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IMU Calibration on a Low Grade Turntable: Embedded Estimation of the Instrument Displacement from the Axis of Rotation

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Abstract—The paper concerns one simple method of calibration of an assembled inertial measurement units (IMU) of different grades on a low grade single axis turntable. The main feature of the method is that it has the weakest possible requirements to the testbench. The method was presented at ICINS 2010, ICINS 2013, and is now being used in industry for 3–4 years. It appears to work well in practice. One of special points of this method is the situation, when the IMU is displaced from the axis of rotation. In some cases this fact can be neglected, but in some cases not. It was shown earlier that parameters of this displacement need not to be measured prior to the experiment, but can be estimated automatically during the data processing, similar to the rest of IMU parameters. So the device can be placed arbitrarily onto the stand, and no care of displacement should be taken while conducting the experiment. This work concentrates primarily on aspects of observability and estimation accuracy of IMU displacement (which influence the accuracy of the calibration itself), and shows more variety of experimental results than in previous publications.

I. INTRODUCTION

Conventional calibration techniques naturally depend on the accuracy of the test bench. They require knowing either some precise positions, or angular rate, or both. But in fact, in a quite a lot of kinds of situations the information given by inertial sensors is sufficient to calibrate themselves without any additional information from other instruments. Under the assumption that sensor errors are small enough, it appears to be possible to reduce errors to the second order of magnitude using the output from IMU only. In most cases the remaining sensor errors become negligible, or at least tolerable. Considering this, we can create a completely testbench-independent algorithm of calibration. This entirely new approach to calibration was proposed by N.A. Parusnikov from MSU [1]. More detailed description of the method, some aspects of its implementation for the case when several IMUs are being calibrated simultaneously, and calibration results, are given below.

II. IDEA AND THE BACKGROUND OF THE METHOD

A. The turntable

In this work, saying the turntable can be of low grade we mean the following:

- it has one single axis of rotation (nearly horizontal);
- no rate measuring (not used if present);
- no rate stabilization required;
- no predefined angular positions or angle measuring;
- no precise instrument alignment;
- no programmable actuator control present (but more convenient in practice);
- the axis of rotation is firmly fixed with respect to the ground, and no vibrating within the desired accuracy of the IMU

Please, see Fig. 1 for the schematic diagram of the calibration experiment. It consists of three cycles of rotation round a nearly horizontal axis, for about 10–20 minutes each. Note that there are no special requirements to these rotations except for a non-zero angular rate. Every cycle is preceded by 1–3 minute static initial alignment.

B. Sensor error model

For inertial sensor errors we use linear model in small terms, assuming that some pre-calibration step was performed. This pre-calibration step can be based on the same experimental data (no additional operations required in a test bench), and it may be performed in different ways, but this is not the subject of this work. Thus, errors are assumed to be small enough that second order terms can be neglected. The model includes accelerometer and gyro small constant biases, errors of scaling factors, and small angles of sensitive axes misalignment. In addition, the model can incorporate dynamic drift (g-sensitivity) coefficients and other parameters.

The true components of unit force vector f_z are being measured by accelerometers, where the subscript z stands for instrumental reference frame. Similarly, the true components of absolute angular rate vector ω_z are being measured by gyros. We use f'_z and ω'_z for corresponding sensor outputs, so that for 6 DoF IMU

$$f'_z = \begin{bmatrix} f'_{z1} \\ f'_{z2} \\ f'_{z3} \end{bmatrix}, \quad \omega'_z = \begin{bmatrix} \omega'_{z1} \\ \omega'_{z2} \\ \omega'_{z3} \end{bmatrix}.$$

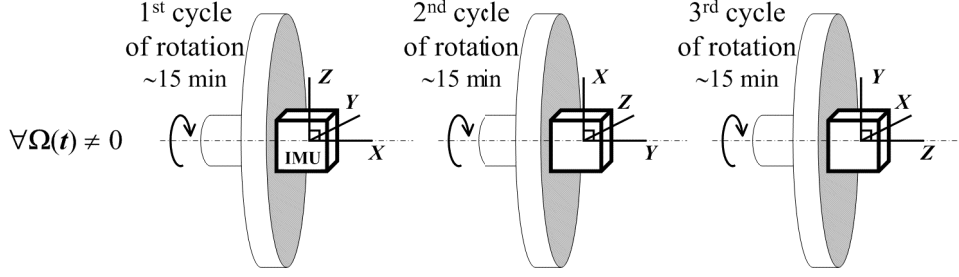


Fig. 1. Schematic diagram of the calibration experiment

The conventional error model described above is now written as follows.

$$\begin{aligned} f'_z - f_z &\equiv \Delta f_z = \Delta f_z^0 + \Gamma f_z + \delta f_z^s, \\ \omega'_z - \omega_z &\equiv -\nu_z = -\nu_z^0 - \Theta \omega_z - D f_z / g - \delta \nu_z^s, \end{aligned}$$

where minus signs in the second equation is just a convention, and error components are grouped in matrices as outlined below. For constant biases

$$\Delta f_z^0 = \begin{bmatrix} \Delta f_{z1}^0 \\ \Delta f_{z2}^0 \\ \Delta f_{z3}^0 \end{bmatrix}, \quad \nu_z^0 = \begin{bmatrix} \nu_{z1}^0 \\ \nu_{z2}^0 \\ \nu_{z3}^0 \end{bmatrix}.$$

For errors of scaling factors (Γ_{ii}, Θ_{ii}) and sensitive axes misalignments in radians ($\Gamma_{ij}, \Theta_{ij}, i \neq j$)

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 0 & 0 \\ \Gamma_{21} & \Gamma_{22} & 0 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{bmatrix}, \quad \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}.$$

Here upper triangular part of Γ is forced to be zero by definition of the instrumental reference frame. This reference has the first axis exactly matched with the sensitive axis of the first accelerometer, and the second axis strictly in the plane of the first and the second accelerometers sensitive axes. Even though this frame is a kind of "virtual" from some point of view, it is this reference that often used in inertial navigation. The matrix D stands for g-sensitivity coefficients (if present), and $\delta f_z^s, \delta \nu_z^s$ — for stochastic errors.

We do not discuss here the temperature variations of sensor output since nothing new to that is going to be introduced in this work. So any of the common approaches can be used to compensate for it. These approaches are usually include experiments at several fixed temperatures and then linear interpolation or polynomial approximation of the results. The possibility of conducting calibration experiments with non-constant temperature is the subject of another ongoing study.

After that, the goal of calibration is to obtain estimates for $\Delta f_z^0, \nu_z^0, \Gamma, \Theta$ and D (with total number of components of 21 or 30). These estimates will allow to compensate sensor output for all error components except stochastic ones.

C. The main concept

The idea of the method is to construct a linear state-space system, containing all desired components in its state vector. This system then can be supplemented with measurements, making it fully observable in experiments described above. These measurements require no additional information, except for precisely known local gravity force vector and Earth's angular rate vector, both in geodetic frame.

In order to derive the linear state-space system we need to introduce some inertial navigation first. We begin with the solution of Poisson attitude equation [3] for the transition matrix L between instrumental ("z") and geodetic ("x") reference frames [4]:

$$\begin{aligned} \dot{L} &= \hat{\omega}_z L - L \hat{u}_x, \quad L(0) = L_0, \\ \hat{\omega}_z &= \begin{bmatrix} 0 & \omega_{z3} & -\omega_{z2} \\ -\omega_{z3} & 0 & \omega_{z1} \\ \omega_{z2} & -\omega_{z1} & 0 \end{bmatrix}, \\ \hat{u}_x &= \begin{bmatrix} 0 & u_{x3} & -u_{x2} \\ -u_{x3} & 0 & u_{x1} \\ u_{x2} & -u_{x1} & 0 \end{bmatrix}. \end{aligned}$$

Here u_x stands for the Earth's angular rate as projected onto the local geodetic frame. Since ω_z is not known, one uses sensor output ω'_z instead of ω_z for computations. As a result, not L , but some different matrix L' is obtained. This matrix differs from the ideal value by small rotation term

$$L' \approx (E + \hat{\beta}_z) L, \quad \hat{\beta}_z = \begin{bmatrix} 0 & \beta_{z3} & -\beta_{z2} \\ -\beta_{z3} & 0 & \beta_{z1} \\ \beta_{z2} & -\beta_{z1} & 0 \end{bmatrix}.$$

The rotation term β_z represents three small angles of rotation round each instrumental axis. For β_z we then have the equation (ignoring higher order terms)

$$\dot{\beta}_z \approx \hat{\omega}'_z \beta_z + \nu_z.$$

This equation relates attitude and gyro errors to each other.

Let's now move to accelerometers. The only force acting on them is the reaction force of the turntable. Every sensor should ideally measure the projection of this force onto the sensitive axis. Assume then, that proof masses of accelerometers are

exactly on the axis of rotation. In this case measured force is certainly the opposite to the gravity force. Real sensors have errors introduced above, so we can write

$$Z \equiv (L')^T f'_z - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \approx \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (L')^T \beta_z + (L')^T \Delta f_z,$$

Here we have some values on the left, which can be computed at any time, and which are linearly related to β_z and Δf_z . The information on the left will now be treated as measurements in our model.

Assuming the state vector X of the system

$$X = [\beta_z^T \nu_z^{0T} (\Delta f_z^0)^T \Gamma_{11} \dots \Gamma_{33} \Theta_{11} \dots \Theta_{33} D_{11} \dots D_{33}]^T,$$

and adding the system dynamic for parameters being calibrated

$$\dot{\beta}_z \approx \hat{\omega}_z' \beta_z + \nu_z, \quad \dot{\nu}_z^0 = \dot{\Delta f}_z^0 = \dot{\Gamma} = \dot{\Theta} = \dot{D} = 0,$$

we obtain linear state-space system with measurements. So that

$$\dot{X} = AX + q, \quad Z = HX + r,$$

where the matrices A and H are known at any time (up to the first order terms), and q and r are stochastic disturbances with some specific characteristics. It appears then, that in the experiment outlined above (see Fig. 1) this system becomes observable. Thus, we can use any optimal estimator (e.g. Kalman filter) to obtain estimates for all desired parameters.

This approach, with slight variations and estimator tuning for different types of IMU, proved its efficiency in several implementations in industry.

III. INSTRUMENT DISPLACEMENT FROM THE AXIS OF ROTATION

A. The effect of the displacement on IMU

Models above were deduced under the assumption that accelerometer proof masses are located on the axis of rotation. In general, it is not exactly the case. Due to inertia there appear an additional reaction which forces the IMU to stay on the turntable and not to move rectilinearly. It is usually divided in two terms: tangential and centripetal force. Denoting the displacement by a vector s in instrumental reference frame we get

$$\hat{\omega}^2 s - \hat{\omega} s.$$

From the equation above we can see that in many cases this contribution can be ignored. For example, if $\|s\|$ is less than 5 cm, $\|\omega\|$ does not exceed 3 degrees (1/20 radians) per second, and $\|\dot{\omega}\|$ is almost negligible, the magnitude of the above error is less than 0.2 mm/s².

But there are also lots of situations when significant displacement of accelerometer proof masses from the axis of rotation is unavoidable. The examples are calibrating several IMUs at the same time on one turntable (see Fig. 2), IMUs with large spatial separation between accelerometers, particular types of turntables, and high spin experiments.

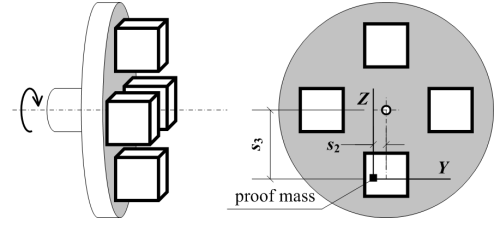


Fig. 2. An example of IMUs displaced from the axis of rotation

B. Taking the displacement into consideration

In some cases parameters of the displacement of accelerometer proof masses are known (e.g. they can be measured directly along axes of the instrumental frame). Then accelerometer outputs are corrected for the above tangential and centripetal terms, and the model works as before.

But more interesting case is when components of s are not known [2]. First of all, we accept that the displacement s_1 along the axis of rotation does not make any significant effect on IMU. So we treat s_1 as 0 and rename $s = [0 s_2 s_3]^T$. After that, we add s_2 and s_3 to the state vector, so that

$$X = [\beta_z^T \nu_z^{0T} (\Delta f_z^0)^T \Gamma_{11} \dots \Gamma_{33} \Theta_{11} \dots \Theta_{33} D_{11} \dots D_{33} s_2 s_3]^T.$$

Then we modify the error model as follows

$$\Delta f_z = \Delta f_z^0 + \Gamma f_z + \hat{\omega}^2 s - \hat{\omega} s + \delta f_z^s.$$

What is interesting here is that substituting the new model in our system gives us the new one, which is also very easy to make observable. It means that we can arrange the rotation of the turntable in a way that components s_2 and s_3 could be estimated well. This was shown by processing data from real calibration experiments. Regarding the angular acceleration $\dot{\omega}$, it can be computed by numerical differentiation for high-grade IMUs. For low-grade IMUs angular accelerations should be avoided in experiment, and then ignored in equations.

C. Data processing and observability issues

Although the state-space model under consideration is linear, it is impossible to examine its observability analytically. That is because the system has an arbitrary function in its structure, namely $\omega_z(t)$. So the only way is to claim some general ideas based on system structure, and then to try making experiments with different rotation rate profiles, and to study the progress of parameter estimation in these experiments. First, it is fairly obvious that it is bad for observability to have the constant $\hat{\omega}^2 s$ term in equations, since it would mix up with Δf_z^0 terms, which is also constant. Thus, the common thing is to have different angular rate magnitudes in an experiment. Second, since the rotation rate is a coefficient of displacement, the accuracy of estimation severely depends on the magnitude of this rate. Higher rates give higher accuracy of estimation.

Three inertial units are in consideration here.

- Ring laser gyros (typ. performance of 0.01–0.03 °/√hr) and quartz flexure suspension accelerometers (0.1 mg)

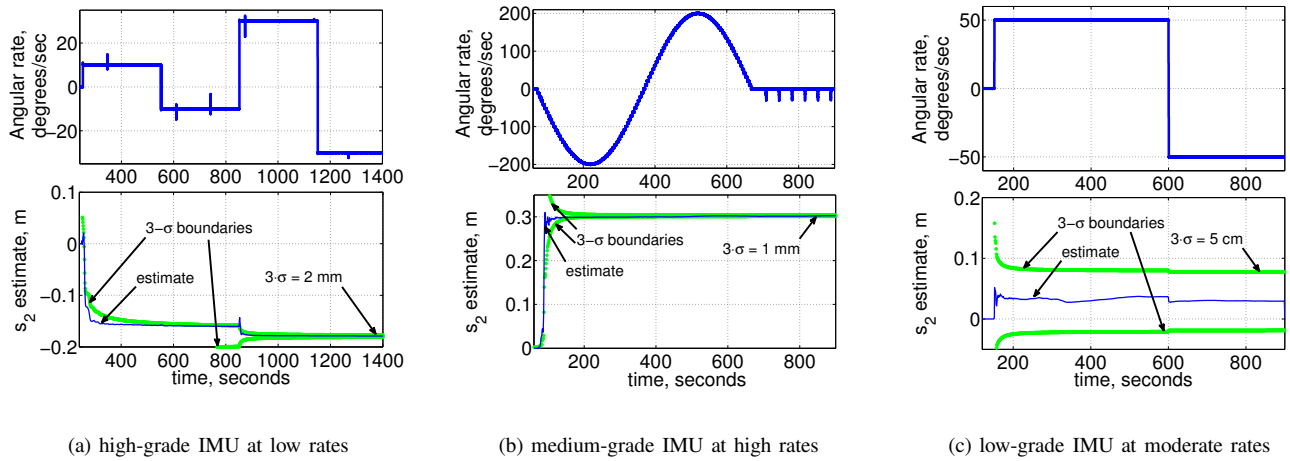


Fig. 3. Estimation progress for different types of IMU at different rotation profiles

assembled in navigation/tactical grade system, referenced below as high-grade IMU.

- Fiber optic gyros ($0.5\text{--}1^\circ/\sqrt{\text{hr}}$) and MEMS accelerometers ($1\text{--}2\text{ mg}$) assembled in tactical grade system, referenced below as medium-grade IMU.
- Hemispherical resonant gyros ($3\text{--}5^\circ/\sqrt{\text{hr}}$) and MEMS accelerometers ($1\text{--}2\text{ mg}$) assembled in tactical/industrial grade AHRS system, referenced below as low-grade IMU.

Sets of data were collected during calibration experiments similar to the described above. Each experiment had different rotation profile and different rate magnitude. All datasets were processed by a software, which incorporates Kalman filtering implemented in C/C++ program on PC. This software was designed in Moscow State University Navigation and Control Lab for calibration data processing. On Fig. 3 there are examples of rotation profiles accompanied by estimation progress with $3\text{-}\sigma$ estimation error covariances given by estimator.

Results are incorporated in Table I. Actually, $3\text{-}\sigma$ values are given here just for common understanding of what orders of accuracy are in question here. In fact, real estimation accuracy corresponds to the accuracy of the whole model, including all other parameters. The point is that residual accelerometer errors caused by errors of estimation are always almost within the calibration accuracy as if there was no any displacement.

IV. CONCLUSION

Summarizing the experience of dealing with a considerable number of different IMUs, we can state the following.

- The above method of IMU calibration on a low grade turntable works well even when the IMU is significantly displaced from the axis of rotation. This case requires to extend the mathematical model of the system.
- Geometrical parameters of the displacement are easily estimated then, up to centimeter or millimeter level depending on IMU type and rotation profile. So there

TABLE I
ESTIMATION RESULTS SUMMARY

IMU grade	High	Medium	Low
Gyro type	RLG	FOG	HRG
Accelerometers	quartz-flexure	MEMS	MEMS
Rate magnitude	30 dps	200 dps	50 dps
Rate change	piecewise 2 magnitudes	sine 10 min period	piecewise 1 magnitude
Displacement	20 cm	30 cm	5 cm
Maximum centripetal term	5 cm/s^2	330 cm/s^2	3.5 cm/s^2
Estimate $3\text{-}\sigma$	2 mm	1 mm	50 mm

is no actual need to measure the displacement prior to the experiment.

- It is recommended for this situation, that the rotation rate should have tens of degrees per second magnitude and this magnitude should be changing. It should have at least 2 different nonzero values (e.g. 10 and 30 dps).
- Angular accelerations should be avoided, since there is no instruments in IMU to measure it correctly.

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