EE128 Feedback Control

Lecture 18, 11/2/2006

- Compensation (6.7)
  - –Lead/Lag compensation

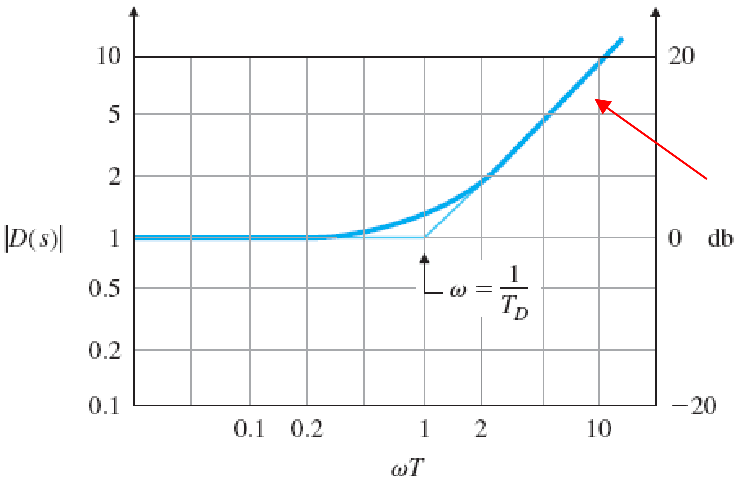
Compensation (6.7)

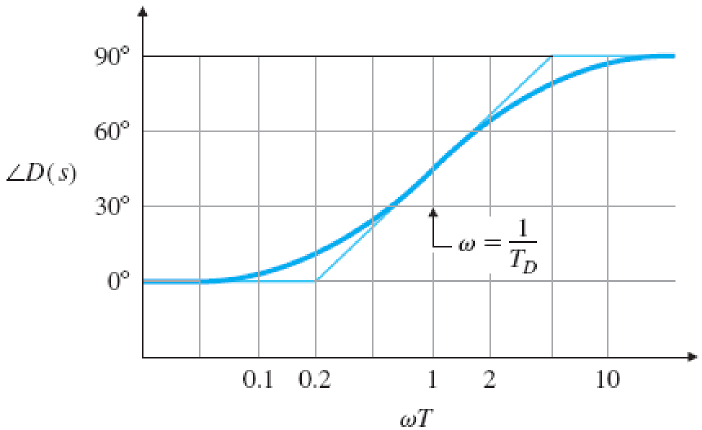
- Goal of compensation: To improve system dynamics in **transient** and **steady-state** responses
- **Lead (PD) compensation:** typically used when substantial improvement in damping is required
- **Lag (PI) compensation:** to improve steady-state response characteristics without changing transient response characteristics significantly
- **Lead+Lag compensation** (i.e. PID): when improvements in both phase margin and low-frequency gain are needed

Compensation (6.7)

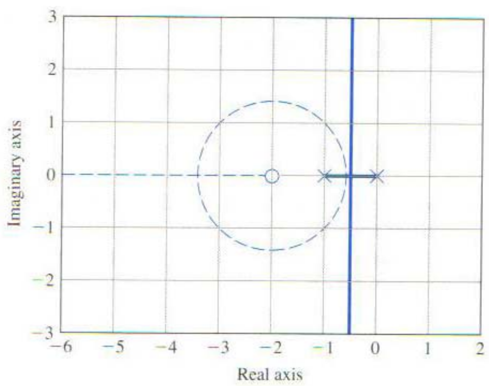
- PD compensation
  - Pure derivative compensation is troublesome
  - Magnitude of compensation grows with increasing frequency
    - High-freq noise amplification problem!
  - Can't be realized with physical elements

$$D(s) = (T_D s + 1)$$



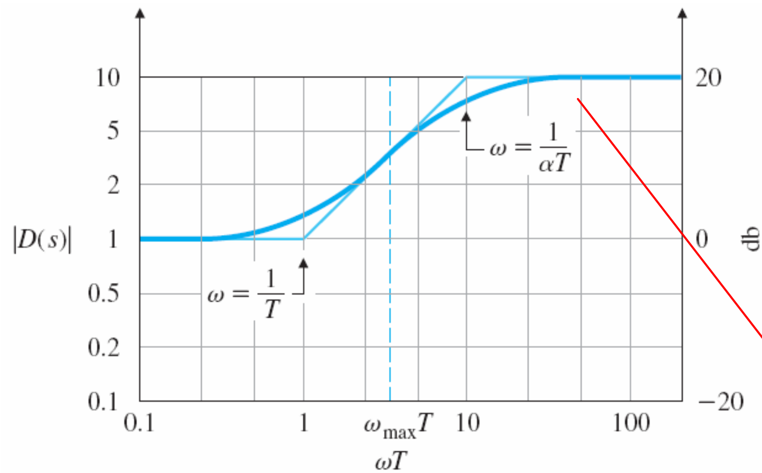


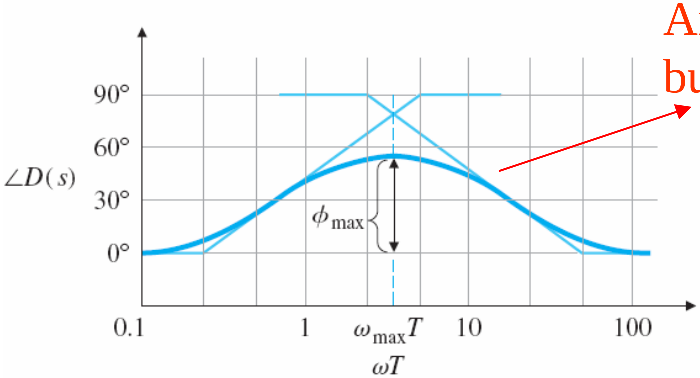
Stabilizing effect of adding a zero in a 2nd order system

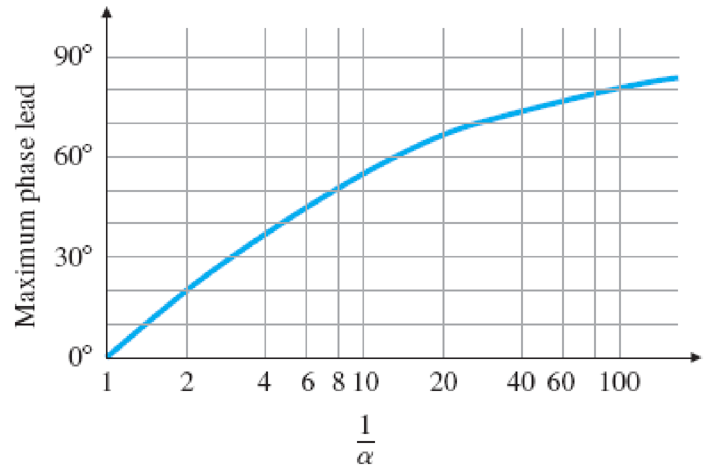


Compensation (6.7)

- Lead compensation







Amplification at higher freqs is limited but phase increase (lead) still occurs

$$D(s) = \frac{Ts+1}{\alpha Ts+1}, \qquad \alpha < 1$$

$$\omega_{max} = \frac{1}{T\sqrt{\alpha}}.$$

$$\sin\phi_{max} = \frac{1-\alpha}{1+\alpha},$$

$$\alpha = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}}.$$

Useful relations

$$D(s) = \frac{s+z}{s+p}.$$

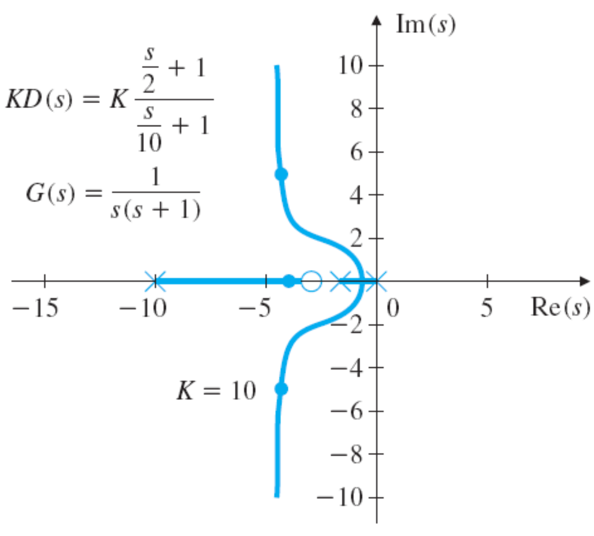
$$\omega_{max} = \sqrt{|z|\,|p|}$$

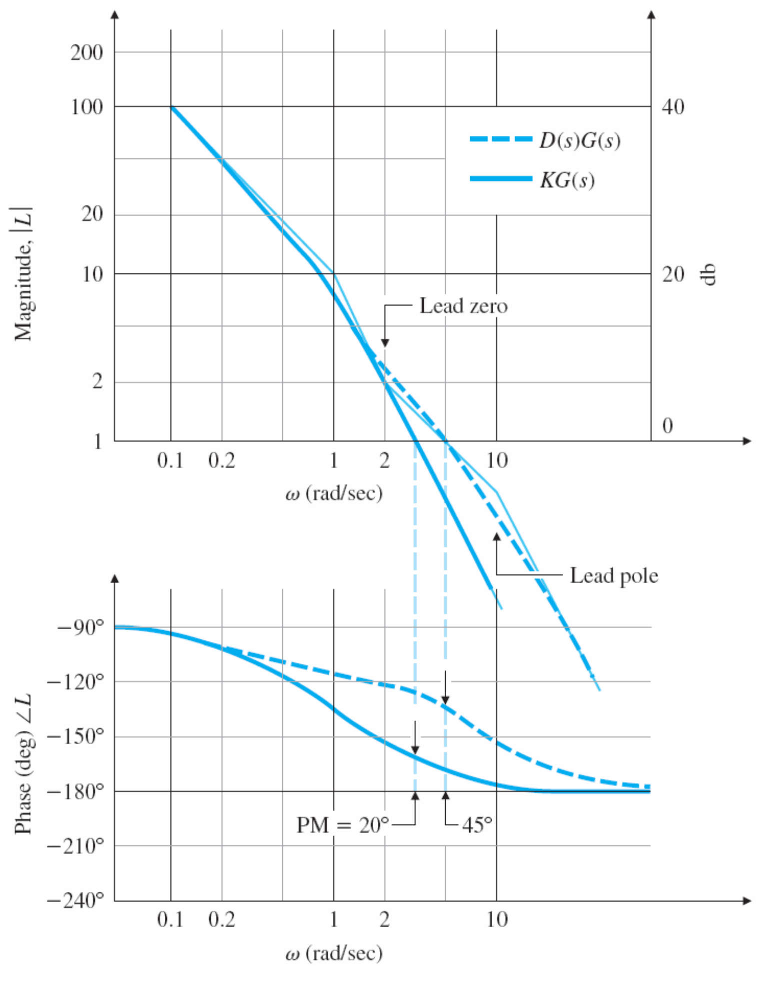
Compensation (6.7)

Example 6.14 (a). Lead compensation for a DC motor

- <u>Specs</u>: Steady-state error < 0.1 for unit-ramp input, overshoot Mp <25%

$$G(s) = \frac{1}{s(s+1)} \rightarrow KD(s) = 10\frac{s/2+1}{s/10+1}.$$





Lead compensation

Design parameters

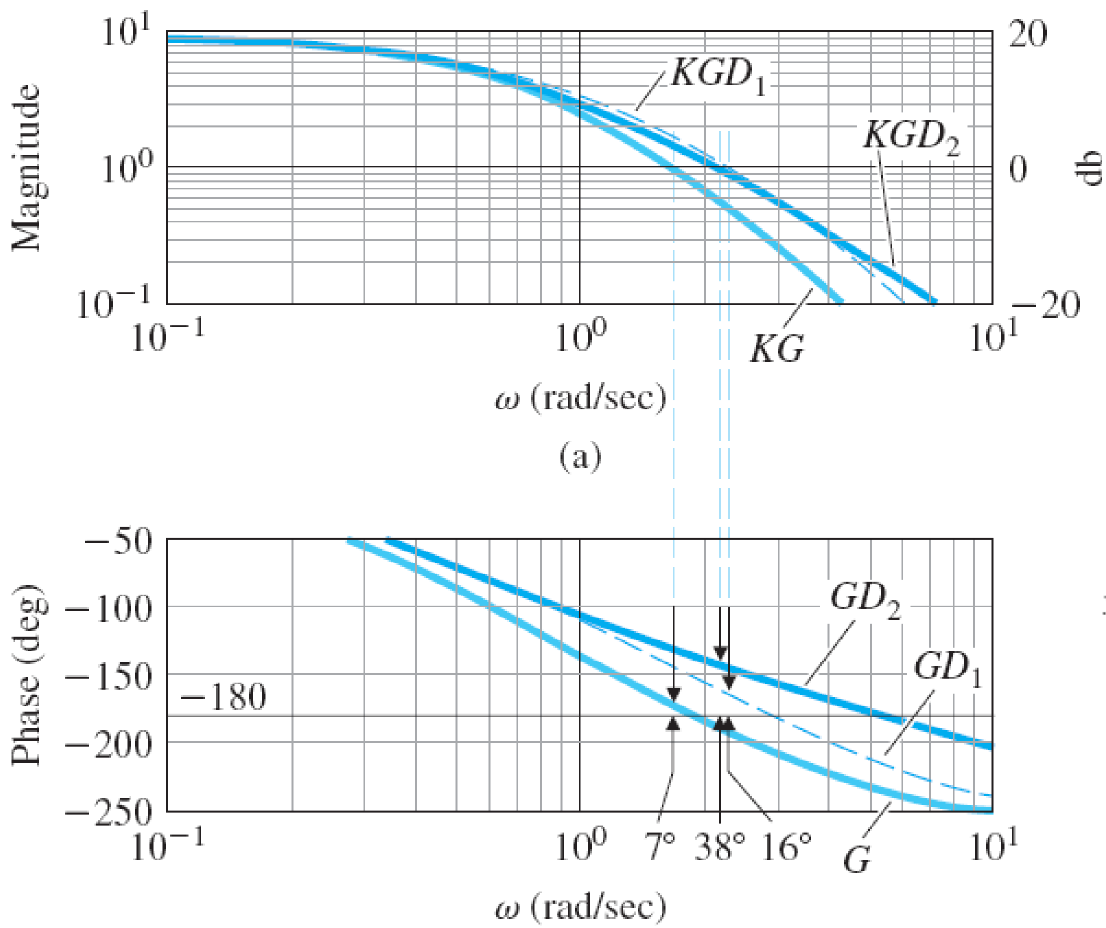
1. **Crossover frequency** (ω_c): determines bandwidth (ω_{BW}), rise time (t_r) and settling time (t_s)
2. **Phase margin** (PM): determines damping ratio (ξ) and overshoot (M_p)
3. **Low-frequency gain**: determines steady state error characteristics

Design procedure

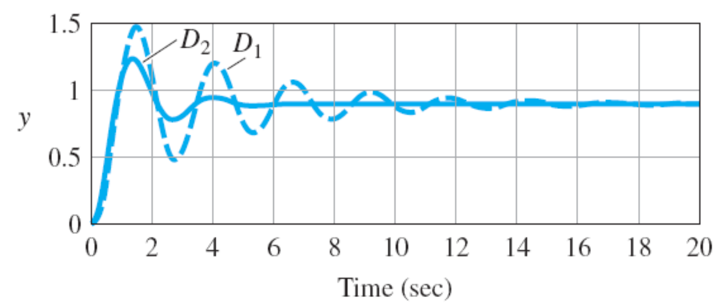
1. Determine open-loop gain K to satisfy error or bandwidth requirements:
   (a) to meet error requirement, pick K to satisfy error constants (K_p, K_v, or K_a) so that e_{ss} error specification is met, or alternatively,
   (b) to meet bandwidth requirement, pick K so that the open-loop crossover frequency is a factor of two below the desired closed-loop bandwidth.
2. Evaluate the phase margin (PM) of the uncompensated system using the value of K obtained from Step 1.
3. Allow for extra margin (about 10°), and determine the needed phase lead $\phi_{\max}$.
4. Determine α from Eq. (6.40) or Fig. 6.53.
5. Pick $\omega_{\max}$ to be at the crossover frequency; thus the zero is at $1/T = \omega_{\max}\sqrt{\alpha}$ and the pole is at $1/\alpha T = \omega_{\max}/\sqrt{\alpha}$.
6. Draw the compensated frequency response and check the PM.
7. Iterate on the design. Adjust compensator parameters (poles, zeros, and gain) until all specifications are met. Add an additional lead compensator (that is, a double lead compensation) if necessary.

Lead compensation

- *Example 6.15: Lead compensator for a temperature control system*
  - Design Lead compensator so that PM>=25° and Kp=9



$$KG(s) = \frac{K}{(s/0.5+1)(s+1)(s/2+1)}$$



Lead compensation

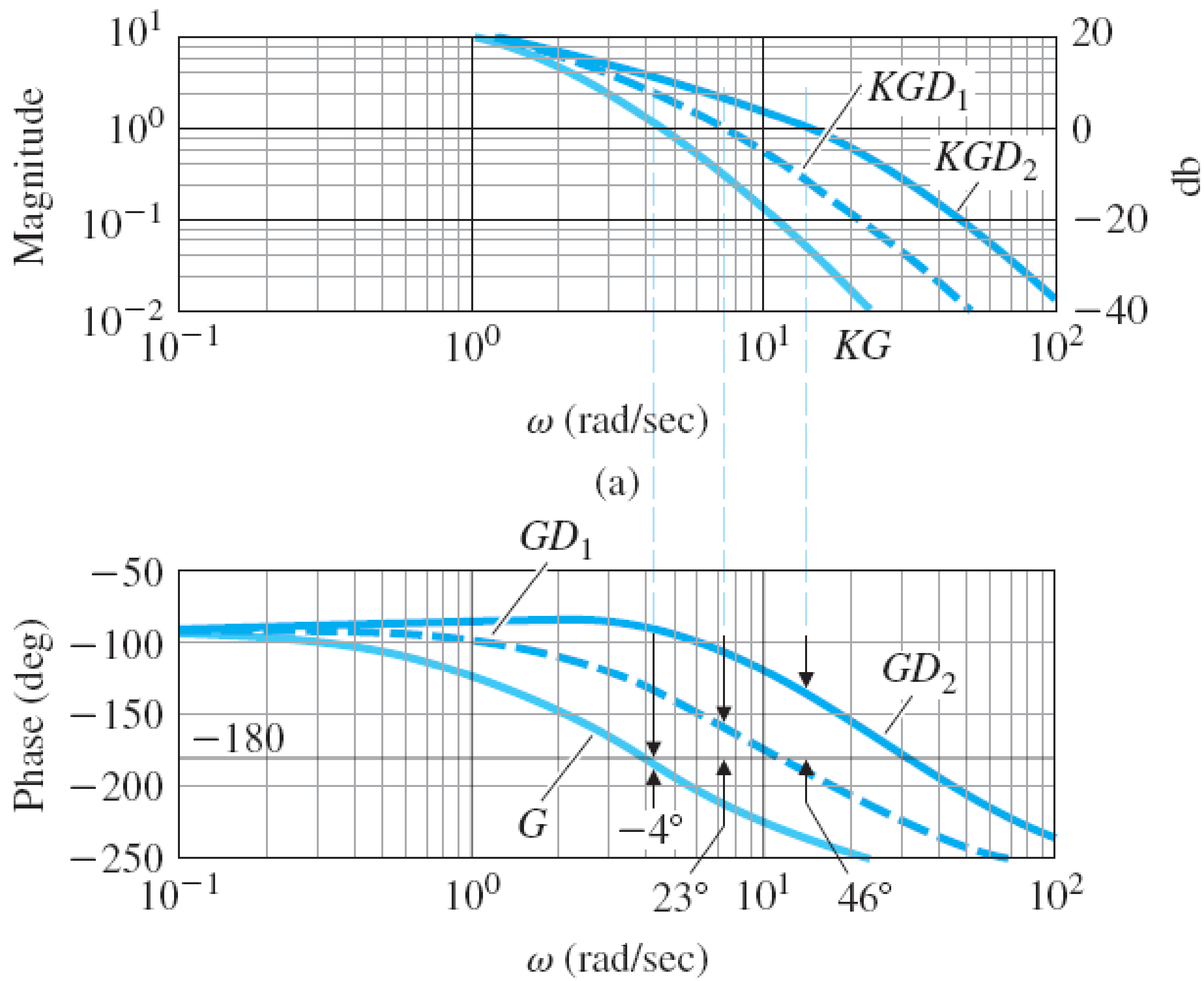
- *Example 6.16: Lead compensator type 1 servo system*
  - Design Lead compensator so that PM=45° and Kv=10

$$KG(s) = K\frac{10}{s(s/2.5+1)(s/6+1)}$$

$$D(s) = \frac{s/2+1}{s/20+1} = \frac{1}{0.1}\frac{s+2}{s+20}.$$

$$D_2(s) = \frac{1}{(0.1)^2}\frac{(s+2)(s+4)}{(s+20)(s+40)}$$

Double lead compensator



Lag compensation

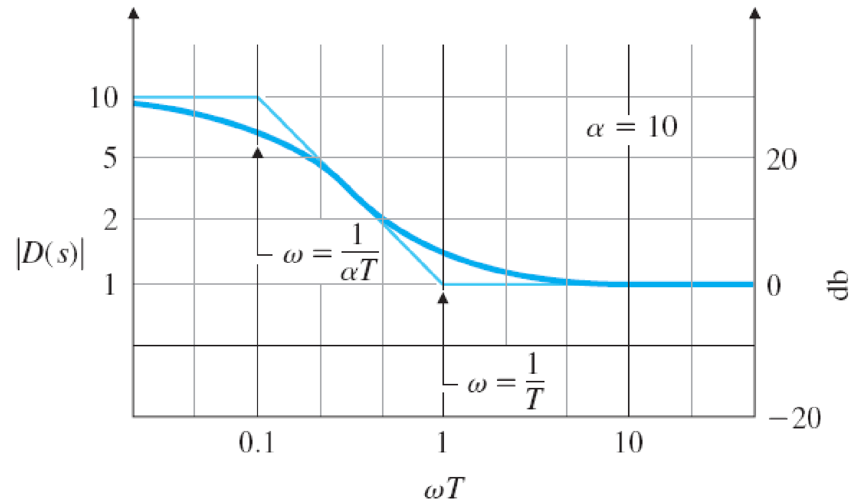
$$D(s) = \alpha \frac{Ts+1}{\alpha Ts + 1}, \qquad \alpha > 1$$

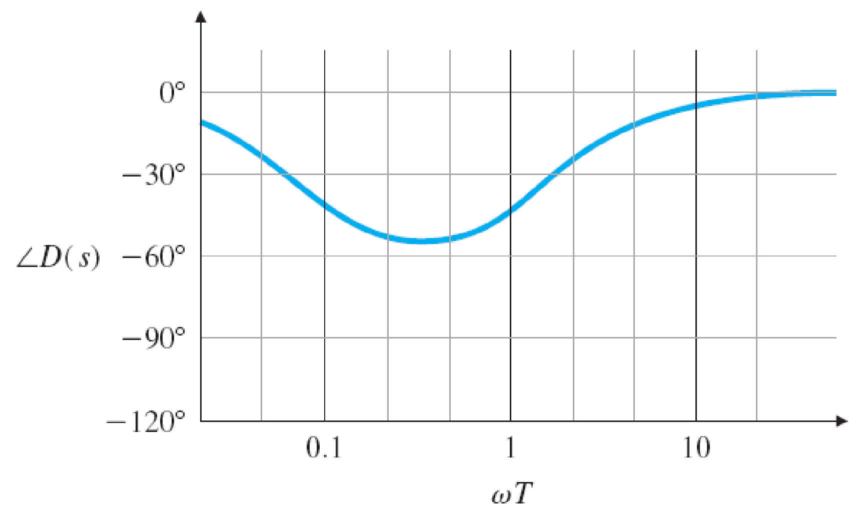
Objective: to improve steady-state response characteristics without changing transient response characteristics significantly (i.e. leaving enough phase margin)

How? Increasing low frequency gain, i.e. open-loop DC gain, by providing additional gain (α)

Problem: Phase decreases (undesired effect)

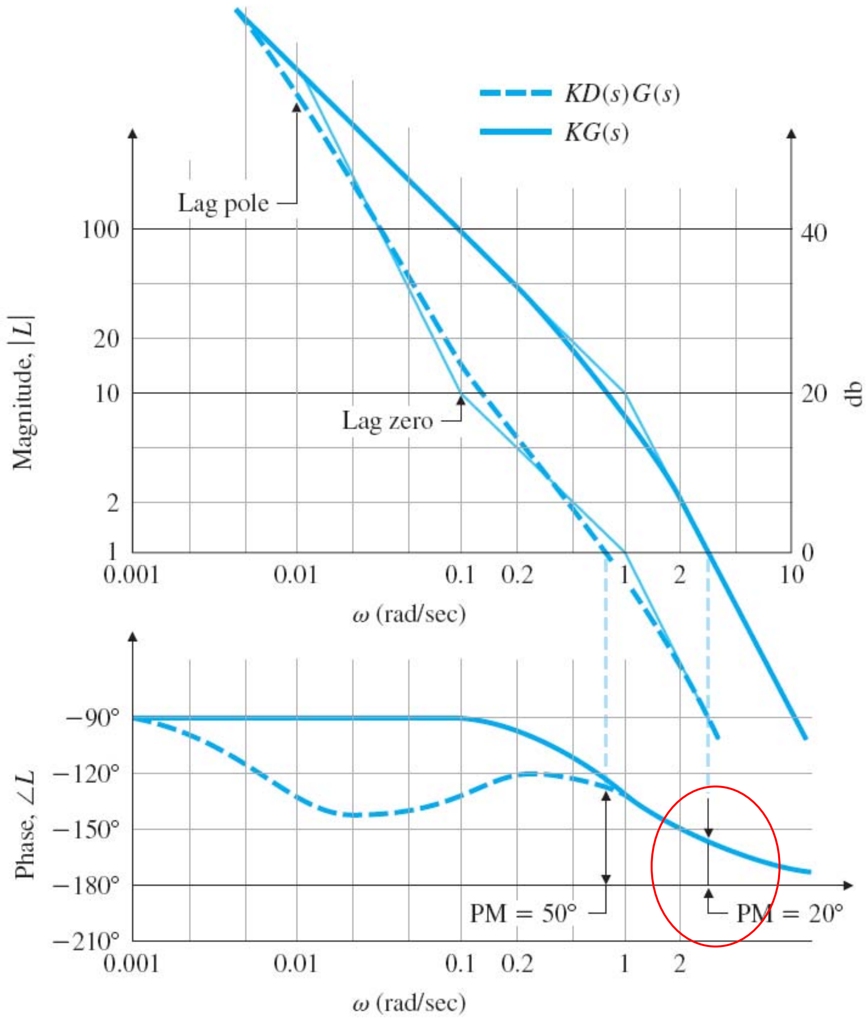
Solution: Pole and zero of D(s) at much lower frequencies than the uncompensated system crossover frequency (ω_c) in order to keep the decreasing effect of PM to a minimum





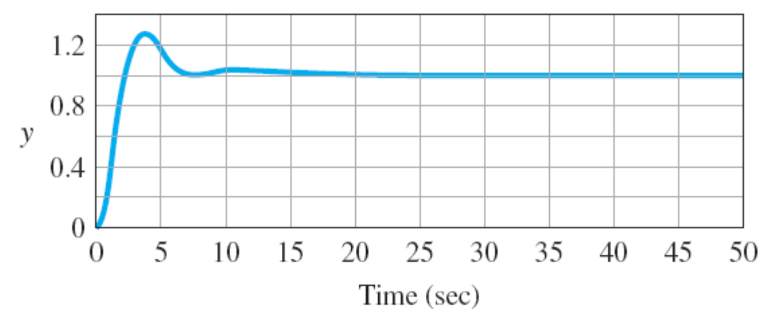
Lag compensation

- *Example 6.18: Lag compensator for a DC motor*
  - Design Lead compensator so that PM=45° and Kv=10



$$G(s) = \frac{1}{s(s+1)}$$

Reasonable performance but 25sec to settle down and Mp a bit large



Lag compensation

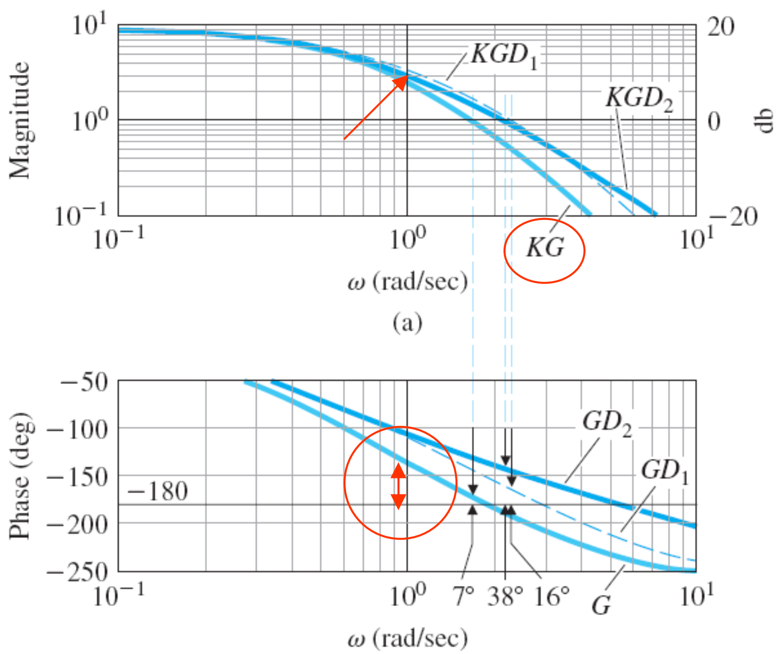
Design procedure

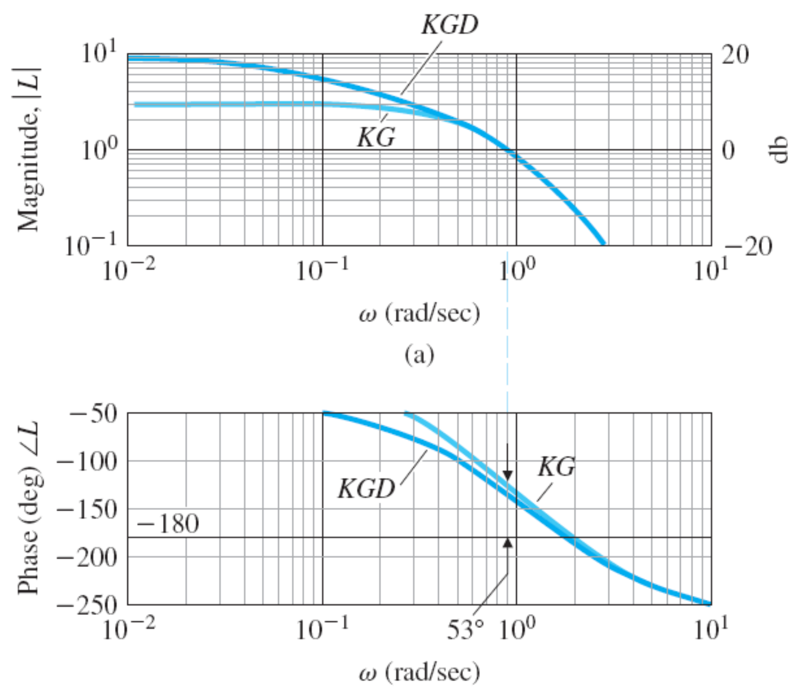
1. Determine the open-loop gain K that will meet the phase-margin requirement without compensation.
2. Draw the Bode plot of the uncompensated system with crossover frequency from Step 1, and evaluate the low-frequency gain.
3. Determine α to meet the low-frequency gain error requirement.
4. Choose the corner frequency $\omega = 1/T$ (the zero of the lag compensator) to be one octave to one decade below the new crossover frequency ω_c.
5. The other corner frequency (the pole location of the lag compensator) is then $\omega = 1/\alpha T$.
6. Iterate on the design. Adjust compensator parameters (poles, zeros, and gain) to meet all the specifications.

Lag compensation

- *Example 6.17: Lag compensator for temperature control system*
  - Design Lead compensator so that PM>=25° and Kp=9

$$KG(s) = \frac{K}{(s/0.5+1)(s+1)(s/2+1)}$$





Reasonable damping for PM=44°