ALTEX Vehicle Hull Strength

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1 Load case

The ALTEX vehicle will have an exterior skin designed to withstand a variety of loads during operation, launch, recovery, and service. In order to arrive at a useful strength specification, we will apply the load condition illustrated in Figure 1. Here, we imagine the vehicle suspended from a single lift point, while

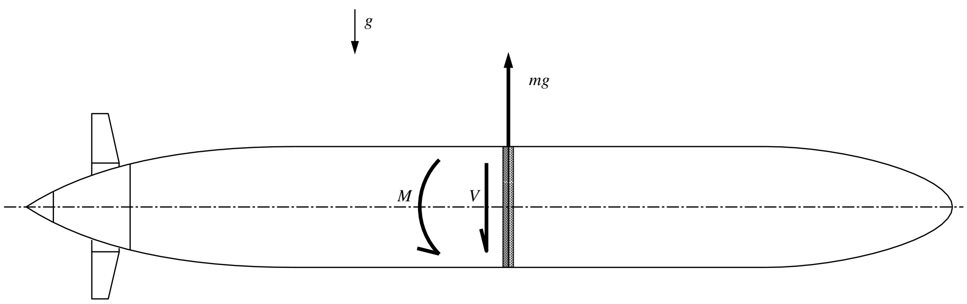

Figure 1: Load case for ALTEX vehicle strength calculation.

fully loaded with water. This condition does not represent the expected method of launching and recovering the vehicle, but should give us an idea of the maximum loads to be expected. The pertinent specifications are:

m_w	=	795 kg	Mass of vehicle filled with water
l	=	4.19 m	Vehicle length
D	=	0.5334 m	Vehicle outside diameter

Assuming the weight distribution is water density times cross sectional area, the maximum moment and shear force is found to be:

$$M = 31.2\text{kNm} \quad (1)$$
$$V = 3.92\text{kN} \quad (2)$$

If the lift point is moving in a sea state or the submarine experiences added mass forces while being submerged, the loads would be of the same character but different in magnitude.

If the vehicle was suspended from two lift point, the moment would be roughly 1/4 as large, whereas shear force would be about half of the above value.

The failure mode that will be considered here is that of elastic buckling. However, plastic deformation could be a concern if we build a thick shell from a weaker material like HDPE.

The canonical buckling case is the Euler column under compression load. It is characterized by the ability to support the load until the load reaches a critical value, at which point the column's equilibrium becomes unstable. Infinitesimal disturbances or imperfections will cause the column to bow out an arbitrary amount. Loads infinitesimally larger than critical will cause complete collapse of the column. A thin circular shell such as the altex hull would have a similar load-displacement history, but the buckling mode is different, with smaller waves emerging in the shell. Think of a compressed soda can or a bent drinking straw.

2 Critical loads

Elastic collapse of thin walled circular cylinders usually deals with three fundamental loads: axial compression, torsion, and radial pressure (Yamaki, 1984). None of these map exactly to our case, so we will have to apply some judgement. For bending, it is normally recommended that the critical compressive stress be the same as for the uniform axial loading case, (Timoshenko, 1936; Kollár and Dulácska, 1984), perhaps with different empirical corrections. Similarly, shear stress can be adopted from the torsion case (Samuelson and Eggwertz, 1992). Let us therefore discuss these fundamental cases. (Samuelson and Eggwertz, 1992) provides a very concise and useful description of the subject.

2.1 Circular cylindrical shell in uniform axial compression

(Timoshenko, 1936) presents a set of governing equations for the axial, tangential, and radial displacement for a thin cylindrical shell under axial compression, based on linearizing assumptions in all dependent variables. Assuming displacements of the type:

$$\sin n\theta \sin \frac{m\pi x}{R}$$

it is possible to write a linear system for the 3 unknown amplitudes. The coefficient matrix contains the axial force, the elastic and geometric properties of the cylinder, and the deflection wave numbers. Setting the determinant of this matrix equal to zero provides a buckling criterion, and the result is

$$\frac{\sigma_x(1-\nu^2)}{E} = \frac{h^2}{12R^2}\frac{n^2+\left(\frac{m\pi R}{l}\right)^2}{\left(\frac{m\pi R}{l}\right)^2} + \frac{(1-\nu^2)\left(\frac{m\pi R}{l}\right)^2}{n^2+\left(\frac{m\pi R}{l}\right)^2} \tag{3}$$

The critical stress is given by the minimum value of σ_x in (m,n) space. This is analogous to the Euler column, where the lowest buckling stress for all the different wavelength deflections is the critical one (which corresponds to the longest permissible wavelengths).

It turns out that σ_{cr} is almost independent of buckling mode, and given by:

$$\sigma_{cr} = \frac{Eh}{R\sqrt{3(1-\nu^2)}} \tag{4}$$

The value of (m,n) is determined by the cylinder dimensions and boundary conditions at the ends of the shell.

Researchers were startled to find that their experimental results showed critical stresses much lower than (4). It turns out that for the axial compression case, nonlinear terms related to the out of plane (non-membrane) stress in the shell drastically reduces the strength, resulting in the load-displacement relationship sketched in Figure 2 (solid line). As indicated by the dashed lines, the critical load achieved by a particular cylinder will be very sensitive to initial imperfections, here represented as zero-load displacements. Therefore, in the design of practical columns, empirical reduction factors must be employed, and manufacturing tolerances must be specified. Interestingly, in torsion, the nonlinear terms are benign, and experimental results agree well with theoretical predictions.

2.2 Circular cylindrical shell in bending

Maximum compressive stress is related to the bending moment through the slender beam approximation:

$$\sigma_x = \frac{M}{\pi R^2 h} \tag{5}$$

Using this stress with the theoretical critical condition in the axial load case, would yield the critical bending moment:

$$M_{cr} = \frac{\pi E R h^2}{\sqrt{3(1-\nu^2)}} \tag{6}$$

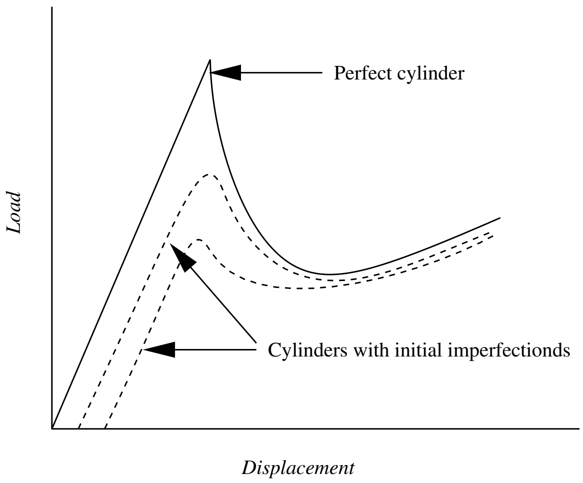

Figure 2: Conceptual load-displacement relationship for circular cylinder shell under axial compression.

As discussed however, this value is optimistic in practice, and reduction factors are needed. Let us look at a few different sources for the practical buckling resistance for a cylindrical shell with the following properties:

$$
\begin{array}{lllrl}
R & = & 0.2667\text{m} & = & 10.5\text{"} & \text{Radius} \\
h & = & 0.00635\text{m} & = & 1/4\text{"} & \text{Thickness} \\
E & = & 2.0\text{GPa} & \approx & 290 \text{ kpsi} & \text{Youngs modulus} \\
\nu & = & 1/3 & & & \text{Poisson Ratio}
\end{array}
$$

$E = 1$GPa represents HDPE, $E = 2.5$GPa is typical for materials like ABS and Polycarbonate, $E = 5$GPa can be obtained with about 20% glass fiber in ABS.

(Samuelson and Eggwertz, 1992) has a form that leads to:

$$
M_{cr} = \alpha\eta\omega\frac{\pi E R h^2}{\sqrt{3(1-\nu^2)}} \tag{7}
$$

where η is the factor for "almost perfect" cylinders:

$$
\eta = \frac{0.83}{\sqrt{1+\frac{r}{100h}}} = 0.679 \tag{8}
$$

and α is intended to account for different manufacturing tolerances. For most fabrication methods it ranges from 0.6 to 1.0. Here we will use the value:

$$
\alpha = 0.9
$$

under the assumption that initial imperfections is not very important for asymmetric loading cases such as bending. ω is another reduction factor, because thin shells under buckling loads quickly reaches the plastic limit in localized regions. If the bending stress remains below a third of the yield stress, we can use:

$$\omega = 0.75$$

This results in:

$$\sigma_{cr} = 13.71\text{MPa}$$
$$M_{cr} = 19.43\text{kNm}$$

(Allmendinger, 1990) recommends a simpler critical stress:

$$\sigma_c r = C_b \frac{Eh}{R} \tag{9}$$

where $C_b(R/h)$ is an empirical factor, equal to about 0.35 for our case. Now:

$$\begin{aligned} \sigma_{cr} &= 16.67\text{MPa} \\ M_{cr} &= 23.6\text{kNm} \end{aligned}$$

(NASA, 1968) recommends that the theoretical result (4) be multiplied by the factor:

$$1 - 0.731(1 - e^{-\sqrt{\frac{R}{4h}}}) = 0.298 \tag{10}$$

which leads to:

$$\sigma_{cr} = 8.68\text{MPa}$$
$$M_{cr} = 12.1\text{kNm}$$

Although the results from these calculations vary a lot, it seems safe to state that the shell needs to be stronger than the reference values above if it is expected to resist the bending loads from a single lift point while fully flooded. Added strength can be achieved by a stiffer material or larger thickness. Alternatively, one might add lengthwise stiffeners, which in the simplest approximation is considered as an increase in effective thickness.

Figure 3 shows how critical stress and moment depends on material stiffness and shell thickness for our cylindrical hull. We can use this figure as a guide in selecting final shell dimensions.

2.3 Circular cylinder in torsion or shear

We will not consider the shear case for now, except note that maximum shear occurs a the water line, where the bending stress is zero. It therefore seems reasonable to treat the two cases separately, without accumulating the effects of the two loads. Also, the water line is reinforced by the frame structure, and should be able to absorb large shear forces.

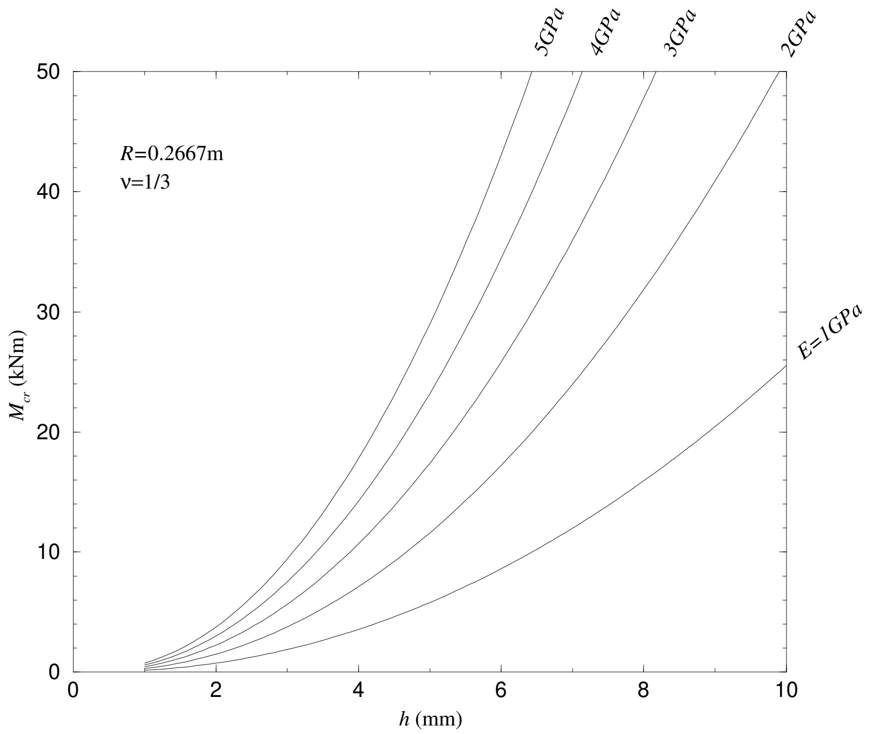

Figure 3: Buckling resistance in bending for a cylindrical shell of radius 0.2667m according to (Samuelson and Eggwertz, 1992), as a function of Young's modulus, E and shell thickness, h.

3 Conclusion

Buckling predictions for circular shells under bending load differ by up to a factor 3. If we expect bending loads to reach the magnitude seen with the vehicle suspended from a single lift point while fully flooded, it is recommended that stiffness be increased compared to that of a 1/4" thick shell made from ABS or similar materials. If the maximum bending moment is not expected to exceed the level experienced with dual lift points, the current dimensions should be adequate.

A Symbols

D	Diameter
E	Young's modulus
h	Shell thickness
l	Length
M	Bending moment
M_{cr}	Critical bending moment
m	Axial buckling wave number
m_w	Mass of vehicle filled with water
n	Circumferential buckling wave number
R	Shell radius
V	Shear force
x	Axial coordinate for circular shell
α	Shell buckling load reduction factor for manufacturing tolerances
η	Basic shell buckling load reduction factor
ν	Poisson's ratio
σ_x	Axial compressive stress
σ_{cr}	Critical axial compressive stress
θ	Circumferential coordinate for circular shell
ω	Plasticity shell buckling load reduction factor

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