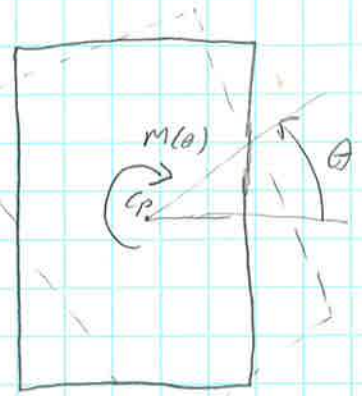
8/16/16 JAH

ESP HOUSING + FRAME PITCH MOTIONS

I^{cp} = mass moment of inertia $[kg\ m^2]$

a^{cp} = added mass $[kg\ m^2]$



$M(\theta)$

ANGULAR POSITION: $\theta(t) = Re\left[\xi e^{i\omega t}\right]$ ξ = COMPLEX PITCH AMPLITUDE

$\dot{\theta}(t) = Re\left[i\omega \xi e^{i\omega t}\right]$

$\ddot{\theta}(t) = Re\left[-\xi \omega^2 e^{i\omega t}\right]$

MOMENTS ACTING ON BODY

STABILITY: $M(\theta) = -\bar{BG}\, mg \sin\theta$

$\approx -\bar{BG}\, mg\, \theta$ $[N\cdot m]$

$= Re\left[-\bar{BG}\, mg\, \xi e^{i\omega t}\right]$

RADIATION

$M_R(t) = -a_{66}\ddot{\theta}(t) - b_{66}\dot{\theta}(t)$

$= Re\left[\xi e^{i\omega t}\left(a_{66}\omega^2 - i\omega b_{66}\right)\right]$

a_{66} = PITCH ADDED MASS

b_{66} = PITCH DAMPING.

EXCITING

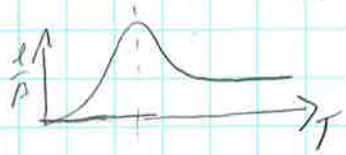
$F_6^E(t) = Re\left[A X_6 e^{i\omega t}\right]$ X_6 = COMPLEX PITCH EXCITING FORCE.

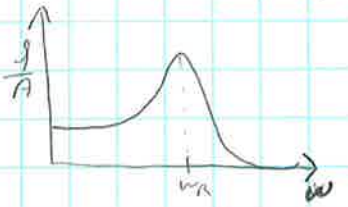
$I\ddot{\theta} = \Sigma M$

$Re\left[-I\xi\omega^2 e^{i\omega t}\right] = Re\left[-\bar{BG}\, mg\, \xi e^{i\omega t} + \xi a_{66}\omega^2 e^{i\omega t} - i\omega b_{66}\xi e^{i\omega t} + A X_6 e^{i\omega t}\right]$

$\xi\left[-(I+a_{66})\omega^2 + \bar{BG}\, mg + i\omega b_{66}\right] = A X_6$

$$\frac{\xi}{A} = \frac{X_6}{-\omega^2(I+a_{66}) + i\omega b_{66} + \bar{BG}\, mg}$$





RESONANCE: $-\omega^2(I+a_{66}) + i\omega b_{66} + \bar{BG}\, mg = 0$

$$\omega = \sqrt{\frac{\bar{BG}\, mg}{(I+a_{66})}}$$

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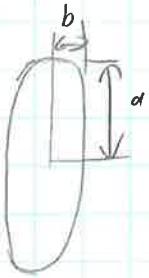
PITCH ADDED MASS (INERTIA)

ESTIMATE: MODEL ESP+ FRAME AS A PROLATE SPHEROID OF SIMILAR LENGTH AND DIAMETER TO DETERMINE APPROXIMATE ADDED-MASS COEFF, THEN APPLY THAT COEFF TO THE PITCH MOMENT OF INERTIA OF THE DISPLACED WATER.



PROLATE SPHEROID $a = .8\,m$, $b = .3\,m$

$b/a = .375$

$$m_{66}' \approx 0.4 = \frac{m_{66}}{\frac{4}{15}\pi\rho ab^2(a^2+b^2)}$$ } NEWMAN PP 147

$m_{66} = 18\ kg\,m^2$ } → FOR ACTUAL PROLATE SPHEROID OF $a = 0.8m$ AND $b = 0.3m$

SINCE THE MOMENT OF INERTIA OF THE WATER DISPLACED BY THE ESP+ FRAME IS DIFFERENT, APPLY m_{66}' TO MOMENT OF INERTIA OF THE WATER DISPLACED.

$I_{66}^{H_2O} = 69\ kg\,m^2$ } AT CENTER OF DISPLACEMENT, WHICH IS CLOSE TO THE PIVOT POINT SO THIS IS OK AS AN ESTIMATE

$m_{66}^{ESP+FRAME} = 0.4\,(69\ kg\cdot m^2) = \underline{28\ kg\cdot m^2}$ ← ESTIMATE OF PITCH ADDED MOMENT OF INERTIA

ESTIMATED PITCH RESONANT FREQUENCY

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$$\omega_R = \sqrt{\frac{\overline{BG}\, m\, g}{(I + m_{66})}}$$

$\overline{BG} = 26.3'' - 15.7''$

$= 10.6''$

$= 0.27m$

$$\omega_R = \sqrt{\frac{0.27m \cdot 362kg \cdot 9.81 m/s^2}{(102.8\ kgm^2 + 28 kgm^2)}} = 2.7\ \frac{rad}{s}$$

$= .4\ Hz$

$T_R = 2.5s$ } RESONANT PERIOD ESTIMATE