Speed calculation methods in electrical drive with non-ideal position sensor

Dominik Łuczak, *Member, IEEE*, Krzysztof Nowopolski, Krzysztof Siembab, Bartłomiej Wicher
Institute of Control and Information Engineering
Poznan University of Technology
Poznan, Poland
Email {dominik.luczak, krzysztof.nowopolski, krzysztof.siembab, bartlomiej.wicher}@put.poznan.pl

Abstract — **In this paper three methods of electric motor angular speed calculation are compared. The source of the measurement signal is a 14-bit absolute encoder. The authors compared the well-known classic methods M with more advanced approaches. First of the complex methods is based on utilization of phase loop lock system in an estimation process using on-line system model. The third method discussed in the paper is focused on application of Chebyshev filter. Short description of all of the mentioned method is followed by simulation and experimental results. The results are presented on an instance of permanent magnet synchronous motor drive, with both speed controller and speed calculation algorithms implemented in a digital signal processor system.**

Keywords—speed measurement, absolute encoder, position sensor, electrical drive, PLL, DSP, Chebyshev filter, low-pass filter, digital filtering, spectral analysis

I. INTRODUCTION

Contemporary absolute encoders available in the market allow to measure the angular position with resolution up to 27 bits per one revolution, even for high-speed applications. It means, that the minimum angular shift detected by the sensor is smaller than 0.01 of angular second. Due to so high resolution, nowadays absolute encoder is treated not only as angular position sensor, but its output signal may be utilized for angular speed calculation, even in high dynamic applications, such as industrial manipulators or CNC machines. Such conception allows to reduce the number of sensors, simplifying mechanical housing and reducing costs. The natural form of output signal transmission is a bus of parallel bits representing present state of encoder photo sensors. Although, serial data transmission standards are available as well – both generic (SPI, I^2C) and dedicated for this kind of sensors (EnDat, Bi-SS).

Absolute encoders, as sensors of all sorts, are exposed to occurrence of disturbances. Possible reasons of these disturbances and their consequences in the output signal are discussed in the section III of the paper. Inaccuracy of encoder operation and noise in position signal are especially harmful, if this signal is used for speed calculation.

In this paper three methods of rotational speed calculation based on real (i.e. burdened with noise and inaccuracy) position signal are presented. A short mathematical introduction (section II) is followed by simulation (section IV) and laboratory tests (section VI). The M method (described more widely in section II.A) is a basic approach, utilized as an element of the two more sophisticated methods described in the paper. T Method (measurement of time between minimum position changes identified by an encoder) was not possible to be implemented in the considered laboratory stand, due to specification of applied sensor interface card of the controller. Therefore, the method T is completely neglected in the paper. The laboratory stand is described shortly in the section V.

II. BASIS OF SPEED MEASUREMENT METHODS

A. M method

There are two basic, commonly-known methods for speed calculation utilizing position signal. Both of them use differentiation of position. Since the vast majority of nowadays measurement and control systems are implemented in digital environment, the continuous derivative need to be replaced by difference quotient $\omega = \frac{d\theta}{dt} \approx \frac{\Delta\theta}{\Delta t}$. In the classic approaches, either constant values of Δt or $\Delta\theta$ is assumed. The first approach is called M method: in the assured period of time the difference of angle position is measured. Obviously, the relative error of M method is connected with position measurement error, that is limited to ± 1[1]:

$$\Delta_M \% = \frac{\pm 1}{\Delta M} \cdot 100\% , \tag{1}$$

where ΔM is difference of angular position in one step expressed as multiply of sensor resolution. Since ΔM is proportional to the angular speed, the relative error of the method is inversely proportional to the velocity. To ensure high accuracy of the method at low speeds it is necessary to choose relatively long sampling time, which makes impossible to obtain good dynamic properties.

B. Filtering in speed measurement

Digital filters are used in drive control systems in order to improve the quality of the speed control [2]–[4]. Low-

978-1-4799-5081-2/14/$31.00

pass filter (LPF) is one of the simple filters allowing damping of the measurement quantization frequency. Chebyshev II filter is a satisfying approximation of a low-pass filter due to its flat shape of the transfer function module in the frequency response. The application of this filter approximation is proposed because the following essential parameters of filtration may be easily determined: maximum attenuation in the passband α_p, the minimum attenuation in the stopband α_z, the edge of the passband f_p, (also called the cut-off frequency), and the edge of the stopband f_z, for which the minimum damping is equal to α_z. In the further discussion the complex operator s is normalized by the arbitrarily chosen angular frequency $\omega_0=2\pi f_0$, obtaining $s'=\dfrac{s}{\omega_0}$.

The angular frequency is computed on the basis of the frequency value: $\omega_p=2\pi f_p$, $\omega_z=2\pi f_z$, $\omega_0=\omega_z$. The stopband ripple factor ε may be defined from the α_z value according to (1) [5].

$$\alpha_z=-10\log\left(\frac{\varepsilon^2}{1+\varepsilon^2}\right),\ \varepsilon=\frac{1}{\sqrt{10^{0.1\alpha_z}-1}} \tag{2}$$

The desired filter parameters are obtained, if the filter order is greater than:

$$N=\left\lceil\frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1\cdot\alpha_z}-1}{10^{0.1\cdot\alpha_p}-1}}\right)}{\cosh^{-1}\left(\dfrac{2\pi f_z}{2\pi f_p}\right)}\right\rceil. \tag{3}$$

General formula for continuous transfer function of the Chebyshev II filter (4) consists of N poles and even number r of conjugated zeros pairs, determined by (5) [5], [6].

$$T_F(s')=\left(\prod_{i\in\{0,\,1,\ldots,N-1\}-\{(N-1)/2\}}\frac{s'-z_i}{z_i}\right)\cdot\left(\prod_{i=0}^{N-1}\frac{p_i}{s'-p_i}\right) \tag{4}$$

$$r=\begin{cases}\dfrac{N}{2}, & \text{if } N \text{ even}\\ \dfrac{N-1}{2}, & \text{if } N \text{ odd}\end{cases} \tag{5}$$

Following relations are utilized in the following part of the paper [5], [6]:

$$D=\frac{\sinh^{-1}\left(\dfrac{1}{\varepsilon}\right)}{N},\ R_1=\sinh(D),\ R_2=\cosh(D),$$

$$\beta_i=\frac{\pi}{2}+\frac{\pi}{2N}+i\frac{\pi}{N}=\frac{\pi}{2}+\frac{\pi+2\cdot i\cdot\pi}{2N},\ i=0,1,..,r-1.$$

The zeros of the continuous transfer function of the Chebyshev II filter were determined using (5). The poles of the Chebyshev II continuous transfer function determined as: $p_i^{R_2}=R_2e^{j\beta_i}$, $p_i^{R_1}=R_1e^{j\beta_i}$, $p_i=\dfrac{p_i'}{\omega_0}$, obtaining (7). From each conjugated pair of zeros and poles, only one zero and one pole are examined and hence $i=0,1,\ldots,r-1$.

$$z_i=\frac{z_i'}{\omega_0}=\frac{1}{j\sin(\beta_i)}=\frac{-j}{\cos\left(\dfrac{\pi+2\cdot i\cdot\pi}{2N}\right)} \tag{6}$$

$$p_i=\frac{1}{Re\{p_i^{R_1}\}+jIm\{p_i^{R_2}\}}=\frac{1}{R_1\cos(\beta_i)+jR_2\sin(\beta_i)} \tag{7}$$

In the case of odd filter order, the additional pole is determined as follows:

$$\beta_{i=r}=\frac{\pi}{2}+\frac{\pi+2\cdot\dfrac{N-1}{2}\cdot\pi}{2N}=\pi,$$

$$p_{i=r}=\frac{1}{R_1\cos(\beta_r)+j\,R_2\sin(\beta_r)}=\frac{1}{-\sinh(D)}.$$

Taking into consideration that zeros and poles of the transfer function (4) are pairs of conjugated complex numbers, the general transfer function formula (4) may be transformed to (8)–(11).

$$T_F(s')=G\cdot T_0\prod_{i=0}^{r-1}\frac{(s'-z_i)(s'-\overline{z_i})}{(s'-p_i)(s'-\overline{p_i})} \tag{8}$$

$$G=\prod_{i=0}^{r-1}\frac{|p_i|^2}{|z_i|^2} \tag{9}$$

Conjugate of a complex zero (6) is equal to $\overline{z_i}=-z_i$ due to the lack of real part. The product of conjugated poles may be treated as a square of complex number module: $p_i\overline{p_i}=|p_i|^2$. The additional pole in odd filter order case is described by the continuous transfer function (10).

$$T_0(s')=\begin{cases}1 & \text{if } N \text{ even}\\ \dfrac{-p_r}{s'-p_r} & \text{if } N \text{ odd}\end{cases} \tag{10}$$

Using properties of complex numbers for zeros and poles of the continuous transfer function of Chebyshev II filter, the final formula (11) is obtained.

$$T_F(s')=G\cdot T_0\prod_{i=0}^{r-1}\frac{s'^2+|z_i|^2}{s'^2-2Re\{p_i\}\,s'+|p_i|^2} \tag{11}$$

Derivation of discrete transfer function was described more widely in [7].

C. PLL speed estimator

Phase-Locked Loop (PLL), operates using feedback and is utilized most commonly for automatic frequency control. The basic version of PLL consists of: phase-detector, low-pass filter, voltage controlled oscillator (VCO), shown in the Fig. 1a.

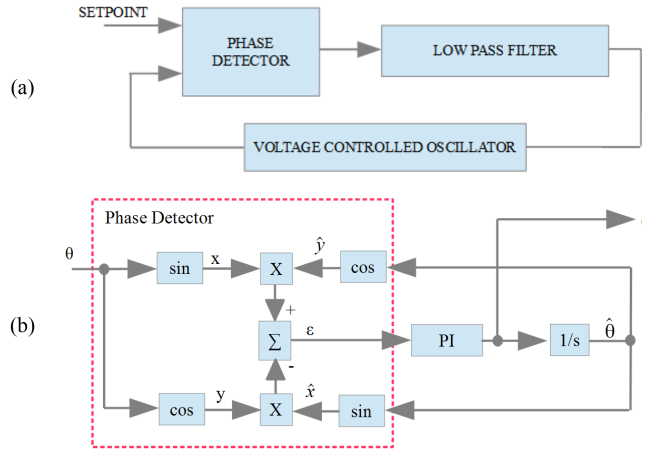

Fig. 1. General (a) and detailed (b) scheme of PLL

The simplest description of PLL operation is tending of the system to remain equal frequency of signals on both inputs of the phase detector. In the state of equilibrium of the PLL, the frequencies on both inputs are equal, and the voltage behind low-pass filter is constant. If change of input signal frequency occur, the difference of both signal phases computed by phase detector after passing the filter, change the frequency of VCO, correcting the frequency. The speed of reaching next state of equilibrium is depended on characteristic of PLL low-pass filter.

PLL may be applied as speed estimator [8], fed by position signal. Block diagram of PLL utilized for speed estimation is shown in the Fig. 1b. Phase detector is the most important part of the scheme. Sine and cosine of angular position measured by an encoder and estimated position from the output of PLL are provided to the inputs of the detector.

$$\begin{cases} x = A\sin\theta \\ y = A\cos\theta \end{cases} \quad \begin{cases} \hat{x} = \sin\hat{\theta} \\ \hat{y} = \cos\hat{\theta} \end{cases}, \tag{12}$$

where θ angular position measured by the encoder, $\hat{\theta}$ estimated position , A gain factor.

The output of phase detector is defined as vector product:

$$\varepsilon(t) = x\hat{y} - y\hat{x}\,. \tag{13}$$

Taking into consideration (12), the following formula may be obtained:

$$\varepsilon(t) = A\left(\sin\theta\cos\hat{\theta} - \cos\theta\sin\hat{\theta}\right) = A\sin\left(\theta - \hat{\theta}\right). \tag{14}$$

For small differences of phase displacement, the output of phase detector may be described as scaled difference of the measured and estimated position values:

$$\varepsilon(t) = A\left(\theta - \hat{\theta}\right). \tag{15}$$

The difference of phases (position error) feeds PI controller. The controller brings the error to zero and removes phase displacement between positions:

$$\varepsilon(t) = 0 \Rightarrow \theta = \hat{\theta}\,.$$

Estimated speed value is provided to the output of the PI controller. The position is estimated on the basis of this value.

Transfer function of linearized open- and close-loop system for Fig. 1 may be described as (15) and (16) respectively.

$$G_{open}(s) = AG_{PI}(s)G_{VCO}(s) = AK_p\frac{1+sT_i}{s^2T_i} \tag{16}$$

$$G_{closed}(s) = \frac{G_{open}(s)}{1+G_{open}(s)} = \frac{AK_ps + \frac{AK_p}{T_i}}{s^2 + AK_ps + \frac{AK_p}{T_i}} \tag{17}$$

The relation (17) may transformed to (18), as in [7]:

$$\frac{\hat{\theta}(s)}{\theta(s)} = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \tag{18}$$

where ω_n – angular frequency of free vibrations, ξ – damping factor. The above transfer function allows to determine parameters of PI controller (19), taking into consideration relation (20).

$$T_i = \frac{2\xi}{\omega_n}, \quad K_p = \frac{2\xi\omega_n}{A} \tag{19}$$

$$\omega_{-3dB} = \omega_n\sqrt{1+2\xi^2+\sqrt{2\left(1+2\xi^2+2\xi^4\right)}} \tag{20}$$

Besides pure PLL method, a more complicated combined method, utilizing both filtered M method and PLL. In this case, PLL is a compensator of the delay cause by low-pass filter. In the following part of paper this method is named PLL+M.

All the methods listed in this section were first tested by simulation in MATLAB/SIMULINK environment. In order to provide good consistency between simulation and tests on laboratory stand, a model of non-ideal encoder was developed.

III. ENCODER SIMULATION MODEL

Real encoders are affected by non-idealities that downgrade the quality of the instrument. The motivation for

presented simulation model of the encoder were the preliminary tests of speed measurement methods conducted at the laboratory stand. The observed measurement noise was significantly bigger than it was expected in M method. Moreover artifacts in calculated speed appeared. The analysis of the signals from the encoder showed, that the fill factor of the square wave is not equal to 50% as it should be. The issue is presented in the Fig. 2. Authors assumed that the model needs to satisfy the following assumptions:
- each bit of the 14-bit encoder shall be independently affected by separate fill factor shift, constant over the full 2π angle,
- the angular period T_α of the waveform generated by each bit of the encoder is related to the actual bit N and could be described by the equation (21),

$$T_\alpha = \frac{2\pi}{2^{N-1}} \tag{21}$$

where N = 1,2..14 is the bit number, first is the most significant,

- the angular period T_α for each bit is divided into two parts T_H and T_L which must satisfy the equation (22):

$$T_H + T_L = T_\alpha . \tag{22}$$

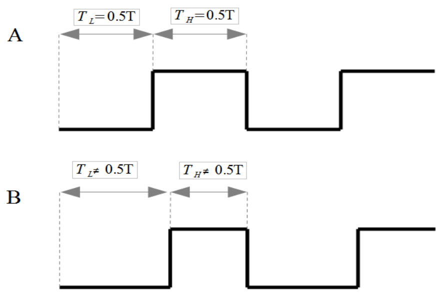

Fig. 2. Waveforms generated by ideal (A) and not ideal (B) encoder while rotation at constant angular velocity

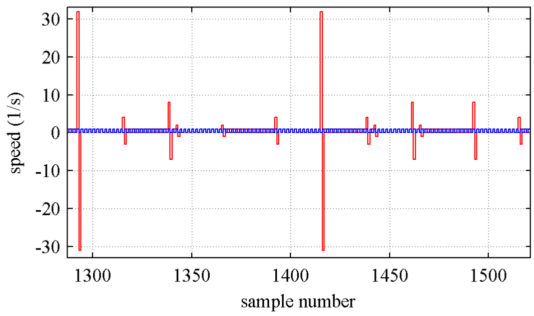

Fig. 3. Speed as a function of sample number during rotation at constant angular speed, red curve - encoder with nonideality, blue curve – ideal encoder

It was expected that the encoder inaccuracy will affect the speed measurement. The phenomenon is presented in Fig. 3. The blue waveform presents the derivative of angular position when ideal encoder is simulated, the red one presents the derivative of angular position when non - ideal encoder is simulated. As it can be seen, with ideal instrument the derivative changes between the values 0 and 1 (what is normal when the angular speed remains constant), and for non -ideal instrument the derivative changes much more.

IV. Simulation

Simulations were performed for three speeds: low 10 rad/s, medium 50 rad/s and high 100 rad/s. During the simulation the model of non-ideal position transducer was used. Results of simulations in the time domain are shown in the Fig. 4. Calculated speed were analyzed with Fourier transform [9]. Spectrums are shown in the Fig. 5.

(a)

(b)

(c)

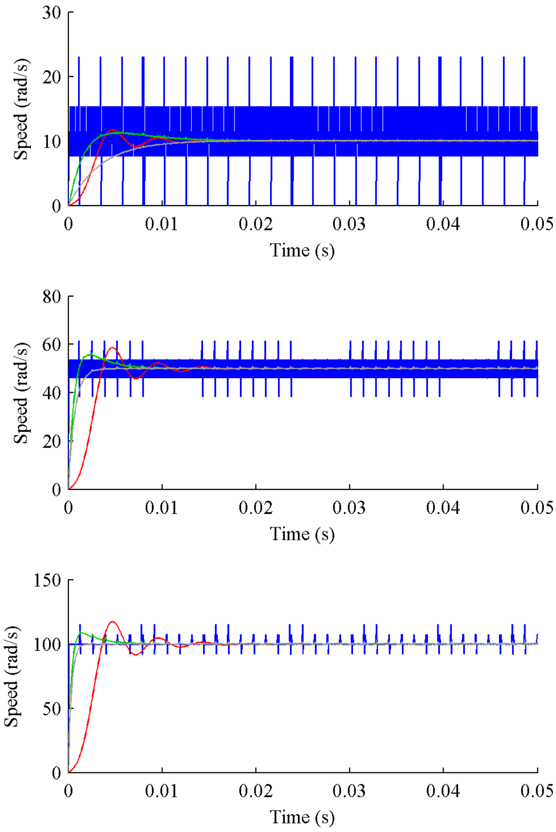

Fig. 4. Results of simulation experiment at angular speed of: (a) 10 rad/s, (b) 50 rad/s and (c) 100 rad/s. Line colors: blue – method M, red – LPF, green – PLL+M, grey – PLL

V. Laboratory Stand (Hardware)

Laboratory stand (Fig. 6) enables implementing methods described in section II. The stand consists of: DSP module (AD SHARC 21369), synchronous motor (3-phase, 0.75kW, 400V), power inverter (IGBT, 3 phase, 700V max, 15A max), absolute encoder (14 bits, Gray code out),- Tektronix DPO 3014 oscilloscope. The DSP enables to put the desired (chosen by the programmer) signals on the 4 analog outputs (e.g. speed, position, voltage, current) and then feed them to the oscilloscope. The frequency of the main control loop was set to 10kHz.

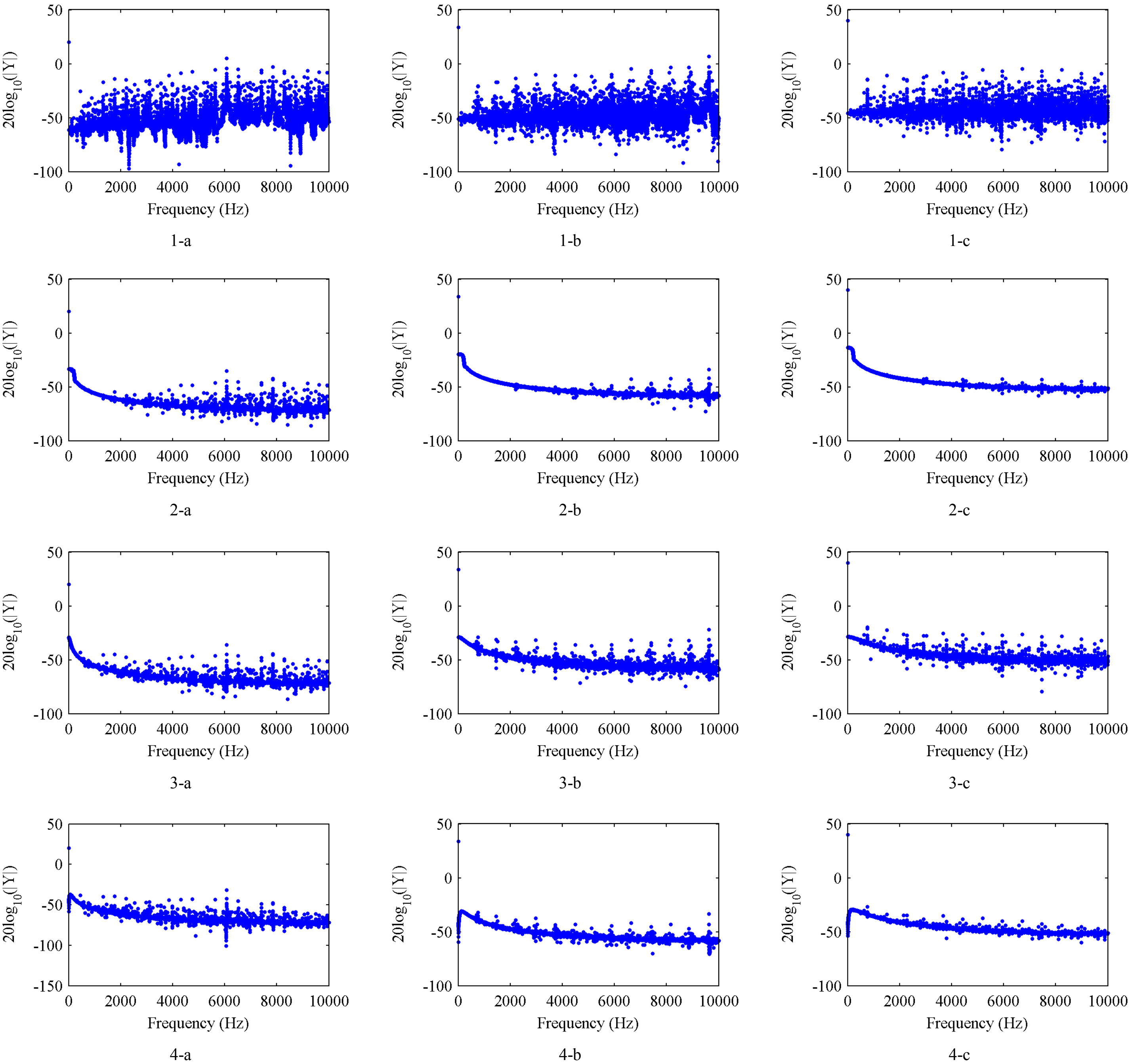

Fig. 5. Spectrum of simulation measured a-low, b-medium and c-high speed obtain by 1-M method, 2-M-method with LPF, 3-PLL+M and 4-PLL

VI. Experimental Results

The following experimental results were obtained in the described laboratory stand. In the Fig. 7a a comparison of three discussed methods in time domain is presented. It can be easily noticed, that the speed calculation results given by M method are very poor, comparing with more advanced approaches. Due to inaccuracy of sensor, relative error of M method is much greater than can be expected by (1). For the applied encoder, at speed of 20 rad/s and position measured by 14-bit encoder, the relative error shall be near to 3%. In the Fig. 7a error reaching 100% can be observed. The variation of signal is too big to ensure any stable operation e.g. in closed-loop speed control system, making the signal useless for further processing. The results of two remaining methods are similar in time domain and ensure satisfying quality.

Differences between methods of LPF Chebyshev II and estimator with PLL are more noticeable in frequency domain. The attenuation in range above 1 kHz is close, the main difference is located between 500 and 1000 Hz.

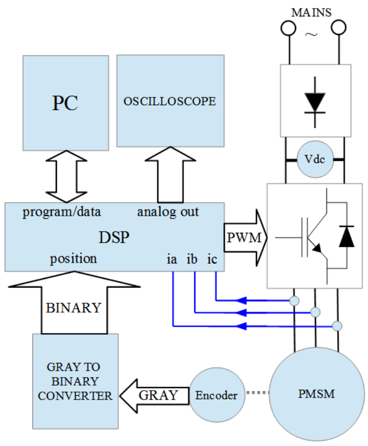

Fig. 6. Laboratory stand overview

Chebyshev filter damping is stronger in this range due to occurrence of two stop-bands.

VII. Conclusions

Both simulation and experimental investigation showed clearly that pure M method is useless in speed measurement system based on non-ideal position sensor. Due to the non-ideality of the transducer the error of the method is much bigger than predicted by method assumptions.

Both low-pass Chebyshev filter and the PLL+M method provide much better, satisfying results. The output signal of these methods is proper, the noise of quantization is removed, higher frequencies are attenuated, since both methods operate as a low-pass filter. However, PLL method removes the delay introduced by LPF due to provided by PI controller lack of time displacement between estimated and measured position.

Since both of them are based on the M method, the decreasing accuracy at lower rotational speeds is inherited. Even at the lowest tested speed, the results given by PLL method are still sufficient. Hence, this method may be recommended for applications with speed measurement calculated from absolute position.

Acknowledgment

The presented results of research, were carried out under the theme No. 04/45/DSMK/0107, which was funded by the Polish Ministry of Science and Higher Education.

References

[1] S. Brock and K. Zawirski, "Cyfrowy pomiar prędkości obrotowej w napędzie elektrycznym," *Pomiary Autom. Robot.*, vol. 8, pp. 10–13, 2005.

[2] S. Brock, "Hybrid PI Sliding Mode Position and Speed Controller for Direct Drive," in *Mechatronics 2013*, T. Březina and R. Jabloński, Eds. Springer International Publishing, 2014, pp. 741–748.

[3] S. Brock and J. Deskur, "The problem of measurement and control of speed in a drive with an inaccurate measuring position transducer," in *10th IEEE International Workshop on Advanced Motion Control, 2008. AMC '08*, 2008, pp. 132–136.

[4] S. Brock and D. Łuczak, "Speed control in direct drive with non-stiff load," in *2011 IEEE International Symposium on Industrial Electronics (ISIE)*, 2011, pp. 1937–1942.

[5] J. Izydorczyk and J. Konopacki, *Filtry analogowe i cyfrowe*. Katowice: Wydaw. Pracowni Komoputerowej, 2003.

[6] T. P. Zieliński, *Cyfrowe przetwarzanie sygnałów: Od teorii do zastosowań*. Wydawnictwa Komunikacji i Łączności, 2007.

[7] D. Łuczak, "Tunable digital filter structures for resonant frequency effect reduction in direct drive," in *2012 8th International Symposium on Communication Systems, Networks Digital Signal Processing (CSNDSP)*, 2012, pp. 1 –6.

[8] B. Fabianski and K. Zawirski, "Switched Reluctance Motor Drive Embedded Control System," in *Mechatronics 2013*, T. Březina and R. Jabloński, Eds. Springer International Publishing, 2014, pp. 339–346.

[9] D. Łuczak, "Frequency analysis of mechanical resonance in direct drive," in *2012 12th IEEE International Workshop on Advanced Motion Control (AMC)*, 2012, pp. 1 –5.

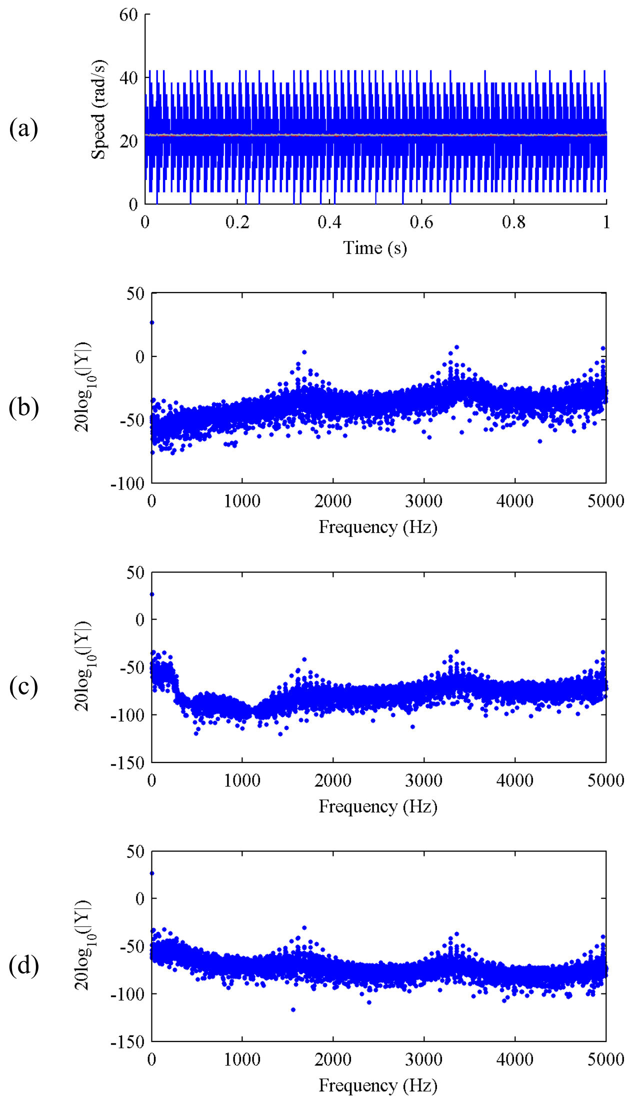

Fig. 7. Speed results of experiment (a), line colors: blue – method M, red – LPF, grey – PLL with M and spectrums of: method M (b), LPF (c), PLL + M (d)