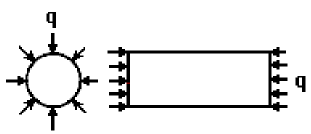
Case 20a Thin tube with closed ends under uniform external pressure, lateral and longitudinal; ends held circular

Thin tube with closed ends under uniform external pressure, lateral and longitudinal



8" NOM. (8.625" OD X 0.322" WALL X 7.981" ID) ALUMINUM 6061-T6 EXTRUDED PIPE SCHEDULE 40

Enter dimensions and properties:

radius: $r := \frac{8.625}{2} \cdot in - \frac{.322}{2} \cdot in$ $r = 4.151 \cdot in$

length: $L := 34 \cdot in$

Modulus of elasticity: $E := 10 \cdot 10^6 \cdot \frac{lbf}{in^2}$

wall thickness: $t := 0.250 \cdot in$

Poisson's ratio: $\nu := 0.33$

$designPressure := 750 \cdot \frac{lbf}{in^2}$

Conditions: For the expressions in the case to be valid, the following condition must be true (i.e., condition = 1):

$\frac{r}{t} = 16.606$ $condition := \frac{r}{t} > 10$ $condition = 1$

Critical pressure: The general expression for q' as a function of n (the number of lobes formed by the tube in buckling) is as follows:

$$q'(n) := \frac{E\cdot\frac{t}{r}}{1+\frac{1}{2}\cdot\left(\frac{\pi\cdot r}{n\cdot L}\right)^2}\cdot\left[\frac{1}{n^2\cdot\left[1+\left(\frac{n\cdot L}{\pi\cdot r}\right)^2\right]^2}+\frac{n^2\cdot t^2}{12\cdot r^2\cdot\left(1-\nu^2\right)}\cdot\left[1+\left(\frac{\pi\cdot r}{n\cdot L}\right)^2\right]^2\right]$$

To determine q', it is necessary to determine which value of n greater than 2 provides the smallest value of q' for the given t/r. To do this, the following is defined:

$nn := 10 \qquad i := 0 .. nn-1 \qquad n_i := i+2$

Find the value of n that corresponds to the smallest computed value of q':

$qq'_i := q'(n_i)$

$index_i := if\left(qq'_i = min(qq'), i, 0\right) \qquad m := max(index) \qquad m = 0$

$$qq'_i = \begin{pmatrix} 1.048\times10^3 \\ 1.9\times10^3 \\ 3.316\times10^3 \\ 5.151\times10^3 \\ 7.397\times10^3 \\ 1.005\times10^4 \\ 1.312\times10^4 \\ 1.659\times10^4 \\ 2.047\times10^4 \\ 2.476\times10^4 \end{pmatrix}\cdot\frac{lbf}{in^2}$$

Compute q' using the correct value of n:

$m := 2$

$q'(m) = 1.048\times10^3\cdot psi$

$SWLoadFactor := \frac{q'(m)}{designPressure} = 1.398$

Values of experimentally determined critical pressure range 20% above and below the theoretical value given by the expression above. A recommended probable minimum critical pressure is

$q' := q'(m)\cdot 0.8 \qquad q' = 838.666\cdot psi$

References:

Ref. 19. Saunders, H. E., and D. F. Windenberg: Strength of Thin Cylindrical Shells Under External Pressure, *Trans. ASME,* vol. 53, no. 15, p. 207, 1931.

Ref. 20. von Mises, R.: Der kritische Aussendruck zylindrischer Rohre, *Z. Ver Dtsch. Ing.,*vol. 58, p. 750, 1914.

Ref. 81. Weingarten, V. I., and P. Seide: Elastic Stability of Thin-walled Cylindrical and Conical Shells Under Combined External Pressure and Axial Compression, *AIAA J.,*vol. 3, no. 5, May 1965.

Ref. 109. Johnson, B. G. (ed.): "Guide to Stability Design Criteria for Metal Structures," 3rd ed., Structural Stability Research Council, John Wiley & Sons, Inc., 1976.

End of Section

Von Mises Formula (Ross; 2nd edition page 101)

$a := r \qquad l := L$

$$PVonMisescr(n) := \frac{E\cdot\frac{t}{a}}{n^2 - 1 + 0.5\cdot\left(\frac{\pi\cdot a}{l}\right)^2}\cdot\left[\frac{1}{\left[n^2\cdot\left(\frac{l}{\pi\cdot a}\right)^2 + 1\right]^2} + \frac{t^2}{12\cdot a^2\cdot\left(1-\nu^2\right)}\cdot\left[n^2 - 1 + \left(\frac{\pi\cdot a}{l}\right)^2\right]^2\right]$$

$PPVonMisescr_i := PVonMisescr\left(n_i\right)$

$PPVonMisescr_0 = 904.752\,\frac{lbf}{in^2}$

$$PPVonMisescr_i = \begin{pmatrix} 904.752 \\ 1.698\times10^3 \\ 3.112\times10^3 \\ 4.947\times10^3 \\ 7.193\times10^3 \\ 9.848\times10^3 \\ 1.291\times10^4 \\ 1.638\times10^4 \\ 2.026\times10^4 \\ 2.455\times10^4 \end{pmatrix}\cdot\frac{lbf}{in^2}$$

$SWLoadFactor := \frac{PPVonMisescr_0}{designPressure} = 1.206$

$0.8\,PVonMisescr(m) = 723.802\cdot\frac{lbf}{in^2}$

Windenburg and Trilling Formula aka David Taylor Model Basin formula (Ross; 2nd edition page 102)

$$PWandTcr := \frac{2.42 \cdot E \cdot \left(\frac{t}{2 \cdot a}\right)^{\frac{5}{2}}}{\left(1-\nu^2\right)^{0.75} \cdot \left[\left(\frac{l}{2 \cdot a}\right) - 0.447 \cdot \left(\frac{t}{2 \cdot a}\right)^{\frac{1}{2}}\right]}$$

$$PWandTcr = 1.033 \times 10^{3} \cdot \frac{lbf}{in^2}$$

$$.0549 \cdot MPa = 7.963 \cdot \frac{lbf}{in^2}$$

$$0.8 \cdot PWandTcr = 826.579 \cdot \frac{lbf}{in^2}$$

Windenburg and Trilling Thinness Ratio (Ross; 2nd edition page 105)

$$d := 2 \cdot a \qquad \sigma yp := 40000 \cdot \frac{lbf}{in^2}$$

$$\lambda := \left[\frac{\left(\frac{l}{d}\right)^2}{\left(\frac{t}{d}\right)^3}\right]^{0.25} \cdot \left(\frac{\sigma yp}{E}\right)^{.5} \qquad \lambda = 1.771 \qquad \frac{1}{\lambda} = 0.565$$