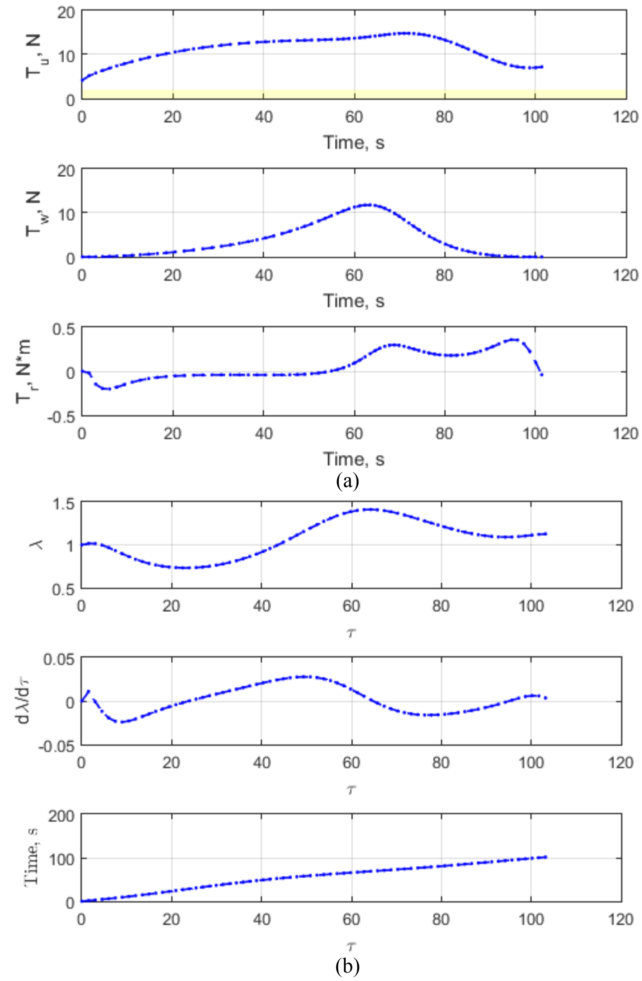

Fig. 3. Time history of control inputs (a), and changes of IDVD's speed factor and mapping between the virtual and physical domains (b).

As can be seen from Fig. 4, the AUV follows the reference 3D path and at the end converges to the DS position. By examining the velocity components, yaw and yaw rate trajectories in Fig. 4, it is inferred that they are realizable for the real vehicle as a result of good match between the IDVD solution and actual one. The jumps at the beginning of the actual trajectories and a slight time delay between the simulated and actual solution, are due to the synchronization of different subsystems in the simulator at the starting time, artefact of the controller and utilization of a simplified AUV model. This issue is more salient at the beginning of the yaw and yaw rate trajectories. The conclusion is that, despite the sharp jump at the beginning of the actual yaw rate trajectory of IDVD and slight chattering along it, there is a good consistency between the IDVD solution and the actual one. There is a slight error at the final condition between the actual and simulated yaw trajectory that leads to the small error of the final yaw.

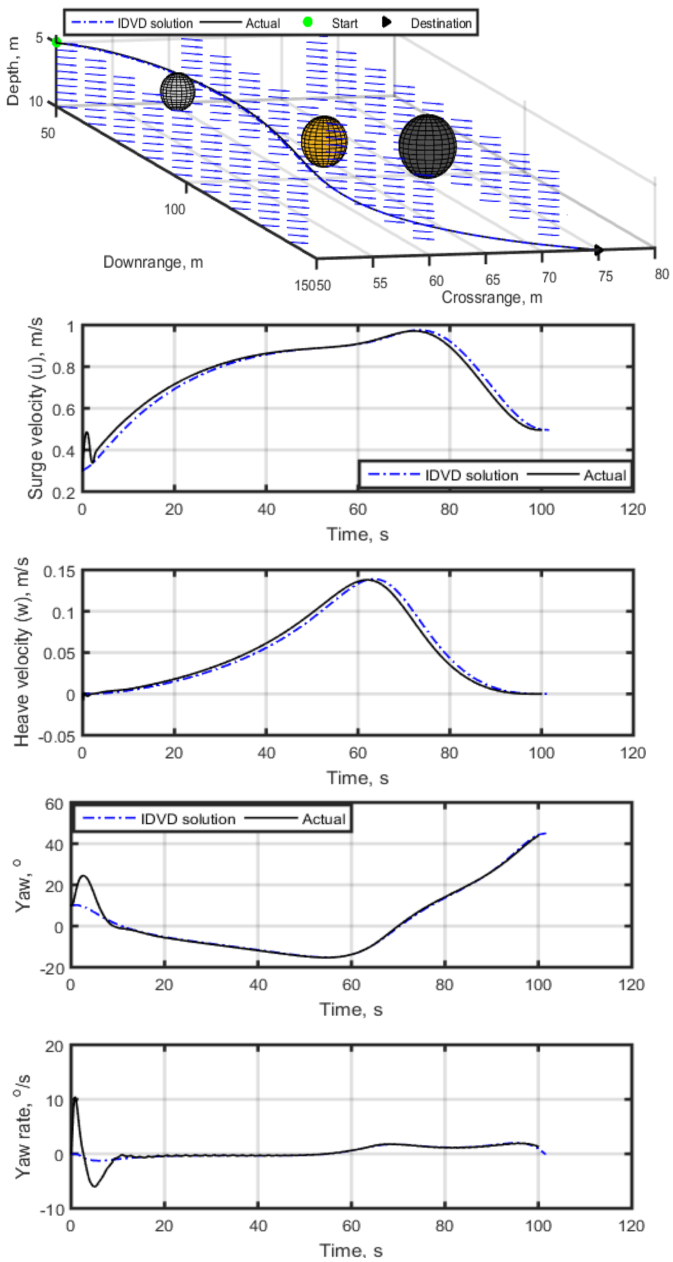

Fig. 4. Comparison of optimized and propagated trajectories for the IDVD method.

VI. Conclusion

By transforming the autonomous underwater docking problem into the context of optimal control theory, this paper presents a novel approach to handle this complex problem. More specifically, a branch of direct methods called IDVD is employed to provide a numerical solution for the underwater docking problem. The spatial manoeuvring capability of the vehicle toward the DS is investigated through the scenario when simultaneous minimization of mission time and vehicle's energy expenditures, with respect to both vehicular and environmental constraints, is carried out. The simulation results indicate that the generated trajectories are smooth and realizable and do not violate the constraints. In essence, the IDVD method provides a feasible quasi-optimal solution, consequently the solution obtained is implementable and tractable for a real vehicle. This is confirmed by propagating the trajectories whithin the AUV simulator. In addition, IDVD features an unprecedented fast convergence of the solution, which enables a real-time implementation. Future work will involve the IDVD implementation on a real underwater vehicle for autonomous docking with the purpose to further validate the efficiency and robustness of the developed guidance system in field trial experiments.

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