

Determining the Modulus of Elasticity in Compression via the Shore A Hardness

Component Design. The modulus of elasticity in compression of elastomers with values of the Shore A hardness between 30 and 95 can be determined with a high degree of accuracy by converting these Shore A values. The formula derived for this purpose establishes a connection between the theoretical principles involved and the technical aspects of the Shore hardness test.

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The deformation behaviour of elastomers is often characterised by specifying the Shore A hardness [1], and less often via the modulus of elasticity. The reason for this is probably because measuring the modulus of elasticity of elastomers by means of the usual methods is not very simple. This fact must be viewed in light of the practical interest to know the modulus of elasticity at least in terms of a useful order of magnitude. After all, the modulus of elasticity is needed above all when calculating a component's design, whether by means of analytical equations or the finite-element method (FEM). Based on this need, the question arises as to whether and how the moduli of elasticity of elastomers can be determined from values of the Shore A hardness.

Starting point for the considerations is the realisation that for elastomers – in contrast to other polymeric and metallic materials – the stiffness is linked to the hardness and, as a consequence thereof, the proposal by Sponagel et al. to determine the modulus of elasticity as a measure of the hardness by means of a modified Shore A test procedure [2]. This primary modification is to employ a cylindrical-tipped indenter in place of the standard truncated cone-tipped indenter when measuring the Shore A value. This requires a measuring instru-

ment that permits exchanging the indenter, which as a rule is not simple. A further drawback is that the Shore A values contained in databases cannot be converted into values of the modulus of elasticity. This was the primary reason for investigating the possibility of determining the modulus of elasticity via the usual Shore A hardness without modifying the indenter.

Shore A Hardness Test: Test Principle

When measuring the Shore A hardness, an indenter in the shape of a truncated cone (Fig. 1) is pressed into the test specimen under the force of a spring with a defined characteristic. The specified minimum dimensions of the test specimen are 35 mm in diameter and, for soft materials, 6 mm thickness. In the event of inadequate thickness, several test specimens may be stacked on top of one another. The depth of indentation measured on a dimensionless scale with 0 corresponding to a depth of 2.5 mm as a minimum and 100 to a depth of 0 mm as a maximum serves as a measure of the hardness [1].

A linear relationship exists between the depth of penetration and the Shore hardness as well as between the depth of penetration and the spring force, as the equations

$$F = C_1 + C_2 \cdot Sh_A \quad [N] \quad (1)$$

for the spring force and

$$w = C_3 \cdot (100 - Sh_A) \quad [mm] \quad (2)$$

for the depth of penetration express. The constants in these have dimensions and values of $C_1 = 0.549 \text{ N}$, $C_2 = 0.07516 \text{ N}$ und $C_3 = 0.025 \text{ mm}$.

Theoretical Background: Theory of Boussinesq

The fact that, when measuring the Shore A hardness, an indenter with a high stiffness is pressed into a test specimen with a low stiffness, thereby deforming it elastically, suggests using the theory of Boussinesq [3] for the theoretical description of the relationships between the load and the deformation. This starts with the action of a single force on the linearly elastic half space and ultimately leads via an analytical path to the associated stress and dis-

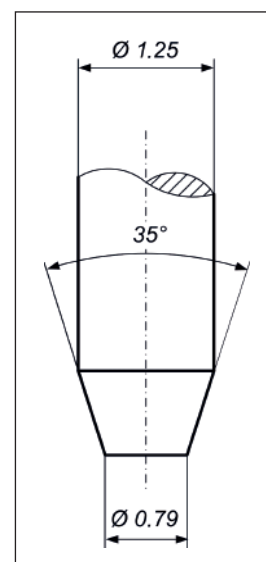


Fig. 1. Indenter for the Shore A hardness test in acc. with DIN EN ISO 868

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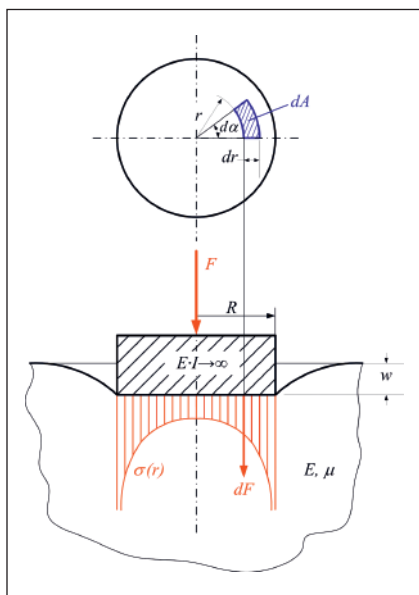


Fig. 2. Elastic half space under load from a rigid indenter

placement functions for this rotationally symmetrical case. Applying these laws to loading of the half space by a rigid indenter with a finite diameter (Fig. 2) yields for the relationship between the loading force F and the depth of penetration by the indenter the equation below [4]

$$w = \frac{F}{2} \cdot \frac{1-\mu^2}{E} \cdot \frac{1}{R} \quad [\text{mm}] \quad (3)$$

where the modulus of elasticity E and Poisson's ratio μ are the elasticity constants of the linearly elastic and isotropic half space, and R designates the radius of the indenter.

Upon inserting (1) and (2) into (3), one obtains a direct relationship between the modulus of elasticity and the Shore hardness in the form

$$E = \frac{1-\mu^2}{2 \cdot R \cdot C_3} \cdot \frac{C_1 + C_2 \cdot Sh_A}{100 - Sh_A} \quad [\text{N/mm}^2] \quad (4)$$

In this model, the rigid indenter represents the Shore indenter quite well, even though the latter has the shape of a truncated cone. The elastic half space, however, represents a very significant idealisation of the test specimen that not only ignores the effect of its finite dimensions, but also the possible deviations from linear deformation behaviour as well as the friction between the indenter and the test specimen. Nevertheless, the above relationship (4) can serve quite well as the basis for describing the actual situation during a Shore hardness test. The stated effects and imperfections can be determined via specific experiments and, when necessary, be taken into account through a suitable correction function.

Experimental Investigations

The experimental investigations aimed at assessing the usefulness of equation (4) as a potential solution and determining quantitatively the effects especially of the finite dimensions of the test specimen and friction. For the purpose of achieving broader support of the results, various commercially available types of cross-linked elastomers with values of the Shore A hardness between 30 and 95 were investigated (Table 1). All measurements were performed at room temperature. Intentionally not included were thermoplastic elastomers, their hardness and stiffness characteristics can lie outside this Shore A hardness range and thus require separate investigation.

Shore A hardness. The digi test 3105 instrument from Zwick/Roell was employed for the hardness measurements. Test durations were established as 30 s in accordance with [2] and 15 s in compliance with the standard [1], respectively. After this period of time, the measured values lay in a sufficiently stationary region. With a scatter of about 1 %, the reproducibility was very good. To clarify the effect of test specimen thickness on the relationship between the Shore A value and the modulus of elasticity, the speci-

men thickness was varied between 6 and 18 mm by stacking up to 3 test specimens. The results indicate that the specimen thickness is of secondary importance (Fig. 3). The possible effect of friction on the measured Shore A values was investigated via measurements with and without a film of low-viscosity oil between the indenter and test specimen. It was found that this effect is negligible, so that the Shore A values could be determined via the usual procedure without any special provisions.

Modulus of elasticity in compression.

To determine the moduli of elasticity in compression, a 250 kN tensile/compression test machine from Schenck/Trebel equipped with the measurement software testXpert from Zwick was employed. The elongation was measured in a non-contact manner by means of video extensometry from Zwick/Roell. A measurement set-up with a total of six layers (Fig. 4) permitted use of the same test specimens employed for the hardness measurements. The elongation was measured for the two innermost layers; the other two layers of the same material were coloured externally with a contrasting pigment in order to visually delineate the region of measurement. The two outer layers that serve to introduce the force were

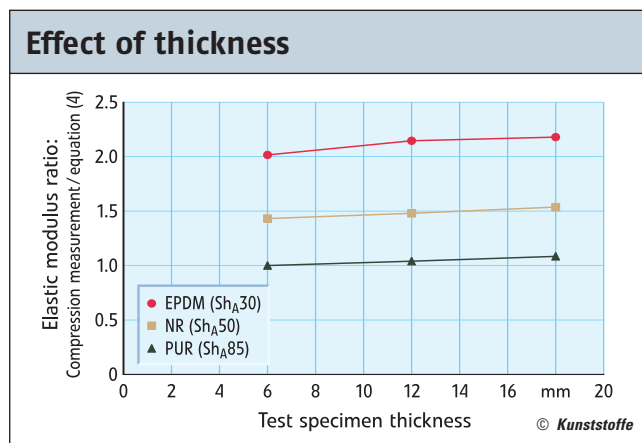


Fig. 3. Effect of test specimen thickness on the ratio of the modulus of elasticity as determined by measurement and as calculated from equation (4)

Material		Supplier	Shore A value	Specimen geometry [mm]	
Type	Designation			Diameter	Thickness
EPDM	3045-30	Maagtechnic	30	40	6
NR	1212-50	Maagtechnic	50	40	6
NBR	6214-70	Maagtechnic	70	40	6
NBR	6214-80	Maagtechnic	78	40	6
EPDM	3213-87	Maagtechnic	85	40	6
CR	5617-90	Maagtechnic	90	40	6
PUR	UK-AA/85ShA	Kundert	85	32	6
PUR	UK-AA/90ShA	Kundert	90	32	6
PUR	UK-AA/95ShA	Kundert	95	32	6

Table 1. Elastomeric materials investigated

chosen to be an elastomer with lower stiffness so as to ensure a uniaxial stress condition and not hinder the transverse contraction in the region of measurement.

With the objective of creating conditions as similar as possible to those for the Shore A hardness measurements, the measured values were likewise determined after testing times of 30 and 15 s at an elongation of 2 %. The selected parameters thus correspond approximately to the usual conditions when determining the modulus of elasticity in a short-time tensile test; the deformation behaviour is practically linear, and the compressive stiffness corresponds largely to the tensile stiffness.

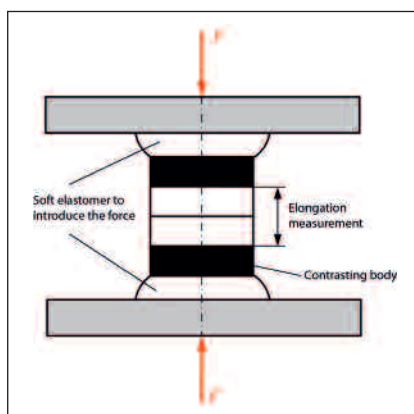
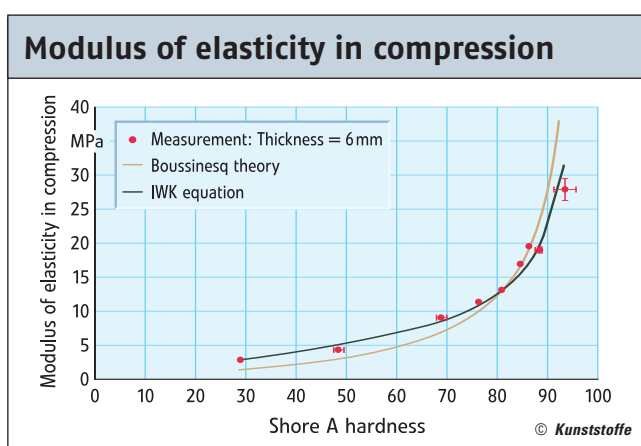


Fig. 4. Test set-up (schematic) for measuring the modulus of elasticity in compression by means of video extensometry

Fig. 5. Modulus of elasticity in compression as a function of the Shore A hardness value



Poisson's ratio. Measurement of the value of Poisson's ratio required in equation (4) was dispensed with, since the effect of any possible deviation from the value $\mu = 0.5$ for incompressible behaviour and characteristic of elastomers is minimal. For $\mu = 0.47$, it is less than 4 %.

Results and Discussion

Comparison of the modulus of elasticity in compression with the Shore A hardness (Fig. 5) shows first of all that the theory of Boussinesq, with equation (4) as the link between both quantities, is adequate. Secondly, the effects not accounted for with this theory can be incorporated via a correction function. By applying the method of least squares, one obtains the following relationship for calculating the modulus of elasticity in compression from the Shore A hardness:

$$E = \frac{1 - \mu^2}{2 \cdot R \cdot C_3} \cdot \frac{C_1 + C_2 \cdot Sh_A}{100 - Sh_A} \cdot (2.6 - 0.02 \cdot Sh_A) \quad [N/mm^2] \quad (5)$$

with the expression in parentheses as the correction function. The relationship

holds for crosslinked elastomeric materials with hardness values of 30 to 95 Shore A and a test specimen thickness of 6 mm. The standard deviation between the calculated value and the measured value is 5.6 % for the testing time of 30 s and 5.4 % for the testing time of 15 s, which can be considered very satisfactory. The constants have the following values: $R = 0.395$ mm, $C_1 = 0.549$ N, $C_2 = 0.07516$ N and $C_3 = 0.025$ mm. The effects of friction between the indenter and test specimen as well as the possible deviation of Poisson's ratio from the ideal value $\mu = 0.5$ are so minimal that they can be neglected without further consideration and thus do not appear in equation (5). The slight effect of test specimen thickness (Fig. 3) can, if necessary, be taken into account via functions that depend on test specimen thickness in place of the constants in the correction function.

Since moduli of elasticity are measured at only slight elongation, the modulus of elasticity in tension hardly differs from the modulus of elasticity in tension according to (5). This is the case even more so, as practically linear deformation be-

haviour was observed in the investigated elongation range of up to 2 %.

In view of the theoretically and experimentally observed clear relationship between the modulus of elasticity in compression and the Shore A hardness, modification of the Shore hardness test procedure is unnecessary. As a consequence, it is possible to convert Shore A hardness values in databases with satisfying accuracy directly into values of the modulus of elasticity and employ them for calculations involving tensile and/or compressive loads. ■

Dedicated to Prof. Dr.-Ing. Dr.-Ing. E.h. Walter Michaeli on the occasion of his 60th birthday.

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